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# The Impact of Information Sharing on Supply Chain Performance

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ABSTRACT

# The Impact of Information Sharing on Supply Chain Performance

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This thesis is motivated by the impact that information technology has had on supply chain management. In particular, information technology has changed the way companies interact with suppliers and customers. For example, in **quick response**, suppliers receive Point-of-Sales (POS) data from retailers and use this information to improve their forecast and better manage production and inventory activities.

Our objective is to study the value of information sharing and how to effectively utilize demand related information in supply chains. For this purpose, we develop and analyze two models, the first one focuses on inventory cost reduction in a two-stage supply chain where the manufacturer has a limited production capacity. The second model characterizes the forecast accuracy improvement in a multi-stage supply chain facing stationary and correlated demand.

The thesis starts by analyzing a periodic review, two-stage production-inventory system with a single capacitated manufacturer and a single retailer facing stochastic demand. The manufacturer receives demand information from the retailer even during time periods in which the retailer does not place orders. Assuming a finite time horizon, we characterize the optimal production-inventory policy for the manufacturer, explore the policy structure, and study the optimal frequency and timing in which information should be shared.

We then analyze a similar model in infinite time horizon. First, we provide a new

and simple proof for the optimality of the cyclic order-up-to policy under average cost criterion. Then, using Infinitesimal Perturbation Analysis (IPA) we quantify the impact of information sharing, as well as the impact of the frequency and timing of information sharing on the manufacturer's performance.

In the last part of the thesis, we consider a distribution system with a single manufacturer, a single distribution center and multiple non-identical retailers in infinite time horizon. The retailers place orders periodically, the distribution center transfers the aggregated orders from the retailers to the manufacturer. Assuming stationary and correlated external demands, we quantify the impact of sharing the order and demand information of individual retailers on the manufacturer's forecast accuracy.

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# Chapter 1

# Introduction

## 1.1 Background and motivation

Information technology is an important enabler of efficient supply chain strategies. Indeed, much of the current interest in supply chain management is motivated by the possibilities introduced by the abundance of data and the savings inherent in sophisticated analysis of these data. For example, information technology has changed the way companies interact with suppliers and customers. Strategic partnering, which relies heavily on information sharing, is becoming ubiquitous in many industries.

As observed by Stein and Sweat (1998), sharing demand related information vertically among supply chain members has achieved huge impact in practice. According to Stein and Sweat, by "exchanging information, such as Point of Sales (POS), forecasting data, inventory level and sales trends, these companies are reducing their cycle times, fulfilling orders more quickly, cutting out millions of dollars in excess inventory, and improving customer service."

To understand the impact of information sharing, consider traditional supply chain strategies. Supply chains are highly complex systems with multiple production and storage facilities. A typical supply chain consists of raw material suppliers, assembly manufacturers, distributors and retailers. It is often managed in a decentralized manner, i.e., each stage is managed based on information received from its immediate suppliers and customers (decentralized information) and the objective of the stage is to maximize profit with no, or very little regards, to its impact on other stages in the supply chain (decentralized control). Thus, each stage makes locally optimal decisions based on the orders placed by its customers, and the replenishment lead time provided by its suppliers.

Such a decentralized information and control system faces significant challenges. For example, ordering information flow may be distorted in the sense that the variation of orders tends to increase as one moves up the supply chain, a phenomenon known as Bullwhip effect. The Bullwhip effect was first observed in practice by companies such as Procter & Gamble and Hewlett-Packard, and later quantified by Lee, Padmanabhan and Whang (1997a, b), and Chen, Drezner, Ryan and Simchi-Levi (2000). Lee et al. identified the sources of Bullwhip effect to be: promotional activities, inflated orders, order batching and price variation. Chen et al. (2000) show that traditional forecasting methods such as moving average and exponential smoothing also contribute to the increase in variability, that is, they also play an important role in the bullwhip effect. They also show that transferring demand information across supply chain partners can significantly reduce the Bullwhip Effect but it will never eliminate it.

The impact of the Bullwhip effect can be very significant. Indeed, the increase in order variability implies that the firm needs to increase safety stock levels, or otherwise service levels decrease. In addition, it is difficult to manage resources, e.g., labor, equipment and transportation, effectively. More importantly, companies are slow to respond to market changes because of the distortion in market signals.

The question, of course, is how to match demand and supply with minimal inventory? In particular, the challenge is to do that in supply chains with long production and transportation lead-times, and short product lifetime. To address these challenges, a number of trends have emerged in supply chain strategies, all of which take advantage of the abundance of information available in today's supply chains:

**Quick Response:** In this strategy, retailers share with the suppliers Point-Of-Sales (POS), inventory levels and forecast data, as well as information on promotional events. With the visibility of current demand and inventory levels, suppliers can better forecast and schedule their production-inventory activities, and provide better service to their customers. Indeed, information sharing can reduce the demand uncertainty to such an extent that suppliers can build inventory well in advance of receiving a promotional order (Fahrenwaid, Wise and Glynn, 2001). Of course, the ability of suppliers to prepare in advance of an incoming order implies that they can reduce lead-times to the retailers. This, together with an improved fill rate, allows retailers to reduce inventory levels and the Bullwhip Effect, see Chen et al. (2000). For example, Milliken and Company, a U.S. based textile and chemicals manufacturer, asked its retail partners not only to provide the manufacturer, Milliken and Company, with demand information, but also to provide the same information to its suppliers, so that Milliken and Company can synchronize its production schedule with its suppliers. This allowed Milliken and Company to reduce replenishment lead-time to its retailers from 18 weeks to 3 weeks (Simchi-Levi, Kaminsky and Simchi-Levi, 1999).

**Collaborative Planning, Forecasting and Replenishment (CPFR):** Many companies not only share information with their supply chain partners, but also jointly make decisions to improve supply chain performance. Specifically, in CPFR, companies share information and also collaborate on forecasts, promotional activities and production strategies. One of the most cited examples is that of Wegmans grocery chain and Nabisco. The two companies use CPFR on 22 items. Nabisco sales force developed a forecast for these items, which was compared with Wegmans' own forecasts. The pilot was successful: Nabisco sales grew by 31%, while Wegmans sales increased by 16% with a surprising 18% decrease in inventory.

Henkel, the worldwide manufacturer of adhesives, consumer brand name products and industrial specialties, collaborates with its customer Eroski, Spain's largest food distribution group, by combining their complementary knowledge of the market. In particular, Eroski brought to the partnership its understanding of sales dynamics and promotions, while Henkel provided an expertise on its products. Thus, Eroski focused more on the impact of promotion on sales, while Henkel focused more on synchronizing demand planning with its production planning. By integrating information of promotion, new product introduction and local activities into one forecast, these companies increased forecast accuracy, improved customer service and reduced inventory level (Fahrenwaid, Wise and Glynn, 2001).

**Vendor Managed Inventory (VMI):** A different type of collaboration between retailers and suppliers based on information sharing is VMI. In this strategy, the supplier determines not only her production schedule, but also the shipments and inventory policies at retail facilities. Thus, VMI is a centralized control strategy, in which the objective is to optimize decisions for the entire supply chain. We refer readers to Simchi-Levi, Kaminsky and Simchi-Levi (1999) for more details.

All these trends have one thing in common: they require retailers to transfer demand information to their suppliers, and sometimes even to their suppliers' suppliers. However, sharing information also posses significant challenges. As reviewed in Lee and Whang (1998), the challenges include: incentive issues, confidentiality of the information shared, anti-trust regulations, reliability and cost of information technology, the timeliness and accuracy of the shared information, and finally the development of capabilities that allow companies to utilize the shared information in an effective way.

In this thesis we focus on **Quick Response**. Our objective is to quantify the benefits of information sharing and identify strategies that allow companies to *utilize information in an effective way.* Evidently, an important related challenge, also addressed in this thesis, is associated with the frequency and timing of information sharing.

## 1.2 Objectives and contributions

The second and third chapters of this thesis are motivated by the Milliken and Company example described earlier. As observed, Milliken and Company reduced replenishment lead-time to the department stores from 18 weeks to 3 weeks by implementing Quick Response. Although the underlying intuition is clear, i.e., demand information can help suppliers better prepare for incoming orders, issues such as when information sharing provides significant cost savings and how manufacturers can use this information most effectively in a make-to stock production system, are not well understood.

To address these issues, we considered a two-stage supply chain with a single manufacturer having a finite production capacity and a single retailer facing independent demand. The manufacturer produces to stock and the retailer manages her inventory using an order-up-to inventory policy. Because many companies place orders periodically while they can share information with their partners continuously, we assume that the manufacturer can receive demand information from the retailer even during time periods in which the retailer doesn't place orders.

Specifically, we assume that the retailer has a fixed ordering interval. That is, every $T$ time periods, e.g., four weeks, the retailer places an order to raise his inventory position to a certain level. The manufacturer receives demand information from the retailer every $\tau$ units of time, $\tau \leq T$. For instance, the retailer places an order every four weeks but provides demand information every week. This is clearly the case in many retailer-manufacturer partnerships in which orders are placed by the retailer at certain points in time but POS data is provided every day or every week. In all these cases, POS data is provided to the manufacturer more frequently than retailer orders. We refer to the time between successive orders as the **ordering period** and the time between successive information sharing as the **information period**. Of course, in most supply chains, information can be shared almost continuously, e.g., every second, while decisions are made less frequently, e.g., every week. Thus, information periods really refers to the time interval between successive use of the information provided.

Our objective is to characterize the benefit of information sharing for the manufacturer, as well as to understand what can be done to make information sharing most beneficial, e.g. how frequently should information be shared and when should it be shared so that the **manufacturer** can realize the potential benefits.

For this purpose, we analyze the model in a finite time horizon and in an infinite time horizon setting. Throughout the thesis, we compare the following two strategies. In the first strategy, referred to as **no information sharing**, the retailer does not share information with the manufacturer except for orders. In the second strategy, referred to as **information sharing with optimal policy**, the retailer shares demand information with the manufacturer at the end of each information period. We assume that the manufacturer knows the external demand distribution for each information period, and uses an **optimal** strategy to schedule production so as to minimize his own expected holding and shortage cost. For the finite horizon model, we also considered a third strategy, referred to as **information sharing with greedy policy**. In this strategy, the retailer shares demand information with the manufacturer just as in the previous strategy, but instead of the optimal policy, the manufacturer uses an heuristic that is easy to implement in practice, based on demand and shortage in the previous information period, as well as his production capacity.

This part of the thesis makes the following contributions:

- For the finite horizon case we characterize the structure of the optimal production inventory policy.

- For the infinite horizon case we characterize the optimal production-inventory policy under both discounted and average cost criteria. In particular, we provide a new and simple proof for the optimality of the cyclic order-up-to policy under average cost criterion.

- Using a computational study, we report on the impact of information sharing on the manufacturer's cost as a function of the production capacity and the frequency and timing in which demand information is transferred from the retailer to the manufacturer.

The fourth chapter of this thesis considers a simple distribution system with a single manufacturer, a single cross-docking distribution center (DC) and multiple retailers. In such a system, the retailers place orders to the DC, the DC aggregates the orders from various retailers and transfers the orders to the manufacturer. This chapter is motivated by our observation that many manufacturing companies generate forecast only based on the history of aggregated orders received from DCs. That is, typically manufacturers do not utilize demand and order information of individual retail stores when generating forecasts. This is the case either because the information is not available to the manufacturer or because the benefits of these information is not clear even if they are readily available.

For example, we are aware of a cosmetic manufacturing company that generates forecast for its manufacturing facilities only based on the aggregated orders placed by distribution centers, which in turn serve some 2000 retail stores. Interestingly enough, in this case the demand and order information of individual retail stores is available to the forecaster but, for some reason, the manufacturer chooses to ignore the available information.

Assuming stationary and correlated external demands, we analyze the following two cases: In the first case, which we refer to as *no information sharing*, the manufacturer only receives the aggregated orders from the distribution center. In the second case, which we refer to as *information sharing*, the manufacturer not only receives the aggregated orders, but also the order and demand information of individual retailers. Our objective is to quantify the impact of information sharing on the manufacturer's forecast accuracy.

This part of the thesis makes the following contributions:

- We analyze the impact of information sharing on the manufacturer's forecast accuracy, and compare it to the forecast accuracy without information sharing.
- Using a computational study, we report on the impact of information sharing on the manufacturer's forecast accuracy as a function of the number of retailers, transportation lead-time and the number of historical data included in determining the forecast.

## 1.3 Literature review

In this section we review the literature on models involving information sharing and its impact on supply chain partners. We focus on

- **Batch Ordering Models**
- **Advance Demand Information Strategies**

- **Correlated Demand Models**
- **Models that involve Promotional Activities**

### 1.3.1 Batch ordering

Chen and Zheng (1997) compare the optimal installation stock and echelon stock policies for a simple two-stage serial system with batch ordering. In an installation stock policy, the inventory policy of each facility is determined so as to minimize the expected supply chain cost, thus the supply chain is under centralized control. But each stage is managed based on the corresponding policy using **only** local information available to this stage (local information). In an echelon inventory policy, while the objective remains the same, each stage is managed using information from all of its downstream facilities (centralized information). Chen and Zheng observe that the value of centralized stock information is insignificant for their test examples. Similarly, Chen (1998) studies multi-stage serial production-inventory systems, and compares the optimal echelon inventory policy (centralized information) with the optimal installation inventory policy (local information). Assuming i.i.d. external demand, Chen shows that the benefit of centralized information is 0-9% with a average of 1.75%, and the benefit increases as lead-times and batch sizes increase.

Cachon and Fisher (2000) study the impact of centralized information on a periodic review single supplier and multi-retailer system under centralized control. They compared the following three strategies: first, no demand and inventory information is shared between the retailers and the supplier, and the supplier uses first come first serve (FCFS) principle to satisfy retailers' orders. In the second strategy, each time retailers place orders, they also transfer inventory information to the supplier so that the supplier can choose an effective inventory allocation scheme. The third strategy allows the retailers to transfer inventory information to the supplier each time period, independent of whether an order is made. This information allows the supplier to better manage its own inventory as well as more effectively allocate inventory among the different retailers. Using a computational study, they report that the gap between the first strategy and the third strategy is 2.2% on average, and can be as high as 12%. Cachon and Fisher concludes that the benefits of information sharing is small, while the benefits of automating transactions maybe much larger since it helps reducing the lead-times and batch sizes.

Gallego, et al. (2001) consider a decentralized controlled system with one supplier and one retailer. In a decentralized control system each party optimizes decisions by looking at its own costs. The supplier, however, is charged with a penalty costs proportional to its backlogged level. The supplier uses a base-stock policy while the retailer uses a $(Q, r)$ policy. All policies are continuously reviewed. With continuous information sharing, the supplier knows exactly the retailer's inventory position at any time. She can reduce her cost by delaying her orders until the retailer's inventory position drop to a certain level. Thus, the supplier can obtain substantial benefits from information sharing, while the retailer maybe slightly better off, or even worse off due to the delayed supplier lead-time.

Another paper focusing on decentralized controlled distribution system is by Aviv and Federgruen (1998). They analyze a single supplier multi-retailer system where retailers face random demand and share inventories and sales data with the supplier. Since it is quite difficult to find the optimal policy for the entire system, these authors use heuristics. Specifically, they analyze the effectiveness of a VMI program where sales and inventory data is used by the supplier to determine the timing and the amount of shipments to the retailers. For this purpose, they compare the performance of the VMI program with that of a traditional, decentralized system, as well as a supply chain in which information is shared continuously, but decisions are made individually, i.e., by the different parties. Their focus in the three systems analyzed is on minimizing long-run average cost. Aviv and Federgruen report that information sharing reduces system wide cost by 0% to 5% while VMI reduces cost, relative to information sharing, by 0.4% to 9.5% and on average by 4.7%. They also show that information sharing provides substantial benefits for the supplier, but almost zero benefits for the retailers.

So far, all the papers either report limited benefits of information sharing for the entire supply chain under centralized control, or substantial benefits but only for the supplier under decentralized control. Cheung and Lee (2002) shows that it is possible to design policies to take advantage of the shared information and bring substantial benefits for both the supplier and the retailers in a decentralized distribution system. Specifically, they show how information sharing can help the supplier better coordinate shipments and rebalance the inventory level among a number of retailers in a single supplier/multi-retailer system. They assume that each partner uses continuous review $(Q, r)$ policies, and the supplier and retailers are located in a close proximity so that a single truck can serve all retailers in one trip. By receiving demand and inventory information from the retailers, the supplier can send out a shipment whenever the accumulative demand from all retailers reaches the truck load. Their computational results show that utilizing the shared information in this way can reduce both the retailers' and the supplier's inventory holding costs without increasing transportation cost.

The papers described above consider the impact of information sharing on the entire system. Gavirneni, Kapuscinski and Tayur (1999) focuses exclusively on the party receiving the information. They analyze a simple two-stage supply chain with a single capacitated supplier and a single retailer. In this periodic review model, the retailer makes ordering decisions every period, using an $(s, S)$ inventory policy, and transfers demand information to the manufacturer every period, independent of whether an order is made. Assuming the retailer can acquire from another supplier any part of the order that manufacturer can not satisfy, they show that the benefit, i.e., the supplier cost savings, due to information sharing, increases as production capacity increases and it ranges from 1% to 35%.

### 1.3.2 Advance demand information strategies

Advance demand information strategies have been used in practice and analyzed in academia for quite some time. Hariharan and Zipkin (1995) studied a single-stage inventory system in which customers place orders well ahead of their due date. Hariharan and Zipkin refer to the time between the placement of the order and the due date as the *customer lead-time*. They show that the customer lead-time has the exact opposite effect as that of replenishment lead-time. That is, the effect of longer customer lead time is identical to the impact of shorter replenishment lead time.

Gallego and Ozer (2001) extend this idea to allow customers to place partial orders ahead of the due date. The customers can modify but cannot cancel the orders as the due date draw closer. The problem is then modeled as a Markov Decision Process (MDP) with state space of multiple dimensions. Assuming unlimited production capacity, they find that a state dependent $(s, S)$ or a base-stock policy are optimal depending on whether the model incorporates setup cost. They quantify the benefits of advance demand information through a numerical study and find that system performance can be improved as customers place orders further into the future. This is, of course, quite intuitive: since future demand is partially known, demand uncertainty is significantly reduced.

Ozer and Wei (2001) extend the results above to include finite production capacity. They report that a state dependent modified base-stock policy is optimal under discounted cost criterion if there is no setup cost. If the setup cost is non-zero, a heuristic policy, first introduced by Gallego and Wolf (2000), is utilized: either produce up-to full capacity or nothing. Using a computational study, Ozer and Wei report the benefits of advance demand information and observe that the benefit increases as capacity utilization increases.

Chen (2001) integrates advance demand information with price discounts by assuming that customers are willing to accept longer delays if the supplier offers price discounts in return. He discusses how such price discounts should be given, how firms should make use of the advance demand information to manage inventory, and what are the advantages and disadvantages of this strategy, both from revenue and inventory cost points of views.

### 1.3.3 Correlated demand processes

All the models reviewed so far assume independent demand processes. A number of recent papers quantify the benefits of information sharing when the external demand is correlated in time.

Lee, So and Tang (2000) consider a two-stage supply chain with a single manufacturer and a single retailer. In their model, external demand follows an $AR(1)$ process, and the supply chain members use periodic review base-stock policies. Observe, that since the demand process is not i.i.d., the retailer's base-stock levels may vary from period to period. They considered a traditional model without information sharing as well as a model in which the retailer transfers demand, inventory policy and forecast data to the manufacturer each time she (the retailer) places an order. They make two important assumptions: (i) the retailer can return excessive inventory to the manufacturer without charge if her inventory position is higher than the target base-stock level, and (ii) the manufacturer is not able to utilize the order history to calculate the actual demand. Under these assumptions, they report that information sharing can help the manufacturer reduce inventory cost substantially. The percentage cost saving increases as demand correlation or transportation lead-time between the supplier and the retailer increases.

Raghunathan (2001) points out the weakness of the assumption that the supplier is not able to utilize the order history to calculate actual demand. He argues that information sharing has zero benefit if the supplier is intelligent enough so that it can retrieve all the demand information from the order history given that it knows the retailer's inventory control policy. Thus, he concludes that only sharing information of unexpected events, e.g. promotion, is beneficial.

Finally, Aviv (2002) explores the value of sharing market signals in a two-stage distribution system, where a single supplier serves multiple retailers. Market signals are defined to be the portion of demand uncertainty observable in advance, e.g., the impact of weather or promotion on demand. Furthermore, the demand process can be characterized by a linear regressional form that depends on early market signals. He discusses and compares the following three supply chain configurations: In the first setting, the supplier and retailers coordinate their policy parameters in order to minimize system-wide costs, but they do not share their observed market signals. In the second setting, the supplier uses VMI without receiving the market signals observed by the retailers. In the third setting, all demand related information are shared between the supplier and the retailers in addition to the VMI strategy. Assuming an $AR(1)$ external demand process, Aviv demonstrates through numerical examples that sharing market signals will likely be more beneficial for the entire supply chain as demand correlation increases, and as companies are able to explain larger portion of demand uncertainty.

### 1.3.4 Other sources

So far customer demand was modeled as exogenously determined stochastic process. Iyer and Ye (2000) study the impact of sharing promotion related information, e.g., the timing of retail promotion, on a decentralized controlled supply chain with a single retailer and a single manufacturer. In their model, the retailer can choose a pricing scheme to maximize her expected profits and customer demand is assumed to be sensitive to price. The manufacturer uses all information available to generate an inventory policy that maximizes her expected profits subject to the service-level requirement. Iyer and Ye observe that (1) if the retailer does not share promotion related information with the manufacturer, the increased fluctuation in demand will decrease the manufacturer's profits; (2) if the retailer shares promotion related information with the manufacturer, then both the retailer and the manufacturer can benefit from promotions.

Finally, it is appropriate to observe that all the literature reviewed so far focuses on information sharing from the downstream facilities to an upstream facility. Recently, Yu and Chen (2001) study the impact of sharing supply related information, e.g., production schedule, from an upstream to a downstream facility. The basic idea is to use information as a tool to reduce supply uncertainty, and hence safety stock at the downstream facility. Specifically, Yu and Chen study a single supplier and single retailer system, in which the supplier makes her production schedule visible to the retailer so that the retailer can better estimate the lead time for each order. They compare this system with a traditional system without information sharing and show that the retailer can achieve substantial cost savings from information sharing.

### 1.3.5 Summary

To summarize, research on the benefits of information sharing and supply chain collaboration is still in its early stage. Significant achievements have been made, while many issues are still not well understood. For instance, what is the impact of information sharing on a supplier with limited production capacity? How can suppliers use the shared information most effectively in a quick response type of partnership? In particular, how often should information be shared? If information cannot be used continuously, when should it be shared? More importantly, how can a capacitated supplier take advantage of the information shared even at times when orders are not placed? And finally, how can suppliers improve their forecast accuracy by using the demand and order information of individual retailers in a multi-retailer distribution system? Some of these questions are answered in this thesis.

## 1.4 Structure of the thesis

The thesis is organized as follows: In Chapter 2, we study a single product, periodic review, two-stage production-inventory system with a single manufacturer and a single retailer in a finite time horizon. The manufacturer has finite production capacity and the retailer faces independent demand. Assuming that the manufacturer can receive demand information from the retailer even during time periods in which the retailer does not place orders, we characterize the optimal production-inventory policy for the manufacturer, and quantify the impact of information sharing on the manufacturer's cost and service level.

In Chapter 3, we analyze a similar model in an infinite time horizon. We provide a new and simple proof for the optimality of the cyclic order-up-to policy under average cost criterion. Using extensive computational analysis, we quantify the impact of information sharing as well as the impact of the frequency and timing of information sharing on the manufacturer's performance.

Finally, in Chapter 4, we consider a single product distribution system with a single manufacturer, a single distribution center and multiple retailers in infinite time horizon. The retailers place orders periodically and use an order-up-to policy to control their inventory. The distribution center serves as a cross docking point and transfers the aggregated orders from the retailers to the manufacturer. Assuming stationary and correlated external demands, we compare a supply chain with information sharing to a supply chain without information sharing. When there is no information sharing the manufacturer's forecast is based on the historical data of aggregated orders received from the DC. In a system with information sharing, the manufacturer's forecast is based on the historical data of external customer's demand as well as orders from each retailer. Using an analytic and a computational study, we quantify the impact of information sharing on the manufacturer's forecast accuracy.

The results described in this thesis appear in Simchi-Levi and Zhao (2001a), Simchi-Levi and Zhao (2001b), and Simchi-Levi and Zhao (2002).

# Chapter 2

# A capacitated two-stage supply chain: finite time horizon

In this chapter, we consider the simple two-stage supply chain in a finite time horizon. Our objective is not only to characterize the benefit of information sharing, but also to understand what can be done to make information sharing most beneficial, e.g. how frequently should information be shared and when should it be shared so that the **manufacturer** can realize the potential benefits.

This chapter is organized as follows: In Section 2.1, we set up the models for the three strategies described in Chapter 1. In addition, we identify the policies used by the manufacturer, discuss their properties and show the value of information sharing. In Section 2.2, the optimal timing of information sharing is discussed. In Section 2.3, we compare the performance of the three strategies using a numerical study. Section 2.4 concludes the chapter.

## 2.1 Models

We consider a single product, periodical review, two-stage system with a single retailer and a single manufacturer. External demand faced by the retailer every information (ordering) period is an i.i.d. random variable. To simplify the analysis, we assume that the retailer controls her inventory position (outstanding order plus on-hand inventory minus backorder) by an order-up-to policy with constant order-up-to level, i.e., in every ordering period, the retailer raises her inventory position to a constant level. All unsatisfied demand at the retailer is backlogged, thus the retailer transfers external demand of each ordering period to the manufacturer. The manufacturer has a production capacity limit, i.e., a limit on the amount the manufacturer can produce per unit of time. The manufacturer runs her production line always at the full capacity limit. Our objective is to compare the performance of the three strategies (see Chapter 1) in a finite time horizon.

The sequence of events in our model is as follows. At the beginning of an ordering period the retailer reviews her inventory and places an order to raise the inventory position to the target inventory level. The manufacturer receives the order from the retailer, fills the order as much as she can from stock, then makes a production decision. If the manufacturer cannot satisfy all of a retailer's order from stock, then the missing amount is backlogged. The backorder will not be delivered to the retailer until the beginning of the next ordering period. Notice that changing the manufacturer's policy may affect the manufacturer's service level, thus the retailer's performance. It will be interesting to study the impact of information sharing on the entire system with both the manufacturer and the retailer. This problem may involve incentive issues and coordination between the manufacturer and the retailer, which we would like address in future studies. In this thesis, we choose to focus exclusively on the manufacturer and assume that the retailer can adjust her order-up-to level to meet her objectives. Finally, transportation lead-time between the manufacturer and the retailer is assumed to be zero. Similarly, at the beginning of an information period, the retailer transfers the POS data of the previous information period to the manufacturer. Upon receiving this demand information, the manufacturer reduces this demand from her inventory position although she still holds the stock, then makes a production decision.

Throughout this chapter, we equally divide each ordering period into an integer number of information periods unless otherwise mentioned. Thus, $N = T/\tau$ is an integer and it represents the number of information periods in one ordering period. We index information periods within one ordering period $1, 2, \ldots, N$ where $N$ is the first information period in the ordering period and 1 is the last. Let $C$ denote the production capacity per information period, $\tau$, while $\overline{C}$ denotes the production capacity per ordering period, $T$. Hence, $\overline{C} = NC$. Finally, $c$ denotes the production cost per item.

Since we calculate inventory holding cost for each information period, we let $h$ be the inventory holding cost per unit product per information period. Let $0 < \beta \leq 1$ be the time discount factor for one information period, evidently, one unit of product kept in inventory for $n$ information periods, $n = N, N-1, \ldots, 1$, will incur a total inventory cost $h_n = h(1 + \beta + \cdots + \beta^{n-1})$. To keep the consistency of notation, let $h_0 = 0$. It's easy to see that the earlier the manufacturer makes a production run in one ordering period, the longer she will carry the inventory, thus the more holding cost she will have to pay. Penalty cost is charged at the end of each ordering period and thus, let $\pi$ be the penalty cost per backlogged item per ordering period. We use $D$ to denote the end user demand in one information period, $\tau$. $D$ is assumed to be i.i.d., with $f_D(\cdot)$ ($F_D(\cdot)$) being the pdf (cdf) function and $\mu$ being its mean.

Finally, $\sum D$ is the total end user demand in one ordering period, $T$.

### 2.1.1 No information sharing

Recall that in this strategy, the retailer does not share information with the manufacturer. Since the retailer uses a constant order-up-to policy and unsatisfied demands are fully backlogged, her order equals to the demand during the last $T$ periods. Thus, we assume that the manufacturer knows the external demand distribution for each ordering period.

Consider a finite horizon model with $M$ ordering periods and $N$ information periods in each ordering period. Ordering periods are indexed in a reverse order, that is, 0 is the index of the last ordering period in the planning horizon, while $M-1$ is the index of the first ordering period. The $i$th information period, $i = 1, 2, \ldots, N$, in ordering period $m$, $m = 0, 1, \ldots, M-1$ is referred to as the $mN+i$ information period.

Let $U'_{mN+i}(x)$ be the minimum expected inventory and production costs from period $mN+i$ until the end of the planning horizon, when we start period $mN+i$ with an inventory position $x$.

It is easy to verify that $W'_{mN+i}(x, y)$, the expected inventory and production cost in information period $mN+i$ given that the period starts with an inventory position $x$ and produces in that period $y-x$, only depends on $i$. So we use $W'_i$ to represent $W'_{mN+i}$ for the $i$th information period, and write it as follows.

$$W'_i(x, y) = \begin{cases} c(y-x) + h_{i-1}(y-x), & i = 2, ..., N \\ c(y-x) + E(L(y, \sum D)), & i = 1 \end{cases}$$

where $L(y, \sum D) = h_N(y - \sum D)^+ + \pi(\sum D - y)^+$, and $E(L(y, \sum D))$ is the expectation of $L(y, \sum D)$ with respect to $\sum D$. In the very first information period, i.e., in information period $NM$, a cost of $h_N x^+$ will be charged for the initial inventory position. In the very last information period, i.e., in information period 1, inventory holding cost for items left at the end of the planning horizon is charged at a level of $h_N$ per unit product.

Let the salvage cost $U_0'(\cdot) \equiv 0$. If the initial inventory position is zero, then,

$$U'_{mN+i}(x) = \begin{cases} \min_{x \le y \le x+C} \{W_i'(x,y) + \beta U'_{mN+i-1}(y)\}, & i = 2, ..., N, \forall m \\ \min_{x \le y \le x+C} \{W_i'(x,y) + \beta E(U'_{mN+i-1}(y - \sum D))\}, & i = 1, \forall m. \end{cases}$$

To find the optimal policy, for $m = 0, ..., M-1$, we rewrite the dynamic program as follows:

$$\begin{aligned} U'_{mN+i}(x) &= -(c + h_{i-1})x + V'_{mN+i}(x), \ \forall i \\ V'_{mN+i}(x) &= \min_{x \le y \le x+C} \{J'_{mN+i}(y)\}, \ \forall i \\ J'_{mN+i}(y) &= \begin{cases} cy + h_{i-1}y + \beta U'_{mN+i-1}(y), & i = 2, ..., N \\ cy + EL((y, \sum D)) + \beta E(U'_{mN+i-1}(y - \sum D)), & i = 1. \end{cases} \end{aligned}$$

We now discuss properties of the above dynamic program. A straightforward analysis of the finite planning horizon, see Federgruen and Zipkin (1986b), shows the following two results:

**Lemma 2.1** *The set $A \equiv \{(x,y) | x \le y \le x + C\}$ is convex. For all $m = 0, ...,$ $M - 1$ and $i = 1, ..., N$ we have:*

*(a) $E(L(y, \sum D)), J'_{mN+i}(y), V'_{mN+i}(x)$ and $U'_{mN+i}(x)$ are convex,*

*(b) $U'_{mN+i}(x) \to \infty$, when $|x| \to \infty$, and*

*(c) if $\beta^{N-1}\pi > c + h_{N-1}$, then $J'_{mN+i}(y) \to +\infty$ when $|y| \to +\infty$.*

See Section 5.1 for a proof.

**Lemma 2.2** *Let $y^*_{mN+i}$ be the smallest value minimizing $J'_{mN+i}$, and $x$ is the inventory position at the beginning of period $mN + i$. Then, $y^*_{mN+i}$ is finite and the* *optimal production-inventory policy is to produce*

$$\begin{cases} 0; & x \geq y^*_{mN+i} \\ y^*_{mN+i} - x; & 0 \leq y^*_{mN+i} - x \leq C \\ C; & otherwise. \end{cases}$$

A third, quite intuitive property, is that given two policies that produce the same amount in a given ordering period, a cost-effective policy will postpone production as much as possible during the ordering period. Of course, this property does not need any proof.

We use dynamic programming methods to solve for $y^*_m$ in single and multiple ordering period cases.

### 2.1.2 Information sharing with optimal policy

In this strategy, the retailer provides the manufacturer demand information every information period and the data is used by the manufacturer to optimize production and inventory costs. We consider the following two cases:

One ordering Period

We start by considering a single ordering period with $N$ information periods. We follow the convention that $N$ is the first information period and 1 is the last information period. Let $I_n$ be the manufacturer *on-hand inventory level* at the beginning of the $n$th information period; $D_n$ represents the demand during the $n$th information period. We use $x_n \equiv I_n - \sum_{i=n+1}^{N} D_i$. Thus, $x_n$ is *inventory position* at the beginning of the $n$th information period. Let $y_n$ be the inventory position at the end of $n^{th}$ information period after production in this period but not taking $D_n$ into account. That is, $y_n$ is equal to $x_n$ plus the amount produced in the $n$th time period.

Let $U_n(x_n)$ be the minimum total inventory and production costs from the beginning of $n$th information period until the end of the planning horizon, given an initial inventory position $x_n$. To simplify notation, we drop the index $n$ from $x_n$, $y_n$ and $D_n$; this will cause no confusion. Clearly,

$$\begin{array}{rcl} U_1(x) & = & \min_{x \le y \le x+C}\{c(y-x) + E(L(y,D))\} \\ U_n(x) & = & \min_{x \le y \le x+C}\{c(y-x) + h_{n-1}(y-x) + \beta E(U_{n-1}(y-D))\}, \\ & & n = 2, \cdots, N-1 \\ U_N(x) & = & \min_{x \le y \le x+C}\{c(y-x) + h_{N-1}(y-x) + \beta E(U_{N-1}(y-D))\} + h_N x^+ \end{array} \tag{1}$$

Unsold product at the end of the ordering period are charged at a rate of $h_N$ dollars per unit. As before, $L(y, D) = h_N(y - D)^+ + \pi(D - y)^+$ and $E(\cdot)$ is the expectation with respect to $D$, the demand in one information period. Observe that the holding cost for $y - x$ items produced in information period $n$ is $h_{n-1}(y - x)$, since these items are kept in inventory from the end of period $n$ until the end of period 1.

Rearranging the equations above, we get:

$$\begin{array}{rcl} U_1(x) & = & -cx + V_1(x) \\ V_1(x) & = & \min_{x \le y \le x+C}\{J_1(y)\} \\ J_1(y) & = & cy + E(L(y,D)) \\ & & \\ U_n(x) & = & -(c + h_{n-1})x + V_n(x) \\ V_n(x) & = & \min_{x \le y \le x+C}\{J_n(y)\} \\ J_n(y) & = & cy + h_{n-1}y + \beta E(U_{n-1}(y-D)) \\ n & = & 2, ..., N-1 \\ & & \\ U_N(x) & = & -(c + h_{N-1})x + h_N x^+ + V_N(x) \\ V_N(x) & = & \min_{x \le y \le x+C}\{J_N(y)\} \\ J_N(y) & = & cy + h_{N-1}y + \beta E(U_{N-1}(y-D)) \end{array} \tag{2}$$

<u>Multiple Ordering Periods</u>

Using the same notation as in the no information sharing model, it is easy to verify that $W_i$, the expected inventory and production cost in the information period

$mN+i$ given that the period starts with an inventory position $x$ and produces in that period $y-x$, can be written as follows.

$$W_i(x,y) = \phi_i(x) + \varphi_i(y), \tag{3}$$

where

$$\phi_i(x) = \begin{cases} -cx, & i=1 \\ -(c+h_{i-1})x, & otherwise, \end{cases}$$

$$\varphi_i(y) = \begin{cases} cy + E(L(y,D)), & i=1 \\ (c+h_{i-1})y, & otherwise, \end{cases}$$

and

$$L(y,D) = h_N(y-D)^+ + \pi(D-y)^+.$$

Thus, the following recursive relation must hold.

$$U_{mN+i}(x) \quad = \quad \min_{x\le y\le x+C}\{W_i(x,y) + \beta E(U_{mN+i-1}(y-D))\},$$

which can be written as,

$$\begin{array}{rcl} U_{mN+i}(x) & = & \phi_i(x) + V_{mN+i}(x) \\ V_{mN+i}(x) & = & \min_{x\le y\le x+C}\{J_{mN+i}(y)\} \\ J_{mN+i}(y) & = & \varphi_i(y) + \beta E(U_{mN+i-1}(y-D)). \end{array} \tag{4}$$

Of course, in the very first information period of the whole planning horizon, we have to add $h_N x^+$ to $U_{MN}(x)$ to account for the holding cost for initial inventory. This is identical to what we did in the no information sharing model.

Similar properties to Lemma 2.1 and Lemma 2.2 can be shown for this model. Specifically,

**Lemma 2.3** *The set $A \equiv \{(x,y)|x\le y\le x+C\}$ is convex. For all $m=0,...,M-1$ and $i=1,...,N$ we have:*

*(a) $E(L(y,D)), J_{mN+i}(y), V_{mN+i}(x)$ and $U_{mN+i}(x)$ are convex,*

*(b)* $U_{mN+i}(x) \to \infty$*, when* $|x| \to \infty$*, and,*

*(c) if* $\beta^{N-1}\pi > c + h_{N-1}$*, then* $J_{mN+i}(y) \to +\infty$ *when* $|y| \to +\infty$*.*

See Section 5.2 for a proof.

**Lemma 2.4** *Let* $y^*_{mN+i}$ *be the smallest value minimizing* $J_{mN+i}$*, and* $x$ *is the inventory position at the beginning of period* $mN + i$*. Then,* $y^*_{mN+i}$ *is finite and the optimal production-inventory policy is to produce*

$$\begin{cases} 0; & x \geq y^*_{mN+i} \\ y^*_{mN+i} - x; & 0 \leq y^*_{mN+i} - x \leq C \\ C; & otherwise. \end{cases}$$

The question is whether one can identify the relationship between the optimal order-up-to-levels of two consecutive information periods. Intuitively, delaying production until close to the end of the ordering period should allow to reduce inventory holding cost. The risk, of course, is that delaying too much may lead to a shortage, due to the limited production capacity. Thus, the next property characterizes sufficient conditions under which postponing production as much as possible is profitable.

**Proposition 2.1** *If* $Pr(D > C) = 0$*, then* $y^*_{mN+i} \leq y^*_{mN+i-1}$*, for* $i = 2, ..., N$*;* $m = 0, 1, \ldots, M-1$*.*

*Proof:* We prove the result for the last ordering period, i.e. $m = 0$. For $n = 2, ..., N$, rewrite equation (2) as following,

$$\begin{aligned} J_n(y_n) &= (1-\beta)cy_n + (h_{n-1} - \beta h_{n-2})y_n + \beta(c + h_{n-2})E(D) + \beta Q_n(y_n) \\ Q_n(y_n) &= E(V_{n-1}(y_n - D)) \\ &= E\{\min_{y_n - D \leq y_{n-1} \leq y_n - D + C}[J_{n-1}(y_{n-1})]\}. \end{aligned}$$

Let $\overline{y}_n$ be the smallest value minimizing $Q_n(y_n)$. Observe that $y^*_{n-1}$, the minimizer of $J_{n-1}(y)$, satisfies

$$Q_n(y^*_{n-1}) = E\{\min[J_{n-1}(y^*_{n-1})]\}.$$

This is true since $Pr(D > C) = 0$, which implies that $y^*_{n-1}$ is feasible in

$$y^*_{n-1} - D \leq y^*_{n-1} \leq y^*_{n-1} - D + C,$$

for all realization of $D$. Hence, $\overline{y}_n \leq y^*_{n-1}$. Furthermore, we notice that the difference between $J_n(y_n)$ and $Q_n(y_n)$ is a linearly increasing function, so the first-order right-hand derivative of $J_n$ is positive at $y^*_{n-1}$ (The first-order right-hand derivative exists for $J_n$ because $J_n$ is convex). Finally, since $J_n$ is convex, the result follows. The proof for all other ordering periods is identical. ∎

In practice, of course, the assumption that $Pr(D > C) = 0$ may not always hold and thus the question is whether one can identify other situations where we can characterize the relationship between $y^*_n$ and $y^*_{n-1}$.

Observe that if $Pr(D > C) > 0$, then $\overline{y}_n \geq y^*_{n-1}$, since $Q_n(y_n) \geq Q_n(y^*_{n-1})$ for $y_n < y^*_{n-1}$. Thus, a result similar to Proposition 2.1 can not be proven.

Since $J_n(y_n), n = 1, 2, ..., N$ is convex, it is continuous and right-hand differentiable. Hence, define $\Delta = \dfrac{d}{dy}$ to be the right-hand derivative. We have

$$\begin{aligned}
\Delta J_n(y_n) &= (1-\beta)c + (h_{n-1} - \beta h_{n-2}) + \Delta\beta Q_n(y_n) \\
&= (1-\beta)c + (h_{n-1} - \beta h_{n-2}) + \beta\textstyle\int_0^{(y_n - y^*_{n-1})^+} \Delta J_{n-1}(y_n - D) f_D(D) dD \\
&\quad +\beta\textstyle\int_{(y_n - y^*_{n-1})^+}^{\infty} \Delta J_{n-1}(y_n - D) f_D(D + C) dD,
\end{aligned}$$

Clearly, if $\Delta J_n(y^*_{n-1}) \geq 0$, then from the convexity and the limiting behavior of $J_n$, we have $y^*_n \leq y^*_{n-1}$. Thus, plug in $y^*_{n-1}$

$$\Delta J_n(y^*_{n-1}) = (1-\beta)c + (h_{n-1} - \beta h_{n-2}) + \beta \int_0^{\infty} \Delta J_{n-1}(y^*_{n-1} - D) f_D(D + C) dD,$$

where $\Delta J_{n-1}(y^*_{n-1} - D) \leq 0$ for $D \geq 0$. Since it's not clear whether $\Delta J_n(y^*_{n-1}) \geq 0$, we use numerical methods to evaluate $\Delta J_{n-1}$ in our computational study.

### 2.1.3 The value of information sharing

In this subsection we quantify the benefits from information sharing in a model with $M$ ordering periods and each of which has $N$ information periods. Our focus is on the extreme case in which production capacity is infinite so that the manufacturer only needs to produce in the last information period. Notice that the sequence of events in our model excludes a make to order policy when production capacity is infinite. That is, in our model, the manufacturer will satisfy the order only from her on-hand stock. If the manufacturer does not have enough stock on hand, she will pay penalty cost for backlogging the missing amount. We can regard this model as the limiting case as the production capacity approaches infinity.

First, consider the no information sharing strategy. The cost function for the last ordering period is

$$B_1'(x) = c(y - x) + L(y, \sum D_{_1}) = -cx + g(y, \sum D_{_1}),$$

where $x$ is the initial inventory position at the beginning of the ordering period, $y$ is the target inventory position, $\sum D_k$ is the total demand in the $k$th ordering period and

$$g(y, \sum D_{_1}) = cy + L(y, \sum D_{_1}).$$

Let $\alpha = \beta^N$ be the time discount factor for one ordering period. Since salvage cost is equal to zero, the total cost in $M$ ordering periods is

$$B_M'(x_M) = \sum_{m=0}^{M-1} \alpha^m [-cx_{M-m} + g(y_{M-m}, \sum D_{_{M-m}})],$$

given that the initial inventory position of the planning horizon is $x_M$. Since $x_m = y_{m+1} - \sum D_{m+1}$ for $m = M-1, \cdots, 1$, a straightforward calculation shows that

$$\begin{aligned} E(B'_M(x_M)) &= -cx_M + E[\textstyle\sum_{m=0}^{M-2}\alpha^m(g(y_{M-m}, \sum D_{M-m}) - c\alpha y_{M-m}) \\ &\quad +\alpha^{M-1}g(y_1, \textstyle\sum D_1) + \sum_{m=0}^{M-1}\alpha^m c \sum D_{M-m}], \end{aligned}$$

where $E(\cdot)$ is the expectation with respect to demand $\sum D_m$, $m = M, M-1, \cdots, 1$. Since our focus is on the trade-off between information and inventory, we ignore production cost in our model. Hence,

$$E(B'_M(x_M)) = E[\sum_{m=0}^{M-1}\alpha^m L(y_{M-m}, \sum D_{M-m})],$$

where $y_m \geq x_m$ for $m = M, M-1, \cdots, 1$.

To simplify the model, we assume that demand has independent and identical increments, and define $D_t$ to be the demand in any time period of length $t$, thus $D_T = \sum D$ and $D_\tau = D$. Let $G(y,t) = E(L(y, D_t))$. Following Heyman and Sobel (1984), it can be shown that if $Pr\{D_T \leq 0\} = 0$, a myopic policy is optimal. Further, let $y_T^*$ be the optimal order-up-to level for the myopic policy. If the initial inventory position $x_M \leq y_T^*$, then $U'_{MN}(x_M)$, the minimum expected inventory cost from information period $MN$ to the end of the planning horizon satisfies

$$U'_{MN}(x_M) = \frac{1-\alpha^M}{1-\alpha}G(y_T^*, T).$$

In order to obtain analytic result, we further assume that demand $D_t$ can be approximated by $N(t\mu, t\sigma^2)$. Notice that in this case $Pr\{D_t < 0\} > 0$. One way of avoiding this problem is to choose $\mu$ and $\sigma$ so that $Pr\{D_t < 0\} \leq \epsilon$, where $\epsilon > 0$ is a very small number.

Let $\Phi(\cdot)$ be the standard normal cumulative distribution function, and $\phi(\cdot)$ be the standard normal density function. Hence,

$$\begin{aligned} G(y,t) &= h_N \int_{-\infty}^{y}(y-\xi)f_{D_t}(\xi)d\xi + \pi\int_{y}^{\infty}(\xi-y)f_{D_t}(\xi)d\xi \\ &= \pi(t\mu - y) + (h_N+\pi)\int_{-\infty}^{y}(y-\xi)f_{D_t}(\xi)d\xi. \end{aligned}$$

We denote $\gamma = \frac{\pi}{\pi + h_N}$, and $z_\gamma = \Phi^{-1}(\gamma)$. From the analysis of the celebrated news vendor problem, we know $G(y,t)$ reaches its minimum at $y_t^* = t\mu + z_\gamma\sqrt{t}\sigma$. Let $\eta = \frac{\xi - t\mu}{\sqrt{t}\sigma}$, hence,

$$\begin{aligned} G(y_t^*,t) &= \pi t\mu - (h_N+\pi)\int_{-\infty}^{z_\gamma}(t\mu + \sqrt{t}\sigma\eta)\phi(\eta)d\eta \\ &= (h_N+\pi)\sqrt{t}\sigma\kappa \end{aligned}$$

where $\kappa = -\int_{-\infty}^{z_\gamma}\eta\phi(\eta)d\eta$.

Next, consider the information sharing strategy. The cost function for one ordering period is

$$c(y - x + D_{T-\tau}) + L(y, D_\tau),$$

where $x$ is defined in the same way as in the no information sharing strategy, $y$ is the target inventory position of the last information period by taking $D_{T-\tau}$ into account, $D_{T-\tau}$ is the realized demand in information periods $N, N-1, \ldots, 2$, and $D_\tau$ is the demand in the last information period. That is, $D_{T-\tau} + D_\tau$ is demand realized in this ordering period. For simplicity, let $D^{'} = D_{T-\tau}$ and $D = D_\tau$.

Assuming zero production cost and following the same procedure, we have

$$E(B_M(x_M)) = E[\sum_{m=0}^{M-1}\alpha^m L(y_{M-m}, D_{M-m})],$$

with $y_m \geq x_m - D^{'}_m$ for $m = M, M-1, \cdots, 1$. Thus, if the initial inventory position $x_M \leq y_\tau^*$, $U_{MN}(x_M) = \frac{1-\alpha^M}{1-\alpha}G(y_\tau^*, \tau)$.

These results lead to the following observations for the model with infinite production capacity:

- Information sharing has the same fill rate as no information sharing.

- The expected cost in the information sharing strategy is proportional to $\sqrt{\tau}$ while expected cost under no information sharing is proportional to $\sqrt{T} = \sqrt{N\tau}$, where $N$ is the number of information periods in one ordering period. Thus, the percentage cost saving due to information sharing (defined as the ratio between cost saving due to information sharing and the cost of no information sharing) is proportional to $1 - \sqrt{(1/N)}$,

- This implies that, if the initial inventory position is low so that we can reach optimal order-up-to level, then 4 information periods will reduce total cost by 50% relative to no information sharing.

### 2.1.4 Analysis of non-dimensional parameters

Our objective in the analysis of non-dimensional parameters is to identify the parameters which may have an impact on the percentage cost reduction due to information sharing. For simplicity, we focus on a single ordering period, but a similar method can be applied to the problem with any number of ordering period.

Dividing both sides of Equation (1) by $h_N N\mu$, we get

$$\frac{U_1(x)}{h_N N\mu} = \min_{\frac{x}{N\mu} \leq \frac{y}{N\mu} \leq \frac{x}{N\mu} + \frac{1}{N}\frac{C}{\mu}} \{\frac{c}{h_N}\frac{(y-x)}{N\mu} + \frac{1}{h_N N\mu}\int L(y,\xi) f_D(\xi) d\xi\}$$

Let $\eta = \frac{\xi}{N\mu}$, $D' = \frac{D}{N\mu}$, it is easy to see that $f_D(\xi) = \frac{d}{d\xi}F_D(\xi) = \frac{d}{d\xi}Pr(\frac{D}{N\mu} \leq \frac{\xi}{N\mu}) = \frac{d}{d\xi}F_{D'}(\frac{\xi}{N\mu}) = \frac{1}{N\mu}f_{D'}(\eta)$. Hence,

$$\frac{1}{h_N N\mu}\int L(y,\xi) f_D(\xi) d\xi = \int ((\frac{y}{N\mu} - \eta)^+ + \frac{\pi}{h_N}(\eta - \frac{y}{N\mu})^+) f_{D'}(\eta) d\eta.$$

Let $x^{'} = \frac{x}{N\mu}$, $y^{'} = \frac{y}{N\mu}$, $c^{'} = \frac{c}{h_N}$, $\rho = \frac{\mu}{C}$, $\pi^{'} = \frac{\pi}{h_N}$, $U_1^{'} = \frac{U_1}{h_N N\mu}$ and $L^{'}(y^{'}, \eta) = (y^{'} - \eta)^{+} + \pi^{'}(\eta - y^{'})^{+}$. We omit $'$ from the notation without creating any confusion, and we can rewrite the non-dimensionalized function $U_1$ as follows:

$$U_1(x) = \min_{x \leq y \leq x + \frac{1}{N}\frac{1}{\rho}} \{c(y-x) + \int L(y, \eta) f_D(\eta) d\eta\}.$$

A similar technique can be applied to $U_n(x)$ for $n = 2, ..., N$. Hence,

$$\begin{aligned} U_n(x) &= \min_{x \leq y \leq x + \frac{1}{N}\frac{1}{\rho}} \{c(y-x) + \frac{h_{n-1}}{h_N}(y-x) + \beta E U_{n-1}(y-D)\}, \\ & \quad n = N-1, ..., 2 \\ U_N(x) &= \min_{x \leq y \leq x + \frac{1}{N}\frac{1}{\rho}} \{c(y-x) + \frac{h_{N-1}}{h_N}(y-x) + \beta E U_{N-1}(y-D)\} + x^{+} \end{aligned}$$

Thus, the percentage cost reduction associated with information sharing relative to no information sharing depends only on the following non-dimensional parameters: $\rho$, $N$, $\pi$, $c$, $\beta$ and $f_D(\eta)$, where $\rho$ is the capacity utilization $\mu/C$, $N$ is the frequency of information sharing, $\pi$ and $c$ are the non-dimensionalized penalty and production costs, and $f_D(\eta)$ is the probability density function of the non-dimensionalized demand. In our computational study, we will focus on the impact of these parameters on the benefit from information sharing.

### 2.1.5 Information sharing and the greedy policy

In this strategy, we apply a simple heuristics that makes production decisions so as to match supply and demand. Such a greedy heuristic represents a special case of the policies commonly used in practice. The reason that we study this policy is to understand the benefits of using information optimally versus heuristically.

Specifically, the manufacturer produces in every information period, $n = N-1, N-2, \ldots, 2$ an amount equal to

$$\min\{C, D_{n+1} - x_n^{-}\},$$

where $x_n^- = \min\{0, x_n\}$ is the shortage level at the beginning of the $n$th information period. In the first information period, i.e., $n = N$, the manufacturer produces

$$\min\{C, -x_N^-\}.$$

Finally, in the last information period, i.e., $n = 1$, the inventory level is raised to a certain level determined by production capacity, inventory at the beginning of the information period, production and inventory holding costs, and the demand distribution. This can be done by solving a news vendor problem with capacity constraint (please Lee and Nahmias 1993 for a review of news vendor problems).

## 2.2 Timing of information sharing

An important question in information sharing is when to share information? To simplify the analysis, we focus on the single ordering period model and assume that the retailer can share information with the manufacturer only once during the ordering period. Intuitively, the higher the production capacity per unit time is, the later the time information may be shared. Of course, the later time information is shared, the more accurate the information on demand during the ordering period but the smaller the remaining production capacity, i.e. the product of the per unit time production capacity and the remaining time until the end of the ordering period. For instance, if production capacity per unit of time is very high, information should be transferred and used almost at the end of the ordering period. As production capacity per unit of time decreases, we expect that it is optimal to share information earlier. Thus, our objective is to find (1) the optimal time to share information, and (2) the parameters which may affect the best timing for sharing information.

In order to find the optimal timing, we need to develop a continuous time model. For this purpose, all notations associated with information period will be changed to per unit of time, while the others remains the same. Hence, $h$ is the inventory holding cost per unit of time; $C$ is the production capacity per unit of time; and $D$ is customer demand per unit of time with mean $\mu$. Finally, we set $\beta = 1$ in this section for simplification. Similar to the discrete time model, production is assumed to take place at the full capacity rate $C$ until the target inventory position is reached.

Consider an ordering interval $[0, T]$ and a given $t < T$, let $T - t$ be the time when information is shared. Thus, $t = 0$ ($t = T$) implies that information is shared at the end (beginning) of the ordering period. We assume that customer demand $D_\tau$ in any time interval of length $\tau$ is $Poisson(\tau\mu)$. This implies that customer demand at any time interval $[t, t + \tau] \subset [0, T]$ of length $\tau$ depends only on $\tau$ and not on $t$, and demand in different time intervals (not overlapping) is independent. The dynamic program is formulated below. Given that information is shared after $T - t$ units of time, let $U_1(x, t)$ ($U_2(x, t)$) be the minimum expected inventory and production costs from the time information is shared (the time the ordering period starts, respectively) to the end of the horizon given an initial inventory position $x$.

$$\begin{array}{rcl} U_2(x,t) & = & \min_{x \le y \le x+(T-t)C}\{c(y-x) + H_2(x,y,t) + E(U_1(y - D_{T-t}, t))\} \\ U_1(x,t) & = & \min_{x \le y \le x+tC}\{c(y-x) + H_1(x,y,t) + E(L(y, D_t))\} \end{array} \tag{6}$$

where

$$\begin{array}{rcl} H_2(x,y,t) & = & T \times h \times x^+ + h \times t \times (E(D_{T-t}) - x) + \frac{h}{2}\frac{(y-x)^2}{C}, \\ H_1(x,y,t) & = & h \times t \times x + \frac{h}{2}\frac{(y-x)^2}{C}, \\ L(y,D) & = & T \times h \times (y-D)^+ + \pi \times (D-y)^+. \end{array}$$

The first term of $H_2$ represents the holding cost for initial inventory. The facts that production line always runs at its full rate $C$, and the manufacturer will postpone production as much as she can, explain the last term of $H_1, H_2$. The middle term of $H_2$ and the first term of $H_1$ come from the fact that the inventory accumulated in the first part of the ordering period (before information sharing) will be carried throughout the second part of the ordering period. Assuming the initial inventory position to be $x$ and the order-up-to level in the first part to be $y$, then it equals $h \times t \times (y - x) = h \times t \times (y - D_{T-t} + D_{T-t} - x) = h \times t \times (y - D_{T-t}) + h \times t \times (D_{T-t} - x)$. Take the expectation with respect to $D_{T-t}$ and notice that $y - D_{T-t}$ is the initial inventory position at the beginning of the second part of the ordering period, then we obtain the expression for $H_1, H_2$.

If $t$ is fixed, then it is easy to show that $V_1(x, y, t) = c(y - x) + H_1(x, y, t) + E(L(y, D_t))$ is jointly convex in both $x$ and $y$ (since the Hessians of $H_1(x, y, t)$, $H_2(x, y, t)$ are positive semi-definite). This observation implies the following.

**Proposition 2.2** *$U_1(x, t)$ and $U_2(x, t)$ are convex in $x$.*

*Proof:* : We start by proving that $U_1(x, t)$ is convex in $x$. Suppose we have $x_1, x_2$, $x_1 \neq x_2$, and $y_1^*, y_2^*$, where $U_1(x_1, t) = V_1(x_1, y_1^*, t)$ with $x_1 \leq y_1^* \leq x_1 + (T - t)C$ and $U_1(x_2, t) = V_1(x_2, y_2^*, t)$ with $x_2 \leq y_2^* \leq x_2 + (T - t)C$. Let $\overline{x} = \lambda x_1 + (1 - \lambda)x_2$ and $\overline{y} = \lambda y_1^* + (1 - \lambda)y_2^*$, obviously, for any $\lambda \in (0, 1)$, $\overline{x} \leq \overline{y} \leq \overline{x} + (T - t)C$. Hence,

$$\begin{aligned} U_1(\overline{x}, t) &= \min\{V_1(\overline{x}, y, t) | \overline{x} \leq y \leq \overline{x} + (T - t)C\} \\ &\leq V_1(\overline{x}, \overline{y}, t) \\ &\leq \lambda V_1(x_1, y_1^*, t) + (1 - \lambda)V_1(x_2, y_2^*, t) \\ &= \lambda U_1(x_1, t) + (1 - \lambda)U_1(x_2, t). \end{aligned}$$

To prove that $U_2(x, t)$ is convex, observe that since $U_1(x, t)$ is convex in $x$, thus

$$V_2(x, y, t) = c(y - x) + H_2(x, y, t) + E(U_1(y - D_{T-t}, t))$$

is jointly convex in $x$ and $y$. Applying the same proof as before, we can show that $U_2(x,t)$ is convex in $x$. ∎

Unfortunately, it is not clear whether or not $E(U_1(y - D_{T-t}, t))$ and $U_2(x,t)$ are convex in $t$. Thus, for any given $t$, we can compute the optimal $y$ efficiently by using Proposition 2.2. But to find the optimal timing of information sharing, we need to discretize the ordering period and use enumeration.

We further analyze the impact of the timing of information sharing when the retailer can transfer demand information to the manufacturer twice an ordering period. Consider an ordering period $[0,T]$, let $0 \le t_1 \le t_2 \le T$ be the times when information is shared, then the following dynamic program holds,

$$\begin{array}{rcl} U_3(x,t_1,t_2) & = & \min_{x\le y\le x+t_1 C}\{c(y-x)+K_3(x,y,t_1)+E(U_2(y-D_{t_1},t_1,t_2))\} \\ U_2(x,t_1,t_2) & = & \min_{x\le y\le x+(t_2-t_1)C}\{c(y-x)+K_2(x,y,t_2)+E(U_1(y-D_{t_2-t_1},t_2))\} \\ U_1(x,t_2) & = & \min_{x\le y\le x+(T-t_2)C}\{c(y-x)+K_1(x,y)+E(L(y,D_{T-t_2}))\}, \end{array}$$

where $K_3(x,y,t_1) = T \times h \times x^+ + \frac{h}{2}\frac{(y-x)^2}{C} + (T-t_1)\times h \times (y-x)$, $K_2(x,y,t_2) = \frac{h}{2}\frac{(y-x)^2}{C} + (T-t_2)\times h\times(y-x)$, $K_1(x,y) = \frac{h}{2}\frac{(y-x)^2}{C}$. Similarly, we can show $U_3(x,t_1,t_2), U_2(x,t_1,t_2)$ and $U_1(x,t_2)$ are convex in $x$. Since it is not clear whether $U_3(x,t_1,t_2)$ is convex in $t_1, t_2$, we will compute the optimal timings by discretization of the ordering period and enumeration.

Finally, we identify the non-dimensional parameters which may affect the optimal timing. We divide Equation (6) by $hT^2\mu$, let $x' = \frac{x}{T\mu}$, $y' = \frac{y}{T\mu}$, $t' = \frac{t}{T}$, $c' = \frac{c}{Th}$, $\rho = \frac{\mu}{C}$, $\pi' = \frac{\pi}{Th}$, $U_1' = \frac{U_1}{hT^2\mu}$, $U_2' = \frac{U_2}{hT^2\mu}$ $H_1' = \frac{H_1}{hT^2\mu}$, $H_2' = \frac{H_2}{hT^2\mu}$, $D' = \frac{D}{T\mu}$ and $L'(y',\eta) = (y'-\eta)^+ + \pi'(\eta - y')^+$. We omit $'$ from the notation without creating any confusion, and rewrite the non-dimensionalized functions $U_1(x,t)$ and $U_2(x,t)$

as follows:

$$\begin{aligned} U_2(x,t) &= \min_{x \le y \le x + \frac{(1-t)}{\rho}} \{c(y-x) + H_2(x,y,t) + E(U_1(y - D_{1-t}, t))\} \\ U_1(x,t) &= \min_{x \le y \le x + \frac{t}{\rho}} \{c(y-x) + H_1(x,y,t) + E(L(y, D_t))\} \end{aligned}$$

where $H_2(x,y,t) = x^+ + t(1-t-x) + \frac{\rho}{2}(y-x)^2$, $H_1(x,y,t) = tx + \frac{\rho}{2}(y-x)^2$, and $L(y,D) = (y-D)^+ + \pi(D-y)^+$. This analysis shows that the non-dimensional optimal timing is a function of only $\rho$, $\pi$, $c$ and $f_D(\cdot)$, of which we will study the effects of $\rho$ and $\pi$ in the following section.

In the case when information can be shared twice in one ordering period, a similar non-dimensional analysis shows that the non-dimensional optimal timings $t_1/T, t_2/T$ are functions of only $\rho$, $\pi$, $c$ and $f_D(\cdot)$.

## 2.3 Computational results

In this Section, we use computational analysis to develop insights on the benefits of information sharing. Our goal is two-fold: (1) determine situations where information sharing provides significant cost savings compared to supply chains with no information sharing; (2) identify the benefits of using information optimally compared to using information greedily. Our focus is on the manufacturer's cost and service level.

According to Section 2.1.4, we examined cases with variations on the following non-dimensional parameters: production capacity over mean demand, the number of information periods in one ordering period, the time when information is shared, coefficient of variation of demand distribution and finally the ratio between penalty cost and inventory holding cost.

We set production cost equal to zero, and focus on holding and penalty costs.

Let inventory holding cost per ordering period to be equal to a constant 0.4 \$ per unit product for all cases. Thus, the inventory holding cost per information period is $0.4/N$ where $N$ is the number of information periods within one ordering period. Penalty cost varies from 1.9 to 7.9 \$ per unit and takes the following values $1.9, 3.4, 4.9, 6.4, 7.9$. In all cases of our computational analysis, the time discount factor $\beta$ is assume to be 1.

Let initial inventory at the beginning of the first ordering period, $x$, to be equal to zero. To simplify the calculation, we use discrete probability distributions for customer demand in one information period. In our study, we consider discrete distributions such as Poisson, Uniform and Binomial. In addition, we also analyze the following discrete distributions: the first, referred to as Disc1, demand takes values from the set $(0, 1, 3, 6)$ with probability $(0.1, 0.3, 0.5, 0.1)$ respectively. In the second, referred to as Disc2, demand takes the same values with probability $(0.05, 0.2, 0.7, 0.05)$, respectively.

The dynamic programming algorithms allow us to find the cost associated with the first two strategies. For the third strategy, the newsboy model allows us to find the optimal order-up-to level in the last information period of every ordering period, while the cost associated with the strategy is estimated through simulation. Finally simulation results provided us with service level for all three strategies. Following convention, we measure service level by type one fill rate, which is defined to be the expected fraction of ordering periods in which no backorders occur.

In the simulation models, each system is simulated $40,000$ times. The fill rate is calculated as follows: let $X_i$ be a random variable taking the value one if demand (at the end of the ordering period) is satisfied with no shortage in the $i$th run, and zero otherwise. Our estimation of the type one fill rate is the sample mean $\overline{X} = \sum_i X_i/n$, where $n$ is 40,000. Since our estimation of the standard deviation of $X_i$ is equal to $\sqrt{\overline{X}(1-\overline{X})}$, which is less than 0.5, thus, the length of a 95% confidence interval of the fill rate is no more than 0.0098. Similarly, when we estimate the average cost for the third strategy using simulation, we can ensure that a 95% confidence interval has a length no larger than 0.098.

The following discussions are based on our computation results for models with one ordering period and four information periods unless otherwise mentioned. For multi-ordering period planning horizon, similar results are obtained.

### 2.3.1 The effect of information sharing on the optimal policy

Table 1: The impact of production capacity

| demand distribution | capacity/ED | Penalty/Holding costs | order-up-to-levels |
|---|---|---|---|
| Poisson(5) | 1.2 | 8.5 | (8,9,9,8) |
| Poisson(5) | 1.6 | 8.5 | (2,5,7,8) |
| Poisson(5) | 2 | 8.5 | (-4,1,5,8) |
| Uniform(0,1,...,9) | 1.22 | 8.5 | (10,10,10,8) |
| Uniform(0,1,...,9) | 1.67 | 8.5 | (4,6,8,8) |
| Uniform(0,1,...,9) | 2.11 | 8.5 | (-2,2,6,8) |
| Binomial(0.5,10) | 1.2 | 8.5 | (6,7,7,7) |
| Binomial(0.5,10) | 1.6 | 8.5 | (0,3,5,7) |
| Binomial(0.5,10) | 2 | 8.5 | (-6,-1,3,7) |

In this subsection we analyze the impact of capacity, penalty cost and demand variability on the optimal policy when information is shared.

Table 1 presents the effect of production capacity for three different distributions of demand in one information period. For each demand distribution, we increase production capacity over average demand (the column capacity/ED) and calculate the order-up-to-level in all information periods. Thus, the last column represents the order-up-to-level for each of the four information periods.

Observe that

- Proposition 2.1 holds for almost all cases except the one for which capacity is very tight, e.g., capacity/ED =1.2.
- As capacity increases, the difference between order up to levels in different information periods increases. The intuition is clear: as capacity increases, the optimal policy delays production as much as possible.
- The order-up-to-levels in the first few information periods may be negative, which implies that the inventory position can be negative.

Table 2: The impact of penalty cost

| Demand distribution | capacity/ED | Penalty/Holding costs | order-up-to-levels |
|---|---|---|---|
| Poisson(5) | 1.6 | 4.75 | (0,3,6,7) |
| Poisson(5) | 1.6 | 12.25 | (3,6,8,8) |
| Poisson(5) | 1.6 | 19.75 | (5,7,9,9) |
| Uniform(0,1,...,9) | 1.67 | 4.75 | (1,4,7,8) |
| Uniform(0,1,...,9) | 1.67 | 12.25 | (5,7,9,9) |
| Uniform(0,1,...,9) | 1.67 | 19.75 | (7,8,9,9) |
| Binomial(0.5,10) | 1.6 | 4.75 | (-1,2,5,6) |
| Binomial(0.5,10) | 1.6 | 12.25 | (1,4,6,7) |
| Binomial(0.5,10) | 1.6 | 19.75 | (2,4,6,8) |

Table 2 analyzes the impact of penalty cost. In this table, we increase the ratio of penalty to holding costs from 4.75 to 19.75 for each demand distribution. The table demonstrates that

- As penalty cost increases, the order-up-to-level increases.
- As penalty cost increases, the difference between order up to levels in different information periods decreases.

Table 3: The impact of demand variability

| Demand distribution | coefficient of variation | order-up-to-levels |
|---|---|---|
| Uniform(3,4,5,6) | 0.25 | (-3,1,4,6) |
| Uniform(2,3,...,7) | 0.38 | (-1,2,5,7) |
| Uniform(1,2,...,8) | 0.51 | (3,5,7,8) |
| Uniform(0,1,...,9) | 0.64 | (7,8,9,9) |

Table 3 presents the impact of demand variability. In this case the capacity over average demand was kept constant, at a level of 1.67 for all cases, while penalty over holding cost was 7.9 for all cases. It is easy to see the demand coefficient of variation has a similar impact as the penalty cost. That is,

- As the coefficient of variation increases, the order-up-to-level increases.
- As the coefficient of variation increases, the difference between order up to levels in different information periods decreases.

In Table 4 we consider two ordering periods with four information periods in each one. We observe that the differences in the order-up-to-levels for the same information periods between two consecutive ordering periods are small relative to the average total demand in one ordering period. For example, in the case of Binomial demand distribution, the maximal difference is 2 while the average total demand in one ordering period is 20.

Table 4: Two ordering periods

| Demand distribution | capacity/ED | Penalty/Holding costs | order-up-to-levels |
|---|---|---|---|
| Poisson(5) | 1.6 | 4.75 | (0,3,6,7,0,3,6,8) |
| Poisson(5) | 2 | 4.75 | (-6,1,4,7,-6,1,4,8) |
| Uniform(0,1,...,9) | 1.67 | 4.75 | (1,4,7,8,1,4,7,8) |
| Uniform(0,1,...,9) | 2.11 | 4.75 | (-5,0,5,8,-5,0,5,9) |
| Binomial(0.5,10) | 1.6 | 4.75 | (-1,2,5,6,-1,2,5,8) |
| Binomial(0.5,10) | 2 | 4.75 | (-7,-2,3,6,-8,-2,3,7) |

### 2.3.2 The effect of capacity

To explore the benefit of information sharing as a function of production capacity, we illustrate in Figure 1 the percentage cost savings from information sharing with the optimal policy relative to no information sharing for five demand distributions. For each demand distribution and each capacity level, we consider the cases where the ratio of penalty cost to holding costs in one ordering period is 4.75. Similar results can be obtained at other values of penalty over inventory holding cost, and we will discuss the impact of penalty cost later. Our computational study reveals that as production capacity increases, percentage cost savings increases. Indeed, percentage cost savings increases from about 8% to about 35% as capacity over mean demand increases from 1.2 to 3. This is quite intuitive, since as capacity increases, the optimal policy would postpone production as much as possible and take advantage of all information available prior to the time production starts. For instance, in case of infinite capacity, it is optimal to wait until the last information period and produce to satisfy all demand realized so far plus an additional amount based on solving a newsboy problem (see Section 2.1.3). Similarly, if there is limited production capacity, then information is not very beneficial since the production quantity is

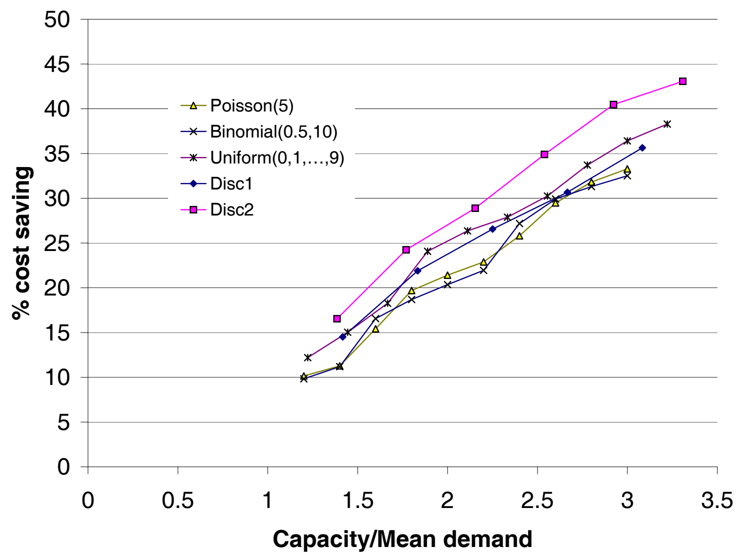


Figure 1: The impact of production capacity.

mainly determined by capacity, not based on realized demand. Finally, from fill-rate point of view, our computational study reveals that information sharing with the optimal policy and the no information sharing strategies have almost identical fill rates.

To explore the effectiveness of information sharing with the greedy policy, we provide in Figure 2 the percentage cost savings of information sharing with the optimal policy relative to information sharing with the greedy policy under similar conditions as above. The Figure illustrates that

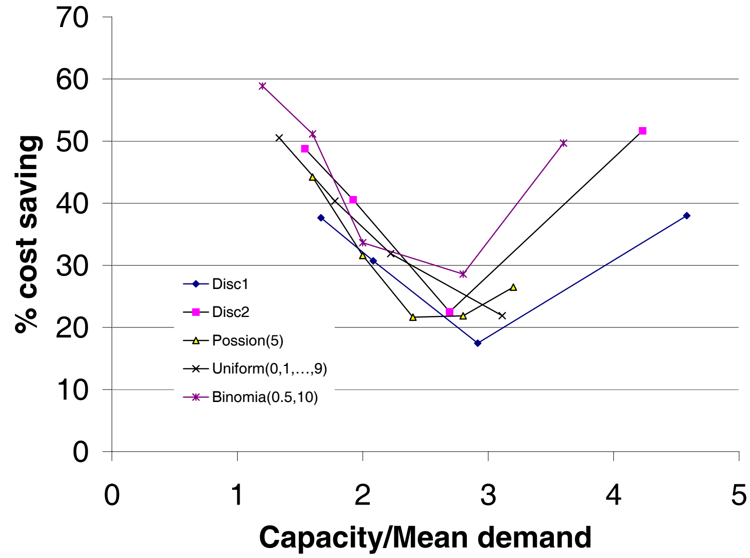


Figure 2: The benefits of using information optimally.

- information sharing with the optimal policy reduces cost by at least 15% relative to information sharing with the greedy policy and the savings can be as much as 50-60%.

- When capacity is tightly constrained, the savings provided by information sharing with the optimal policy is relatively high. This is because the greedy policy only responds to demand and does not build safety stock until the last information period. In the last information period, if capacity is very tight, the greedy policy may not be able to build as much safety stock as needed, thus results in heavy penalty cost. On the other hand, the optimal policy can

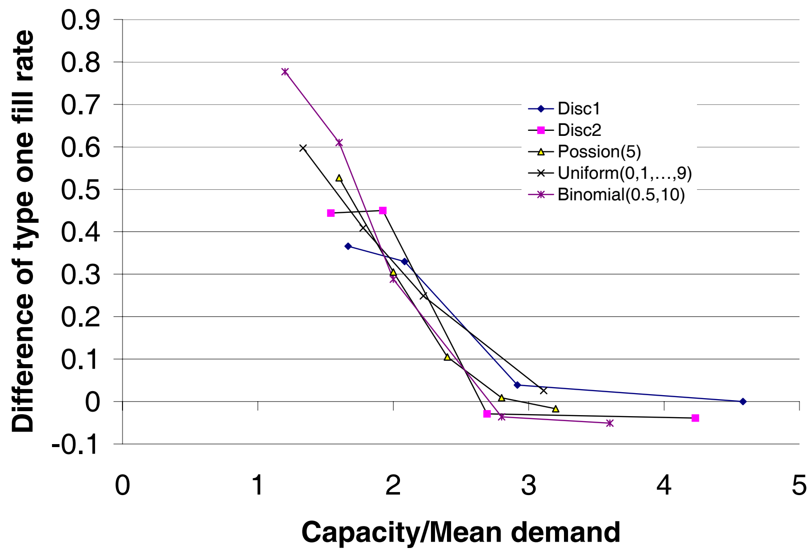


Figure 3: Fill rates: the optimal policy vs. the greedy policy.

start building safety stock from the beginning of the ordering period by taking advantage of excessive capacity in all information periods.

- As capacity increases, the benefit from information sharing with the optimal policy relative to information sharing with the greedy policy decreases first and then increases again. This is true, since as capacity becomes very large relative to average demand, information sharing with the optimal policy will postpone production as much as possible, while information sharing with the greedy policy will build inventory starting from the beginning of ordering periods, thus results in heavy inventory holding cost.

Figure 3 shows the difference between type-one fill rates for information sharing with the optimal policy and information sharing with the greedy policy as a function of production capacity for various demand distributions. The figure demonstrates that when capacity is relatively tight, the difference in the fill-rates may be substantial. However, as capacity increases, the two strategies have almost identical fill-rates.

### 2.3.3 The effect of penalty cost

To explore the benefit of information sharing as a function of the penalty cost, we present in Figure 4 the percentage cost savings with information sharing relative to no information sharing. The Figure illustrates the percentage cost savings as a function of the ratio between penalty cost and inventory holding cost for various capacity levels. Demand distribution in one information period is assumed to be $Uniform(0, 1, \cdots, 9)$.

The Figure 4 illustrates that

- When capacity is tightly constrained (e.g. capacity/mean demand =1.2), the percentage cost saving could decrease as penalty cost increases. This is explained as follows: when capacity is tightly constrained, the total cost for both the no information sharing and information sharing strategy increases quite fast as penalty costs increases. Thus, the percentage saving decreases.

- When capacity is not tightly constrained, the benefit from information sharing increases initially as penalty cost increases. As Kapuscinski and Tayur (1998) points out, benefit from information sharing eventually decreases as penalty cost becomes very large.

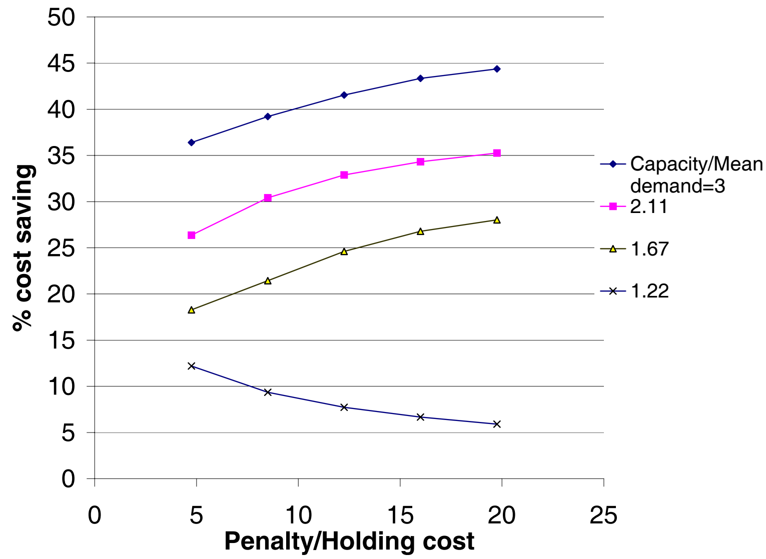


Figure 4: The impact of penalty cost

### 2.3.4 The effect of the number of information periods

To explore the benefit of information sharing as a function of the number of information periods, we present in Figures 5 the percentage cost savings with information sharing relative to no information sharing for two production capacity levels. The number of information periods, $N$, was 2,4,6 and 8 while the length of the ordering period was assumed to be constant in all the models. The demand distribution during the entire ordering period is assumed to be $Poisson(\lambda)$ with $\lambda = 24$, hence demand in a single information period follows $Poisson(\lambda/N)$. Similarly, the total production capacity, and the inventory holding cost per item, in the entire ordering

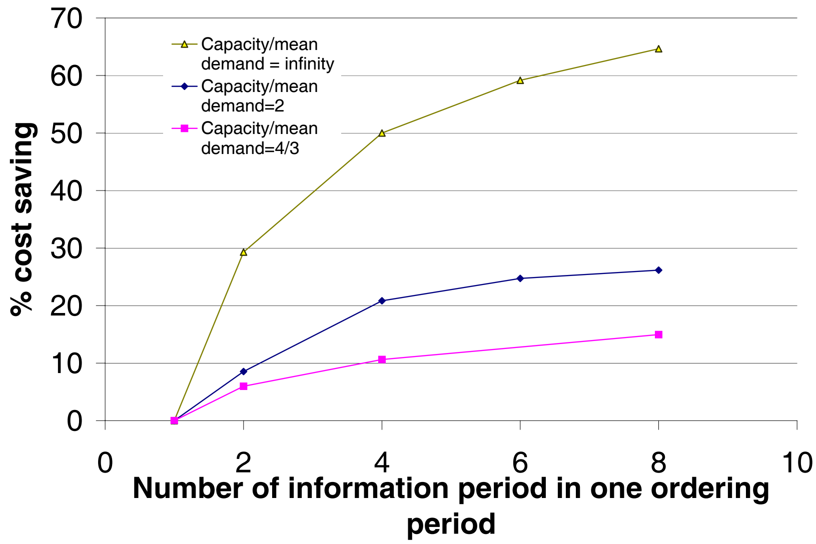


Figure 5: The impact of information sharing frequency.

period are kept constant and are equally divided among the different information periods. The ratio of penalty to holding cost is 4.75. Figure 5 illustrates that

- As the number of information periods increases, the percentage savings increases.

- Most of the benefits from information sharing is achieved within about 4 information periods. That is, the marginal benefit is a decreasing function of the number of information periods. Specifically, the benefit achieved by increasing the number of information periods from 4 to 8 is relatively small.

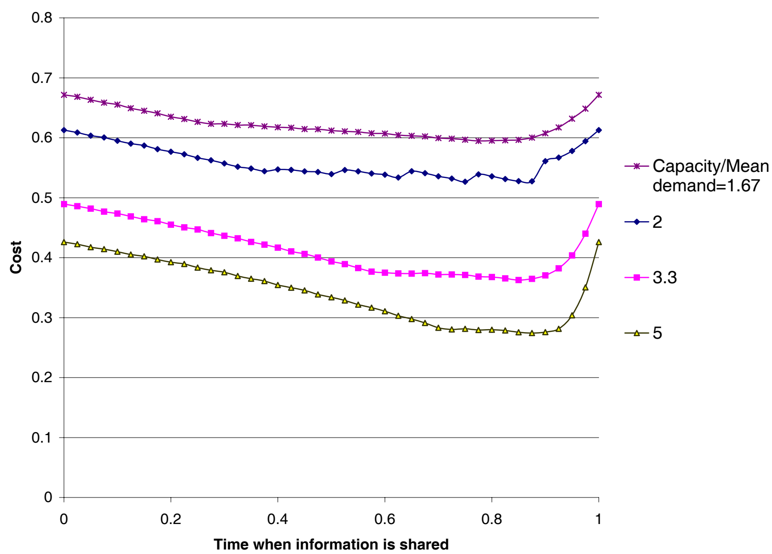


Figure 6: The impact of the timing of information sharing.

- Define the maximum potential benefit from information sharing to be the percentage cost reduction when the manufacturer has unlimited capacity. A manufacturer with production capacity twice as much as mean demand can achieve a substantial percentage of the maximum potential benefit, e.g., when the frequency of information period is 4, the manufacturer can obtain about 50% of the cost benefit that a manufacturer with unlimited capacity can achieve.

### 2.3.5 Optimal timing for information sharing

In this subsection we analyze the impact of the time(s) when information is shared on the manufacturer's total inventory and penalty costs.

Sharing information once in one ordering period

Figure 6 presents the manufacturer's total cost as a function of the time when information is shared, assuming that the manufacturer can only share information once in one ordering period. The Figure provides the normalized manufacturer's cost as a function of a normalized time. That is, time is normalized and is measured from 0 to 1, while the cost is normalized by the cost of carrying one ordering period's expected demand for one ordering period. Thus, 0 in x coordinate implies that information is shared at the beginning of ordering period, and 1 means that information is shared at the end of the ordering period and hence can not be utilized. Demand distribution is assumed to be $Poisson(10)$ and penalty over holding cost equals 4.

In Figure 7 we study the impact of the production capacity and penalty cost on the optimal timing of information sharing. These figures illustrate that

- As information sharing is delayed, the manufacturer's total cost first decreases and then increases sharply. The cost reaches its maximum when information is shared at the beginning or end of one ordering period.

- The optimal timing for information sharing is not in the middle of ordering period for any combination of production capacity and penalty cost, rather, it's in the later half of the ordering period.

- Both the production capacity and penalty cost have minor effects on the optimal timing of information sharing. For all combination of production capacity and penalty cost, the optimal timing is somewhere between 0.75 to 0.9 of normalized time, and very close to 0.8 on average.

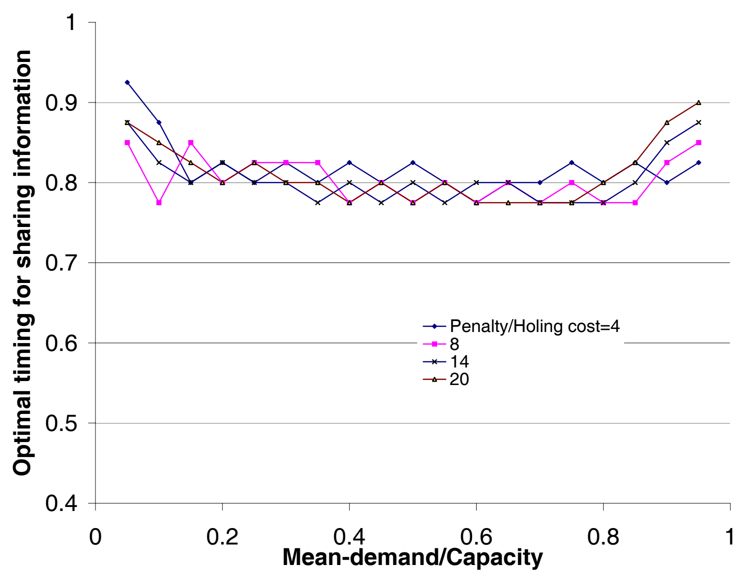


Figure 7: Optimal timing for information sharing with different penalty cost.

- When capacity is very large, it is appropriate to postpone the time when information is shared to the last production opportunity in this ordering period; interestingly, this is also the right thing to do when capacity is tightly constrained, i.e., postpone the time when information is shared until the last production opportunity. This contradicts our initial expectation (Section 2.2); one possible explanation is that when capacity is very tight, the production schedule mainly depends on capacity instead of information. Thus, early demand information will not provide much help for the manufacturer, a better choice for her is to build as much safety stock as she can until the last production opportunity, when she can check demand information to see whether

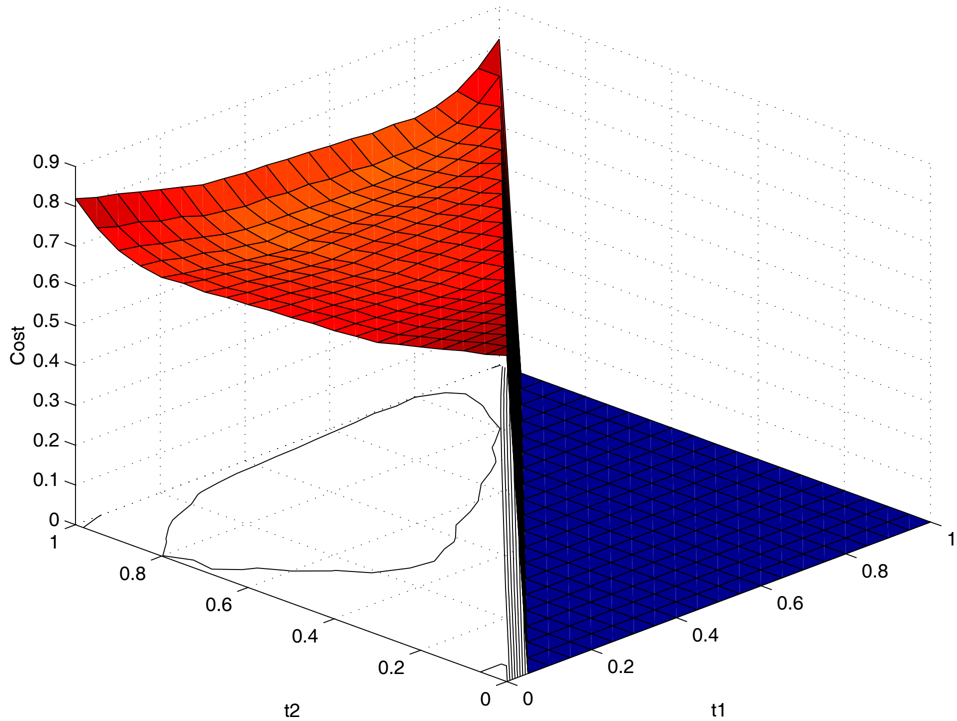


Figure 8: The impact of the timing of information sharing.

she needs to produce more or not, before the demand is finally realized.

Sharing information twice in one ordering period

Figure 8 presents the manufacturer's total cost as a function of the times when information is shared, assuming that the manufacturer can share information twice in one ordering period. Similar to the previous Section, the Figure provides the normalized manufacturer's cost as a function of a normalized time. Demand distribution is assumed to be $Poisson(10)$, penalty over holding cost equals 4, and the production capacity is twice as much as the mean demand. Figure 9 illustrates the manufacturer's cost as a function of $t_1/T$ for given $t_2/T$.

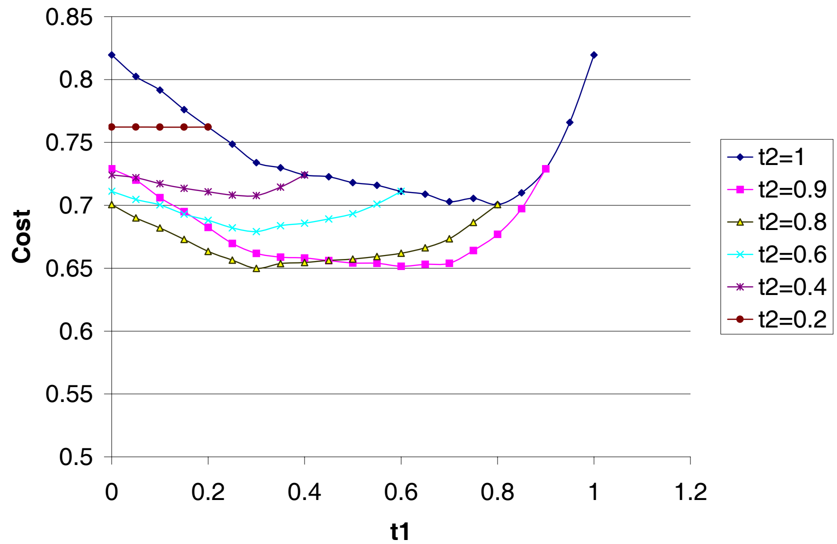


Figure 9: The impact of the timing of information sharing.

In Figure 10 we study the impact of the production capacity and penalty cost on the optimal timings of information sharing. The higher curves are the optimal timing for the second information sharing, and the lower curves are the optimal timing for the first information transferring. These figures illustrate that

- Sharing information twice an ordering period may help the manufacturer achieve substantially more benefits than sharing information once, e.g. in Figure 9, the manufacturer's maximum cost saving increases from 14.6% (once an ordering period) to 20.7% (twice an ordering period).

- The optimal timing of the first information sharing varies significantly as the

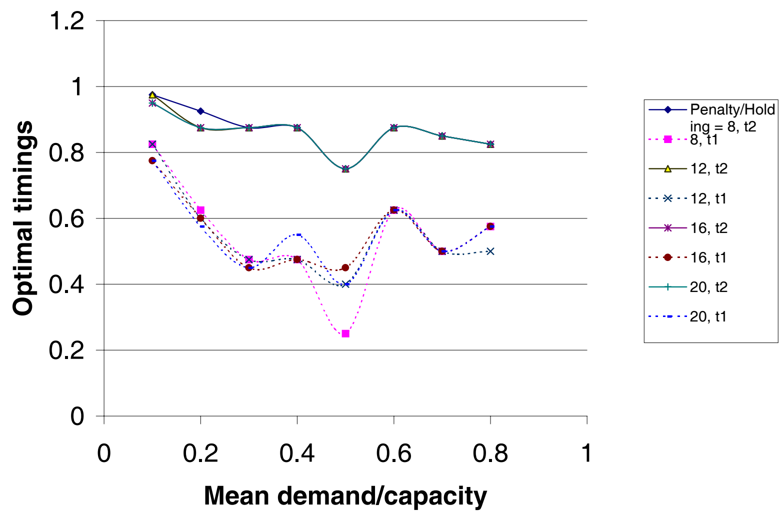


Figure 10: The impact of capacity and penalty cost on the optimal timing.

ratio between mean demand and production capacity changes, and it approaches the end of the ordering period as the ratio tends to zero. The ratio between penalty and holding cost also has an impact on this optimal timing, especially when capacity utilization is neither very high nor very low.

- The optimal timing of the second information sharing varies slightly for different ratios between the mean demand and production capacity, and approaches the end of the ordering period as the ratio tends to zero.

- The optimal timing of the second information transferring is unlikely to be in the first half of the ordering period for all combination of parameters. On the other hand, the optimal timing of the first information sharing can be either in the first or the second halves of the ordering period. Indeed, in our computational results the optimal timing of the first information sharing varied from 0.22 to 0.82.

## 2.4 Conclusion

In this chapter, we consider a two-stage supply chain with a single retailer facing i.i.d demand and a single manufacturer with finite production capacity. In the model, the manufacturer receives demand information from retailer even during time periods in which the retailer does not order. By analyzing the model in finite time horizon, we study the value of information sharing for the manufacturer as well as how the manufacturer can use the shared demand information effectively.

As for the optimal inventory control policy with information sharing, we find the following property holds under certain conditions: the optimal order-up-to-levels in any ordering period increase as we move from the first information period to the last one. From our computational analysis, we find that even if these conditions do not hold, the property is still true in most cases except when production capacity is tightly constrained.

We demonstrate, through an extensive computational study, the potential benefits of sharing demand information in terms of the manufacturer's cost and service level. For instance, if the manufacturer has a large capacity, information sharing can be very beneficial. Indeed, the manufacturer can cut down inventory cost while maintaining service level to the retailer by using information effectively. One interesting observation is that the manufacturer can realize most of the benefits from information sharing if the retailer shares demand information with the manufacturer only a few times in each ordering period.

If the retailer has only one opportunity to share information with the manufacturer, then the best timing to share information is in the later half of the ordering period. Parameters such as the production capacity and penalty cost have limited impact on the optimal timing.

Finally, although the greedy policy is easy to implement, it may not perform well in manufacturer's costs and fill rate.

In this chapter we focus exclusively on the benefit of information sharing for the manufacturer. In fact, information sharing can be also beneficial to the retailer if the retailer and the manufacturer share the benefits. For example, the manufacturer can provide a better service level to the retailer in exchange for the demand information. As we discuss in Section 2.1, this represents a very interesting future research direction. Another future research direction is the impact of information sharing on multi-product capacitated production systems in finite time horizon.

# Chapter 3

# A capacitated two-stage supply chain: infinite time horizon

In this chapter we extend the analysis of the previous chapter to the infinite time horizon model. Specifically, we study the value of information sharing in a two-stage supply chain with a single manufacturer and a single retailer in infinite time horizon, where the manufacturer has finite production capacity and the retailer faces independent demand. As before, the manufacturer receives demand information even during periods of time in which the retailer does not order. Allowing for time varying cost functions, our objective is to characterize the impact of information sharing on the manufacturer's cost and service level.

The optimal control of a periodic review production-inventory system in infinite time horizon is a classical Markov decision problem with infinite state space and unbounded cost function. The literature on these problems is quite voluminous, see e.g., Heyman and Sobel (1984), Bertsekas (1987) and Puterman (1994) for general theory of Markov decision process; Karlin (1960a, b), Zipkin (1989), Sethi and Cheng (1997), and Song and Zipkin (1993) for systems with time varying parameters; and

Federgruen and Zipkin (1986a) and (1986b) for systems with production capacity constraints.

The work of Aviv and Federgruen (1997) and Kapuscinski and Tayur (1998) is closely related to our research. These authors analyzed periodic review production-inventory systems with capacity constraints and time varying parameters. They show that a modified cyclic order-up-to policy, i.e., a modified order-up-to policy with periodically varying order-up-to levels, is optimal under both discounted and average cost criterion. This is done by employing the optimality conditions developed by Sennott (1989) and Federgruen, Schweitzer and Tijms (1983).

In the next Section we describe our model and identify the main differences between our model and results and those of Aviv and Federgruen (1997) and Kapuscinski and Tayur (1998).

## 3.1 The model

Consider the infinite time horizon version of the model analyzed in Chapter 2. In this periodic review model, a single capacitated manufacturer can receive demand information from a single retailer every $\tau$ units of time, while the retailer places orders every $T$ time periods, $\tau \leq T$. The external demand faced by the retailer is assumed to be independent, but not necessarily identical, and the retailer uses an order-up-to inventory policy to control her inventory. In what follows, we focus on the differences between the model that we discuss here and the model that we analyze in Chapter 2.

Let $N = T/\tau$ be an integer which represents the number of information periods in one ordering period. We index information periods within one ordering period

$1, 2, \ldots, N$ where 1 is the first information period in the ordering period and $N$ is the last. We use $D_i$ to denote the end user demand in information period $i$, $i = 1, \ldots, N$. $D_i$ is assumed to be independent, its distribution only depends on $i$, and its mean is defined as $ED_i$. To simplify the analysis, we assume that costs do not change from information period to information period; later we will demonstrate that our results can be easily extended to include cases where costs change periodically within one ordering period but are the same across different ordering periods.

We start by considering a finite time horizon model with $M$ ordering periods. Index the ordering periods from 0 to $M-1$ where the 0 ordering period is the first one and the $M-1$ ordering period is the last one. The finite horizon starts in the first ordering period at the beginning of the $j$th information period, $1 \leq j \leq N$. Consider the $i$th information period, $i = 1, 2, \ldots, N$, in ordering period $m$, $m = 0, 1, \ldots, M-1$. Of course, $mN + i \geq j$. We refer to this information period as the $mN + i$ information period. For instance, information period $mN + 1$ is the first information period in the $m$th ordering cycle. We refer to this indexing convention as a forward indexing process.

Define $S_i$ to be the state space for inventory position $x$ at the beginning of the $i$th information period, and $y - x$ is the amount produced in that information period, $y \in A_x$ and $A_x$ is the set of feasible actions. Let $\xi(x, y, D)$ to be the transition function and $D$ be the demand. In our case, $A_x = [x, x+C]$, and $\xi(x, y, D) = y - D$.

It is easy to verify that $g_{mN+i}(x, y)$, $i = 1, 2, \ldots, N$,$m = 0, \ldots, M-1$, the expected inventory and production cost in information period $mN + i$ given that the period starts with an inventory position $x$ and produces in that period $y - x$

items, can be written as follows.

$$g_{mN+i}(x,y) = \begin{cases} c(y-x) + E(L(y,D_N)), & i = N \\ c(y-x) + h_{N-i}(y-x), & otherwise, \end{cases}$$

where $E(\cdot)$ is the expectation with respect to $D_N$. Since $g_{mN+i}(x,y)$ depends only on $i$ and not $m$, we can write $g_{mN+i}(x,y) = r_i(x,y)$, $\forall m = 0,1,\ldots,M-1$ and all $i =, 1,\ldots,N$. Furthermore, $r_i(x,y)$ can be written as the sum of a function of $x$, $(\phi_i(x))$ and a function of $y$, $(\varphi_i(y))$ where

$$\phi_i(x) = \begin{cases} -cx, & i = N \\ -(c+h_{N-i})x, & otherwise, \end{cases}$$

$$\varphi_i(y) = \begin{cases} cy + E(L(y,D_N)), & i = N \\ (c+h_{N-i})y, & otherwise, \end{cases}$$

and

$$L(y,D) = h_N(y-D)^+ + \pi(D-y)^+.$$

Finally, we assume the salvage cost function $g_{MN+1} = 0$.

We define the optimal policy under discounted cost criterion as follows. Let $\sigma = \{\sigma_1, \sigma_2, \ldots\}$ be any feasible policy where $\sigma_k$ is a function depending on the initial inventory position in period $k$, i.e. $\sigma_k = \sigma_k(x)$ and $\sigma_k(x) \in A_x$ for all $k$. Let $\Pi$ be the set of all feasible policies. Define the expected discounted cost from the $i$th information period in the first ordering period until the end of the horizon when $M \to \infty$, as

$$U_i^\beta(x,\sigma) = E(\lim_{M\to\infty} \sum_{k=i}^{MN} \beta^{k-i} g_k(x_k, \sigma_k(x_k))|x_i = x),$$

where $E(\cdot)$ denotes the expectation with respect to demand in all information periods, and $x_i$ is the initial inventory position of the $i$th information period for

$i = 1, 2, \ldots, N$. A policy $\sigma^* = \{\sigma_1^*, \sigma_2^*, \ldots\} \in \Pi$ is called optimal under the discounted cost criterion if for all $x \in S$ and $i$,

$$U_i^\beta(x, \sigma^*) = \inf_{\sigma \in \Pi} U_i^\beta(x, \sigma).$$

To simplify the notation, we define $u_i^\beta(x) = U_i^\beta(x, \sigma^*)$.

Similarly, the optimal policy in infinite time horizon under average cost criterion can be defined. Following Heyman and Sobel, the performance measure for any feasible policy $\delta = \{\delta_1, \delta_2, \ldots\} \in \Pi$ under average cost criterion is defined as follows,

$$G_i(x, \delta) = lim_{M \to \infty} \sup\left(\frac{E\{\sum_{k=i}^{MN} g_k(x_k, \delta_k(x_k))|x_i = x\}}{MN - i + 1}\right).$$

As before, a policy $\delta^*$ is optimal if it minimizes $G_i(x, \delta)$ for all $x$ and $i$ over $\Pi$.

It is easily seen that, in our model of information sharing, except for those periods in which orders are placed, the cost function $\varphi_i(y)$ can go to negative infinity as $y \to -\infty$. This implies that the results obtained by both Aviv and Federgruen and Kapuscinski and Tayur cannot be applied to this model since they all assume $\varphi_i(y)$, for all $i$, are bounded from below. To further explain the difference between this model and previous models, consider following three cases. First, if the manufacturer only has inventory holding cost but no penalty cost for all periods, then a finite optimal policy does not exist, since producing nothing in all periods is clearly the optimal policy. Second, if the manufacturer has both inventory holding cost and penalty cost for all periods, then we have the models studied by Aviv and Federgruen (1997) and Kapuscinski and Tayur (1998). Finally, if in some periods the manufacturer only has inventory holding cost, while in other periods, she has both holding and penalty cost, then it is not clear whether there exists a finite optimal policy if the manufacturer has finite production capacity.

Thus, the objective of this chapter is two fold: First, characterize the finite optimal policy for the information sharing model under both discounted and average cost criterion. Second, identify the conditions under which information sharing is most beneficial, that is, characterize how frequently information should be shared and when it should be shared so that the **manufacturer** can maximize the potential benefits.

For this purpose, we first develop a new method in Section 3.2 to prove that the steady-state average cost is finite for any finite cyclic order-up-to policy under certain non-restrictive conditions. Then, in Section 3.3.1 we characterize the conditions for cyclic order-up-to policy to be optimal under the discounted cost criterion, and prove that a cyclic order-up-to policy is also optimal under average cost criterion (Section 3.3.2). Next, extensive computational study is conducted in Section 3.4, using the **Infinitesimal Perturbation Analysis** (IPA), to characterize the effects of frequency and timing of information sharing. Finally, we conclude the chapter with a discussion of our results and contributions in Section 3.5.

## 3.2 Properties of cyclic order-up-to policy

Consider the information sharing problem with the cost function $r_i(x, y) \sim O(|x|^\rho) + O(|y|^\rho)$, where $\rho$ is a positive integer. Define a cyclic order-up-to policy as a policy with different order-up-to levels for different information periods, but these levels are the same for the same information period in different ordering periods. That is, the order up to level in information period $mN + i$ is the same for all $m$ but may be different for different $i$, $i = 1, 2, \ldots, N$.

In this section we study the Markov processes associated with any cyclic order-up-to policy and identify conditions under which they are positive recurrent and have finite steady-state average cost. The conditions are similar to those identified by Aviv and Federgruen and Kapascinski and Tayur but the analysis is quite different.

Let $D_i$, $i = 1, \cdots, N$ be the random variable representing demand in information period $mN+i$ for all $m$, demand is assumed to be discrete. Consider a cyclic order-up-to policy with levels $a_1, a_2, \cdots, a_N$, and define the shortfall processes $\{s_{mN+i}, m = 0, 1, \cdots\}$ for different $i = 1, \cdots, N$ as $s_{mN+i} = a_i - y_{mN+i}$, if we are in period $mN+i$ and $y_{mN+i}$ is the inventory position at the end of this period before demand is realized. Hence the dynamics of shortfall are,

$$s_{mN+i+1} = \begin{cases} (a_{i+1} - a_i) + s_{mN+i} + D_i - C, & \text{if } (a_{i+1} - a_i) + s_{mN+i} + D_i > C, \\ 0, & \text{if } 0 \leq (a_{i+1} - a_i) + s_{mN+i} + D_i \leq C, \\ (a_{i+1} - a_i) + s_{mN+i} + D_i, & \text{if } (a_{i+1} - a_i) + s_{mN+i} + D_i < 0. \end{cases}$$

Observe that the shortfall can be negative when initial inventory is higher than the order-up-to level in this period. If excessive stock is returned when the inventory position is higher than the order-up-to level, then the dynamics of shortfall processes $\{s^r_{mN+i}, m = 0, 1, \cdots\}$ for $i = 1, \cdots, N$ is $s^r_{mN+i+1} = (a_{i+1} - a_i + s^r_{mN+i} + D_i - C)^+$. We refer to this policy as an **order-up-to policy with returns**.

Aviv and Federgruen show that if (1) $E(D_i^l) < \infty$ for all positive integers $l \leq \rho+1$ and $i = 1, \cdots, N$, (2) $E(\sum_{i=1}^N D_i) < NC$, then for any finite order-up-to policy, the shortfall process has a finite set of states such that it can be reached with finite expected cost from any starting state. Kapascinski and Tayur prove that in steady state, $E(|x_i|^\rho)$, $E(|s_i|^\rho)$ are finite for order-up-to zero policy under similar conditions as those proved by Aviv and Federgruen, namely (1) $E(D_i^{2\rho+2}) < \infty$ for $i = 1, \cdots, N$, (2) $E(\sum_{i=1}^N D_i) < NC$.

Using a new and simpler method based on Foster's criterion and one of its extension, we will prove that $\{x_{mN+i}, m = 0, 1, \cdots\}$ (defined to be the inventory positions at the beginning of $mN + i$th information period), $\{y_{mN+i}, m = 0, 1, \cdots\}$ (inventory positions at the end of $mN + i$th information period before demand realized) and $\{s_{mN+i}, m = 0, 1, \cdots\}$ (shortfalls) generated by any finite cyclic order-up-to policy give rise to Markov Chains with single irreducible and positive recurrent class and finite steady-state average cost under the same condition as Aviv and Federgruen; that is we require (1) $E(D_i^l) < \infty$ for all positive integer $l \leq \rho + 1$ and $i = 1, \cdots, N$, and (2) $E(\sum_{i=1}^{N} D_i) < NC$.

Our method is to first show that single irreducible and positive recurrent class and finite steady state average cost hold true for order-up-to a constant (zero) policy under these conditions. We then extend the results to any finite order-up-to policy with returns. Finally, we show that properties such as positive recurrence and finite steady-state average cost can be transferred from a system using a cyclic order-up-to policy with returns to a similar system without returns.

We begin our analysis by presenting new proofs for the positive recurrence and finite steady-state average cost of order-up-to zero policy. To simplify the analysis, let's assume $Pr(D_i < 0) = 0, \forall i$.

**Proposition 3.1** *Given an order-up-to zero policy, the inventory positions* $\{x_{mN+i}, m = 0, 1, \cdots\}$, $\{y_{mN+i}, m = 0, 1, \cdots\}$ *and the shortfall process* $\{s_{mN+i}, m = 0, 1, \cdots\}$ *for* $i = 1, \cdots, N$, *generate Discrete Time Markov Chains (DTMC) with single irreducible and positive recurrent class if* $\sum_{i=1}^{N} ED_i < NC$.

*Proof:* See Section 5.3 for details. ∎

The proof that the steady-state average cost associated with an order-up-to zero policy is finite is based on the following generalization of Foster's criterion.

**Lemma 3.1 A generalization of Foster's Criterion** *Consider an irreducible and aperiodic Markov chain* $\{X_n, n = 0, 1, \cdots\}$ *with a single period cost function* $r(\cdot)$*, which is continuous and bounded from below. Assume there exists a potential function* $V(\cdot)$ *mapping the state space* $S$ *to* $[0, \infty)$*, and a constant* $\eta$ *such that*

$$E\{V(X_{n+1}) - V(X_n)|X_n = x\} \leq -r(x) + \eta, \forall x \in S.$$

*Then given an initial state* $x_0$ *with* $V(x_0) < \infty$*, the Markov chain* $X_n$ *has finite steady-state average cost if* $X_n$ *is positive recurrent.*

*Proof:* See Section 5.3 for details. ∎

This Lemma is a variation of the generalization of Foster's criterion by Meyn and Tweedie (1993) (see Theorem 14.0.1 (f-regularity)). As we will see later, our generalization allows us to prove the following.

**Lemma 3.2** *Consider an order-up-to zero policy. If* $E((D_i)^k) < \infty$ *for all integer* $0 \leq k \leq \rho + 1$, $\forall i$ *and* $\sum_{i=1}^{N} ED_i < NC$*, then in steady state*

*(1)* $E(|s_i|^\rho) < \infty$, $E(|x_i|^\rho) < \infty$ *and* $E(|y_i|^\rho) < \infty$ *for all* $i = 1, \cdots, N$,

*(2) for* $0 < \beta < 1$, $E(\sum_{n=0}^{\infty} \beta^n |y_n|^\rho) < \infty$ *and* $E(\sum_{n=0}^{\infty} \beta^n |x_n|^\rho) < \infty$ *for any initial inventory position* $x_0$ *and initial information period* $i$.

*Proof:* See Section 5.3 for details. ∎

Proposition 3.1 and Lemma 3.2 can be extended to any finite cyclic order-up-to policy with returns as follows,

**Proposition 3.2** *Consider any finite cyclic order-up-to policy with returns. If* $\sum_{i=1}^{N} ED_i < NC$ *and* $E((D_i)^k) \leq \infty$ *for any positive integer* $k \leq \rho + 1$ *and* $\forall i$*, then*

*(1) each shortfall process* $\{s^r_{mN+i}, m = 0, 1, \cdots\}$, $i = 1, \cdots, N$ *gives rise to a Markov chain with single irreducible and positive recurrent class,*

*(2)* $E(|x^r_i|^\rho) < \infty$ *and* $E(|y^r_i|^\rho) < \infty$ *for all* $i$*,*

*(3) for* $0 < \beta < 1$, $E(\sum_{n=0}^{\infty} \beta^n |y^r_n|^\rho) < \infty$ *and* $E(\sum_{n=0}^{\infty} \beta^n |x^r_n|^\rho) < \infty$ *for any initial finite inventory position* $x^r_0$ *and initial information period* $i$*.*

*Proof:* Assume $a_i, i = 1, \cdots, N$, to be the order-up-to levels. We only need to transform this policy to an order-up-to 0 policy, and then apply Proposition 3.1 and Lemma 3.2.

Consider $y^r_{mN+i}$, the inventory positions at the end of period $mN + i$ before demand is realized. For simplicity, we drop subscript $mN$. The system dynamics is $y^r_{i+1} = \min\{y^r_i - D_i + C, a_{i+1}\} = a_{i+1} + (y^r_i - a_{i+1} - D_i + C)^-$, where $x^- = \min\{0, x\}$. Let $z^{'}_i = y^r_i - a_i$ and $D^{'}_i = D_i + (a_{i+1} - a_i)$, then $z^{'}_{i+1} = (z^{'}_i - D^{'}_i + C)^-$ and $\sum_{i=1}^{N} D^{'}_i = \sum_{i=1}^{N} D_i < NC$. For $z^{'}_i$ and $D^{'}_i$, this is order-up-to zero policy, where demand $D^{'}_i$ can be negative, but bounded from below.

Notice that the shortfall processes associated with this order-up-to zero policy have state space $\{0, 1, 2, \cdots\}$ even if demand can be negative. Using similar proofs as those of Proposition 3.1 and Lemma 3.2, we can show that the same results hold for this order-up-to zero policy if there exists a positive constant $d_i$ so that $Pr\{D_i < -d_i\} = 0$, $\forall i = 1, 2, \cdots, N$. ∎

We are ready to study the gap between the inventory position processes without returns, $y_{mN+i}$, and with returns, $y^r_{mN+i}$, assuming they start with the same initial inventory level and face the same stream of demand.

**Proposition 3.3** *Consider two inventory systems with cyclic order-up-to levels $a_1$, ..., $a_N$, for $N \geq 2$, one system without returns and the second one with returns. If the two systems start with the same initial state $x_0 \leq \max\{a_1, \cdots, a_n\}$, and face the same stream of random demand, then the stochastic process $\{z_{mN+i} = y_{mN+i} - y^r_{mN+i}, m = 0, 1, \cdots\}$ has following properties for all $i$:*

*(1) $z_{mN+i} \geq 0$ for all $m$,*

*(2) $z_{mN+i} \leq \max\{a_1, \cdots, a_N\} - \min\{a_1, \cdots, a_N\}$ for all $m$.*

*Proof:* The proof is by induction. Clearly, $z_1 = 0$. Consider period $mN + i$ and assume $0 \leq z_{mN+i} \leq \max\{a_1, \cdots, a_N\} - \min\{a_1, \cdots, a_N\}$. We distinguish between the following two cases. In the first case, $a_i \leq a_{i+1}$ and in the second case $a_i > a_{i+1}$.

$\underline{a_i \leq a_{i+1}}$: There are two sub cases to consider. (1) $x_{mN+i+1}$ and $x^r_{mN+i+1}$ are no larger than $a_{i+1}$. In this case $y_{mN+i+1} = \min\{x_{mN+i+1} + c, a_{i+1}\}$ and $y^r_{mN+i+1} = \min\{x^r_{mN+i+1} + c, a_{i+1}\}$ and hence

$$0 \leq y_{mN+i+1} - y^r_{mN+i+1} \leq x_{mN+i+1} - x^r_{mN+i+1} = y_{mN+i} - y^r_{mN+i}.$$

(2) $x_{mN+i+1} > a_{i+1} \geq x^r_{mN+i+1}$. This can only occur if $N > 2$, and $a_{i+1} < \max\{a_1, \cdots, a_N\}$. Clearly, $y_{mN+i+1} = x_{mN+i+1}$ and $y^r_{mN+i+1} = \min\{x^r_{mN+i+1} + c, a_{i+1}\}$ which implies that

$$0 \leq y_{mN+i+1} - y^r_{mN+i+1} \leq x_{mN+i+1} - x^r_{mN+i+1} = y_{mN+i} - y^r_{mN+i}.$$

$\underline{a_i > a_{i+1}}$: In this case there are three possible sub cases. (1) $x_{mN+i+1}$ and $x^r_{mN+i+1}$ are no larger than $a_{i+1}$. (2) $x_{mN+i+1} > a_{i+1} \geq x^r_{mN+i+1}$. (3) $x_{mN+i+1}$ and $x^r_{mN+i+1}$ are larger than $a_{i+1}$. The proof of the first two sub cases is identical to the proof in the previous case. Consider (3) and observe that in this case

$y_{mN+i+1} = x_{mN+i+1}$ and $y^r_{mN+i+1} = a_{i+1}$ and hence

$$0 \le y_{mN+i+1} - y^r_{mN+i+1} = x_{mN+i+1} - a_{i+1} \le \max\{a_1, \cdots, a_N\} - \min\{a_1, \cdots, a_N\}.$$

∎

**Remark:** If $x_0 > \max\{a_1, \cdots, a_N\}$, then the state space for $\{z_{mN+i}, m = 0, 1, \cdots\}$ is $\{0, 1, \cdots, x_0 - \min\{a_1, \cdots, a_N\}\}$, $\forall i$.

**Lemma 3.3** *Consider two arbitrary irreducible DTMCs $\{x_n, n = 0, 1, \cdots\}$ and $\{y_n, n = 0, 1, \cdots\}$ starting with same initial state. If their difference process $\{z_n = x_n - y_n, n = 0, 1, \cdots\}$ has finite state space $S_z$, and if $x_n$ is positive recurrent and has certain finite steady-state moments, then $y_n$ is also positive recurrent and the same steady state moments of $y_n$ are finite.*

*Proof:* First, we show that positive recurrence is transferable from $y_n$ to $x_n$. Define $S_x, S_y$ to be the state space for $\{x_n, n = 0, 1, \cdots\}$ and $\{y_n, n = 0, 1, \cdots\}$. Assume $y_n$ is positive recurrent, using contradiction, we assume $x_n$ is transient or non-recurrent for all of its states. Since $y_n = x_n - z_n$ and $y_0 = x_0$, we have

$$\begin{array}{rcl} Pr\{y_n = i|y_0\} & = & Pr\{y_n = i|x_0\} \\ & = & \sum_{k \in S_z} Pr\{x_n = i + k, z_n = k|x_0\} \\ & = & \sum_{k \in S_z} Pr\{z_n = k|x_n = i + k, x_0\} Pr\{x_n = i + k|x_0\} \\ & \le & \sum_{k \in S_z} Pr\{x_n = i + k|x_0\}. \end{array}$$

Since $S_z$ has only a finite number of states, and $x_n$ is transient or non-recurrent, so $Pr\{y_n = i|y_0\} \to 0$ as $n \to \infty$ for $i \in S_y$. This contradicts to the assumption that $y_n$ is positive recurrent (see Kulkarni pg 80 Theorem 3.4, and Kemeny, Snell and Knapp 1966 pg 36 Proposition 1-61).

Second, assume $y_n$ has finite steady state moments $E(|y|^l)$ for $0 < l \le \rho$, where $\rho$ is a positive integer. Consider

$$\begin{array}{rl} & \sum_{i\in S_x} |i|^\rho Pr\{x_n = i|x_0\} \\ = & \sum_{i\in S_x} |i|^\rho \sum_{k\in S_z} Pr\{y_n = i-k, z_n = k|x_0\} \\ = & \sum_{i\in S_x} |i|^\rho \sum_{k\in S_z} Pr\{z_n = k|y_n = i-k, y_0\} Pr\{y_n = i-k|y_0\} \\ \leq & \sum_{k\in S_z} \sum_{i\in S_x} |i-k+k|^\rho Pr\{y_n = i-k|y_0\}, \end{array}$$

Taking the limit $n \to \infty$ on both side, we obtain

$$E(|x|^\rho) \leq |S_z|(E(|y|^\rho) + c_1 E(|y|^{\rho-1}) + \cdots + c_\rho) < \infty.$$

where $c_1, \cdots, c_\rho$ are positive finite constants, and $|S_z|$ is the size of the state space for $z_n$. ∎

This Lemma is quite intuitive; it provides a method to simplify Markov chains with infinite state space and complicated dynamics. A consequence of Lemma 3.3, Proposition 3.3 and 3.2 is,

**Proposition 3.4** *Consider any finite cyclic order-up-to policy $\sigma$. If $\sum_{i=1}^N ED_i < NC$, and $E((D_i)^k) \leq \infty$ for all $i$ and positive integers $k$ such that $k \leq \rho + 1$, then*

*(1) each shortfall process $\{s_{mN+i}, m = 0, 1, \cdots\}$, $i = 1, \cdots, N$ gives rise to a Markov chain with single irreducible and positive recurrent class,*

*(2) $E(|x_i|^\rho) < \infty$ and $E(|y_i|^\rho) < \infty$ for all $i$.*

*(3) For $0 < \beta < 1$, $E(\sum_{n=0}^\infty \beta^n |x_n|^\rho) < \infty$ and $E(\sum_{n=0}^\infty \beta^n |y_n|^\rho) < \infty$ for any finite initial inventory position $x_0$ and initial information period $i$.*

*(4) For $0 < \beta < 1$, we have*

$$U_i^\beta(x, \sigma) = E(\lim_{M\to\infty} \sum_{k=i}^{MN} \beta^{k-i} g_k(x_k, \sigma_k(x_k))|x_i = x) < \infty.$$

The next step is to show that the long-run average cost convergences to a finite value.

**Proposition 3.5** *Consider the information sharing model, if* $\sum_{i=1}^{N} ED_i < NC$*, and* $E((D_i)^l) < \infty$ *for any positive integer* $l \leq \rho + 1$ *and all* $i$*. Using any finite cyclic order-up-to policy, we have*

$$lim_{M\to\infty}\frac{E\{\sum_{k=i}^{MN} g_k(x_k, y_k)|x_i = x\}}{MN - i + 1}$$

*converges to a finite value independent of initial period* $i$ *and initial state* $x$*.*

*Proof:* From Proposition 3.4, we have that in steady state $\sum_{i=1}^{N} E(r_i(x, y)) < \infty$ due to the assumption that $r_i(x, y) \sim O(|x|^\rho) + O(|y|^\rho)$. The proposition also implies that the shortfall processes and thus the inventory positions before and after demand arrives, $y_{mN+i}$ and $x_{mN+i+1}$, give rise to Markov chains with single irreducible and positive recurrent class. Without loss of generality, assume that the Markov chains are ergodic (Kulkarni 1995, Theorem 3.16). Let the steady state distribution for $(x, y)$ in the $i$th information period be $p^i_{(x,y)}$, where $(x, y) \in \Omega^i$ (the feasible region of $(x, y)$), then $E(r_i(x, y)) = \sum_{(x,y)\in\Omega^i} p^i_{(x,y)} r_i(x, y)$ must converge since the summation is over at most countable number of positive values and it is bounded from above. Finally, applying the Proposition 1-61 (arithmetic average) of Kemeny, Snell and Knapp, the long-run average cost converges and equals to the steady state average cost. ∎

## 3.3 A Markov decision process

Our objective in this section is to discuss the discounted and average cost criterion and present optimal policies for the information sharing model. For discounted cost criterion, we will mainly follow the ideas of Heyman and Sobel; while for average cost criterion, we will take the advantage of the special cost structure of the problem, and develop a simple proof based on the idea of Blackwell optimal policy.

### 3.3.1 Discounted cost criterion

In this subsection, we present conditions for cyclic order-up-to policy to be optimal under discounted cost criterion. These conditions are due to Heyman and Sobel (1984) and Federgruen and Zipkin (1986b). Their variations are discussed in Aviv and Federgruen (1997) or Kapuscinski and Tayur (1998).

It is well known that for infinite horizon problems, if single-period cost functions are bounded from below, then the optimal cost function $u_i^\beta(x)$ (see Section 3.1 for definition) satisfies the following Bellman's equation,

$$u_i^\beta(x) = \min_{x \le y \le x+C}\{r_i(x,y) + \beta E(u_{i-1}^\beta(y - D_i))\},$$

for $i = 1, 2, \ldots, N$, where subscript 0 refers to $N$. A finite horizon dynamic programming provides a successive approximation method to find the optimal policy (Bertsekas 1987).

We index periods in a reverse order starting at the end of the planning horizon. Let $m = 0$ be the last ordering period while $m = M - 1$ be the first ordering period. We set $i = N$ for the first information period and 1 for the last information period in any ordering period. Thus, period $mN + i$ represents the $i^{th}$ information period in the $m - 1^{th}$ ordering period. Finally, let $g_{mN+i}(x, y) = r_i(x, y)$ denote the single period expected cost in this information period. We refer to this indexing as backward index, and we will only use backward index in this subsection.

Let $U_{mN+i}^\beta(x)$ be the minimum expected total costs if there are $mN + i$ periods remaining in the planning horizon, starting with an initial state $x$. Let the salvage cost $U_0^\beta \equiv 0$, and hence,

$$\begin{aligned} U^\beta_{mN+i}(x) &= Min_{y\in A_x}\{g_{mN+i}(x,y)+\beta E(U^\beta_{mN+i-1}(y-D_i))\} \\ &= Min_{y\in A_x}\{r_i(x,y)+\beta E(U^\beta_{mN+i-1}(y-D_i))\} \end{aligned}$$

where $E(\cdot)$ is the expectation with respect to $D_i$. Since $r_i(x,y)$ can be written as the sum of a function of $x$, $(\phi_i(x))$ and a function of $y$, $(\varphi_i(y))$ where

$$\phi_i(x) = \begin{cases} -cx, & i=1 \\ -(c+h_{i-1})x, & otherwise, \end{cases}$$

$$\varphi_i(y) = \begin{cases} cy+E(L(y,D_1)), & i=1 \\ (c+h_{i-1})y, & otherwise, \end{cases}$$

Thus, the following recursion must hold.

$$\begin{aligned} U^\beta_{mN+i}(x) &= \phi_i(x)+V^\beta_{mN+i}(x) \\ V^\beta_{mN+i}(x) &= Min_{x\le y\le x+C}\{J^\beta_{mN+i}(y)\} \\ J^\beta_{mN+i}(y) &= \varphi_i(y)+\beta E(U^\beta_{mN+i-1}(y-D_i)). \end{aligned} \tag{1}$$

Observe that in the very first information period (i.e., information period $(M-1)N+N=MN$) of the entire planning horizon, we have to add $h_N x^+$ to $U^\beta_{MN}(x)$ to account for the holding cost of initial inventory.

The dynamic programming model has the following properties:

(a) Cost function $r_i(x,y)$ for each information period $i=1,\cdots,N$ is positive and convex in $y$. Thus, the following property, proved in Kapuscinski and Tayur (1998), holds, $U^\beta_{mN+i}(x) \geq U^\beta_{(m-1)N+i}(x)$ for any $x$, $m=1,\cdots,M$ and $i=1,\cdots,N$.

(b) $r_i(x,y)=\phi_i(x)+\varphi_i(y)$ for all $i$, and there exists a positive integer $\rho$ so that $\phi_i(x)\sim O|x|^\rho$ and $\varphi_i(y)\sim O|y|^\rho$. From Proposition 3.4, the total expected discounted cost using cyclic order-up-to policy is finite if $\sum_{i=1}^N ED_i < NC$, and $E((D_i)^l) < \infty$ for any positive integer $l \leq \rho+1$, $\forall i$. Which implies $U^\beta_{mN+i}(x)$ converge point-wise to a finite value for any finite $x$ and for all $i$ (Heyman and Sobel Theorem 8-13). Let $U^\beta_i(x)$ denote the convergence point of $U^\beta_{mN+i}(x)$.

(c) Obviously, $A_x = [x, x+C]$ is convex and finite for all $x$ if we consider denumerable state space. Thus, following the proof of Heyman and Sobel Theorem 8-14, we obtain,

**Proposition 3.6** *$U_i^\beta(x)$ satisfies the Bellman's equation*

$$U_i^\beta(x) = Min_{y \in A_x}\{r_i(x,y) + \beta E(U_{i-1}^\beta(\xi(x,y,D_i)))\}$$

*for $i = 1, \cdots, N$, where subscript 0 refers to $N$.*

The next property of the dynamic programming model is:

(d) $J_{mN+i}^\beta(y)$ is convex in $y$, for all $m, i$.

The main difficulty in proving that an order-up-to policy is optimal is that the function $\varphi_i(y)$ maybe unbounded from both above and below for some time periods. To overcome this difficulty, we need to aggregate $N$ consecutive information periods and identify conditions under which the cost function for all these periods tends to positive infinity as action variables approach either positive or negative infinity.

For this purpose, we re-arrange Equation (1) to get,

$$\begin{aligned} U_{mN+i}^\beta(x) &= Min_{y \in A_x}\{r_i(x,y) + \beta E(U_{mN+i-1}^\beta(y - D_i))\} \\ &= Min_{y \in A_x}\{\phi_i(x) + \varphi_i(y) + \beta E(U_{mN+i-1}^\beta(y - D_i))\} \\ &= \phi_i(x) + \min_{y \in A_x}\{J_{mN+i}^\beta(y)\} \end{aligned}$$

where $J_{mN+i}^\beta(y) = \varphi_i(y) + \beta E(U_{mN+i-1}^\beta(y - D_i))$. Hence,

$$J_{mN+i}^\beta(y) = w_i(y) + \beta E\{Min_{y' \in B_y^i} J_{mN+i-1}^\beta(y')\}, \ \forall i$$

where $B_y^i = [y - D_i, y - D_i + C]$,

$$w_i(y) = \begin{cases} (1-\beta)cy + (h_{i-1} - \beta h_{i-2})y + \beta(c + h_{i-2})ED_i, & i \neq 1, \\ (1-\beta)cy + E(L(y, D_i)) - \beta h_{N-1}y + \beta(c + h_{N-1})ED_i, & i = 1. \end{cases}$$

Aggregate $N$ consecutive information periods starting from the $i^{th}$ information period in one ordering period until the $(i+1)^{th}$ information period in the next ordering period, where $i = N-1, N-2, \cdots, 1$. When $i = N$, the aggregation starts from the $N^{th}$ information period in one ordering period until the $1^{st}$ information period in the same ordering period. We use here a cyclic order so that all indices $j$ refer to the $i^{th}$ information period if $j \ mod \ N = i$. Define

$$\begin{aligned} W_i(y_i) \;\; &= \;\; w_i(y_i) + \beta E\{Min_{y_{i-1} \in B^i_{y_i}} w_{i-1}(y_{i-1}) + \beta E\{Min_{y_{i-2} \in B^{i-1}_{y_{i-1}}} w_{i-2}(y_{i-2}) \\ &\quad + \cdots + \beta E\{Min_{y_{i-N+1} \in B^{i-N+2}_{y_{i-N+2}}} w_{i-N+1}(y_{i-N+1})\} \cdots\}\}, \end{aligned}$$

which is the total cost in terms of $y_i$ for these $N$ information periods. Following the same analysis as in Chapter 5 Section 5.2, we find that if $\beta^{i-1}\pi > c + h_{i-1}$, $W_i(y) \to +\infty$ as $|y| \to +\infty$, for $i = 1, 2, \cdots, N$. Finally, if $\beta^{N-1}\pi > c + h_{N-1}$, $W_i(y) \to +\infty$ as $|y| \to +\infty$ for all $i = 1, 2, \cdots, N$, and thus $lim_{|y| \to +\infty} J_{mN+i}(y) \to +\infty$ for all $m$ and $i$.

From these analysis, we have the following results,

**Theorem 3.1** *For Markov decision process defined in Equation (1), if*

*(a)* $\sum_{i=1}^{N} ED_i < NC$, *and* $E((D_i)^l) < +\infty$ *for any positive integer* $l \leq \rho + 1$, $\forall i$,

*(b)* $\beta^{N-1}\pi > c + h_{N-1}$,

*then*

*(1) Order-up-to policy is optimal for any* $m$ *and* $i$,

*(2) Optimal order-up-to levels* $y^*_{mN+i}$ *are bounded as* $m \to +\infty$,

*(3)* $J_{mN+i}(y)$ *convergences to* $J_i(y)$ *for all* $y, i$, *and every limit of* $y^*_{mN+i}$ *is a minimal point for* $J_i(x)$.

*(4) Cyclic order-up-to policy is optimal under discounted cost criterion.*

*Proof:* Since $J^{\beta}_{mN+i}(y)$ is a convex function of $y$, and $lim_{|y|\to+\infty} J^{\beta}_{mN+i}(y) \to +\infty$ for all $m$ and $i$, the order-up-to policy is optimal for all $m$ and $i$. Since $U^{\beta}_{mN+i}(x)$ is bounded from above for any $m$ and finite $x$, order-up-to levels are finite as $m \to +\infty$ because of (b) (see the proof of Theorem 2 in Aviv and Federgruen). Notice that $U^{\beta}_{mN+i}(x)$ is nondecreasing and converges to $U^{\beta}_{i}(x)$, which implies $J^{\beta}_{mN+i}(y)$ is nondecreasing and converges to, say $J^{\beta}_{i}(y)$, due to the Monotone Convergence Theorem. Hence, (3) is true (see the proof of Theorem 2 in Aviv and Federgruen). Finally, (1), (2) and (3) implies that cyclic order-up-to policy is optimal under the discounted cost criterion. ∎

For the capacitated inventory system in which $\varphi_i(y), \forall i$ is unbounded from above, but bounded from below, Aviv and Federgruen (1997) find that under certain condition, the optimal order-up-to levels under the discounted cost criterion are uniformly bounded for all $0 < \epsilon \leq \beta < 1$. Here we extend their result to the case where $\varphi_i(y)$ is unbounded from both above and below for some $i$.

**Proposition 3.7** *For the Markov decision process defined by Equation (1), if the conditions of Theorem 3.1 are satisfied, then the optimal order-up-to levels* $y^*_i, i = 1, 2, \ldots, N$ *under the discounted cost criterion are uniformly bounded both from above and from below for any* $0 < \epsilon \leq \beta < 1$*. Furthermore, the bounds are independent of* $\beta$*.*

*Proof:* see Section 5.3 for details. ∎

As we shall see in the next subsection, this property allows us to extend the optimality of cyclic order-up-to policy from discounted cost criterion to average cost criterion in a simple way.

### 3.3.2 Average cost criterion

The following lemma applies the varnishing discount method to show sufficient conditions for the optimality of cyclic order-up-to policy under average cost criterion.

**Theorem 3.2** *Consider the information sharing model and suppose:*

*(a) Cyclic order-up-to policy is optimal under the discounted cost criterion,*

*(b) For all $0 < \epsilon \leq \beta < 1$, the optimal order-up-to levels under discounted cost criterion are uniformly bounded both from above and from below,*

*(c) The long-run average cost of any finite cyclic order-up-to policy converges to a finite value,*

*then cyclic order-up-to policy is optimal under the average cost criterion.*

*Proof:* Consider a sequence of $\beta_1, \beta_2, \cdots, \beta_j, \cdots \uparrow 1$ as $j \to \infty$. Since the optimal order-up-to levels under the discounted cost criterion are uniformly bounded from both above and below for all $0 < \epsilon \leq \beta < 1$, there must exist an finite cyclic order-up-to policy $f$ and a subsequence $j_n \to \infty$, so that $f$ is the optimal policy for all $U_i^{\beta_{j_n}}(x)$, $\forall i = 1, 2, \ldots, N$.

Since we can show that for any finite cyclic order-up-to policy $\sigma$,

$$G_i(x, \sigma) = lim_{M \to \infty} \frac{E\{\sum_{k=i}^{MN} g_k(x_k, \sigma_k(x_k)) | x_i = x\}}{MN - i + 1}$$

converges to a finite value which is independent of the initial period $i$ and initial state $x$ (see Proposition 3.5), then by Tauberian Theory (Heyman and Sobel, pg 172),

$$\begin{aligned} G_i(x, f) &= lim_{\beta_{j_n} \uparrow 1} (1 - \beta_{j_n}) U_i^{\beta_{j_n}}(x) \\ &\leq lim_{\beta_j \uparrow 1} sup((1 - \beta_j) U_i^{\beta_j}(x)) \\ &\leq G_i(x, \delta), \ \forall \delta \in \Pi; x, i \end{aligned}$$

The last inequality is justified by the Lemma A2 of Sennott (1989). ∎

Finally, we combine Theorem 3.2, Theorem 3.1, Proposition 3.7 and 3.5, to get

**Proposition 3.8** *In the information sharing model, if*

*(a)* $\sum_{i=1}^{N} ED_i < NC$, *and* $E((D_i)^l) < +\infty$ *for any positive integer* $l \leq \rho + 1, \forall i$,

*(b)* $\beta^{N-1}\pi > c + h_{N-1}$,

*then cyclic order-up-to policy is optimal under the average cost criterion.*

## 3.4 Computational results

In this section, we report on an extensive computational study conducted to develop insights about the benefits of information sharing. Our goal is to determine situations where information sharing provides significant cost savings compared to supply chains with no information sharing. In what follows, we determine the optimal order-up-to levels under average cost criterion.

We examine the cases with variation on the following parameters: production capacity, the number of information periods in one ordering period, and the time when information is shared. In all the numerical studies, we set the production cost $c = 0$ and focus on holding and penalty costs. The inventory holding cost per ordering period is set to be a constant 4 $ per unit product for all cases. Thus, the inventory holding cost per information period is $4/N$ where $N$ is the number of information periods within one ordering period. Finally, the initial inventory position, $x$, at the beginning of the first ordering period is set to be zero without loss of generality.

### 3.4.1 Computational method

To compute the optimal order-up-to levels and cost, we apply the Infinitesimal Perturbation Analysis (IPA) (Fu 1994 and Glasserman and Tayur 1995) for the production-inventory systems analyzed in this chapter.

The Model with Information Sharing

For capacitated, multi-echelon production-inventory systems with linear ordering cost, Glasserman and Tayur (1995) develop the estimators for the derivatives of cost function with respect to order-up-to levels, and prove these estimators converge to the correct value for the finite horizon problems and infinite horizon problems under discounted and average cost criteria. Since our models are special cases of their general systems, their convergence results apply to our models. More detailed description of the sample path derivatives, method validation and simulation are included in Section 5.4.

The Model with No Information Sharing

In the model with no information sharing, the retailer only places orders at the end of each ordering period to the manufacturer without transferring demand information anytime during the ordering period. Since the retailer uses an order-up-to policy with constant order-up-to level, the order placed by the retailer is equal to the total demand in one ordering period. Thus, we assume that the manufacturer knows the demand distribution in one ordering period. To make this model comparable to the model with information sharing, we assume that the manufacturer also charges inventory holding cost per information period.

The model with no information sharing can be considered as a special case of the information sharing model. Indeed, consider an instance of the model with no information and construct an information sharing model in which demand in every information period within an ordering period is exactly zero except in the last information period. Demand in this information period equals total demand during that ordering period. This information sharing model has the same dynamic programming formulation as the model with no information sharing. Thus, the dynamic program designed to solve the information sharing model can be applied to solve the model in which information is not shared. Finally, cyclic order-up-to policy is optimal for the model with no information sharing under both discounted and average cost criterion.

To find the best order-up-to levels for the model with no information sharing, we use the same computational method as in the model of information sharing. Please see Section 5.5 for more details.

### 3.4.2 The effect of capacity

To explore the benefit of information sharing, we examine in Figure 11 the percentage cost savings from information sharing relative to no information sharing as a function of the production capacity. The demand distribution of one information period are Poisson(5), Uniform(0,10) and truncated Normal(5,4). Where truncated Normal distribution is defined as follows: when the realization of the random variable is negative, we set it to be zero. For each demand distribution and each capacity level, we consider the cases where the ratio of penalty cost to holding costs in one ordering period is 4.75, and there are 4 information periods in each ordering period. The computational study reveals that as production capacity increases, the percent-

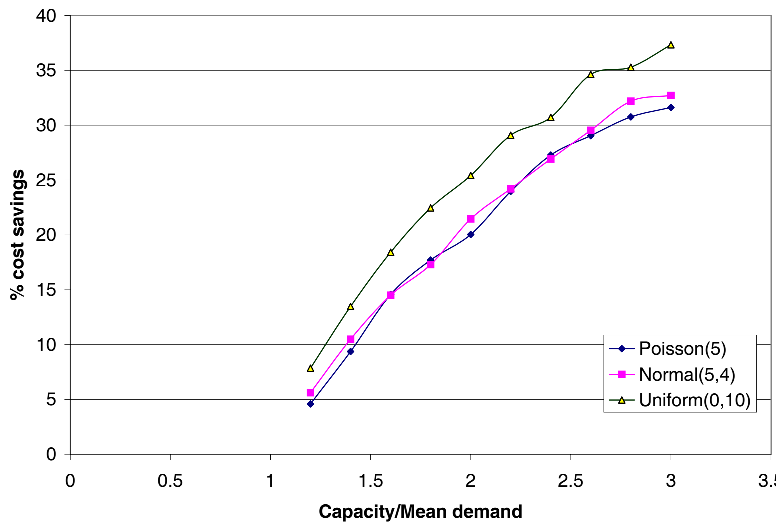


Figure 11: The impact of the production capacity

age cost saving increases. Indeed, percentage cost saving increases from about 5% to about 35% as capacity over mean demand increases from 1.2 to 3. This is quite intuitive, since as capacity increases, the optimal policy would postpone production as much as possible and take advantage of all information available prior to the time production starts. For instance, in case of infinite capacity, it is optimal to wait until the last information period and produce to satisfy all demand realized so far plus an additional amount based on solving a newsboy problem. Similarly, if the production capacity is very limited, then information is not that beneficial since production quantity is mainly determined by capacity, not based on realized demand. Finally, from fill-rate point of views, our computational study reveals that information sharing and no information sharing have almost identical fill rates.

### 3.4.3 The effect of frequency of information sharing

To understand the impact of information sharing frequency, we display in Figure 12 the percentage cost savings from information sharing as a function of the number of information periods in one ordering period, for two production capacity levels. The number of information periods, $N$, was 2,4,6 and 8 while the length of the ordering period was assumed to be constant in all cases. The demand distribution during the entire ordering period is assumed to be Poisson with parameter $\lambda = 18$, hence demand in a single information period is Poisson with parameter $\lambda/N$. Total production capacity and inventory holding cost per item in the entire ordering period are kept constant and equally divided among the different information periods. Finally, the ratio of penalty to holding costs is set to be 4.75. Figure 12 illustrates that

- As the number of information periods increases, the percentage savings increase.

- Most of the benefits from information sharing is achieved within a few information periods, e.g., 4. That is, the marginal benefit is a decreasing function of the number of information periods. Specifically, the benefit achieved by increasing the number of information periods from 4 to 8 is relatively small.

- Define the maximum potential benefit from information sharing to be the percentage cost reduction when the manufacturer has unlimited capacity, i.e. capacity equals M (see Chapter 2, Section 2.1.3). A manufacturer with production capacity twice as much as mean demand can achieve a substantial

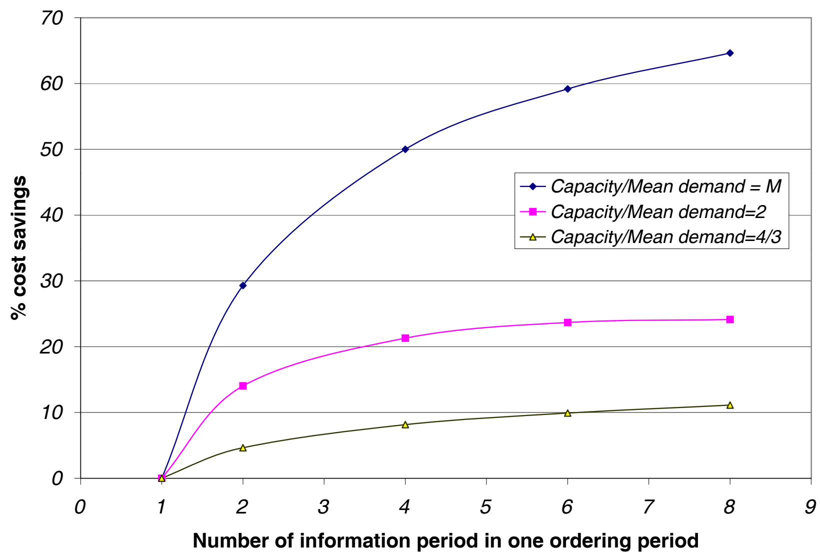


Figure 12: The impact of the frequency of information sharing

percentage of the maximum potential benefit, e.g., when the frequency of information period is 4, the manufacturer can obtain almost 50% of the benefit that a manufacturer with unlimited capacity can achieve.

### 3.4.4 Optimal timing for information sharing

In this subsection, we study the impact of the time when information is shared, given that the retailer only shares demand information once with the manufacturer in one ordering period. For this purpose, we equally divide one ordering period into 10 intervals, and compute the total cost for the manufacturer when the retailer shares demand information with her at one of these intervals. The sample path

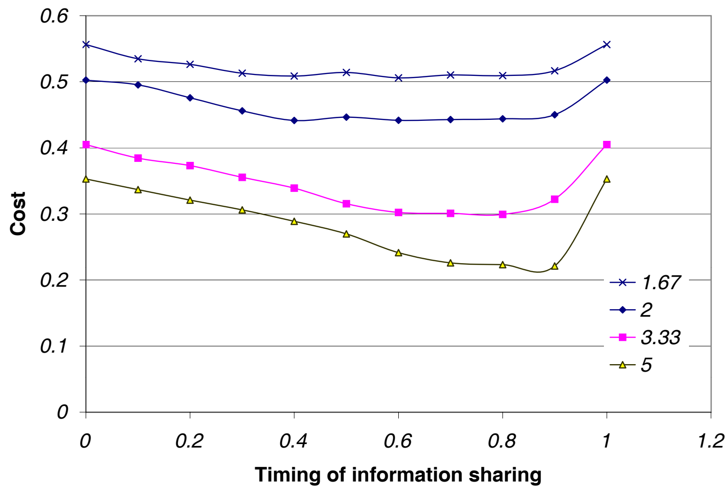


Figure 13: The impact of the timing of information sharing.

derivatives are computed in a similar way as in the model of information sharing. More details can be found in Section 5.6.

Figure 13 presents the total cost of the manufacturer as a function of the time when information is shared. The figure provides normalized cost as a function of normalized time. That is, time is normalized and is measured from 0 to 1, while cost is normalized by $h_N \cdot N \cdot ED$. Thus, 0 in $x$ coordinate implies that information is shared at the beginning of ordering period, and 1 means that information is shared at the end of the ordering period and hence can not be used. Demand distribution is assumed to be Poisson(24) and the ratio of penalty to holding costs is 4. This figure illustrates that

- As information sharing is delayed, manufacturer's cost first decreases and then increases sharply. Cost reaches its maximum when information is shared at the beginning or end of one ordering period.

- When capacity is very large relative to mean demand, e.g. capacity over mean demand equals 5, it is appropriate to postpone the time of information sharing to the last production opportunity in this ordering period, e.g., 0.9 of one ordering period. On the other hand, when capacity is relatively tightly constrained, i.e., capacity over mean demand $= 2, 1.67$, manufacturer's cost is less sensitive to the timing of information sharing. For instance, in these two cases, cost keeps almost constant when the time of information sharing varies from 0.4 to 0.9.

## 3.5 Conclusion

In this chapter, we analyzed the value of information sharing in a two-stage supply chain with a single manufacturer and a single retailer. The manufacturer has finite production capacity and she receives demand information from the retailer even during periods of time in which the retailer does not make ordering decisions. We prove that cyclic order-up-to policy is optimal under both discounted and average cost criterion.

For this purpose, we show that for any finite cyclic order-up-to policy, the associated inventory positions and shortfalls give rise to Markov chains with single irreducible, positive recurrent class and finite steady-state average cost under certain non-restrictive conditions. The proof is based on an extension of the Foster's criterion. Then, we prove that cyclic order-up-to policy is optimal under average cost criterion by showing that the optimal order-up-to levels under discounted cost criterion are uniformly bounded for $0 < \epsilon < \beta < 1$.

Using extensive computational study, we demonstrate the potential benefits of information sharing on the manufacturer's cost and service level. For instance, by using information effectively, the manufacturer can cut down inventory cost by 5-35% while maintaining or increasing service level to the retailer. One interesting observation is that the manufacturer can realize most of the benefits from information sharing if the retailer shares demand information with the manufacturer only a few times in each ordering period.

Finally, we analyze the optimal timing of information sharing. We show that if the retailer has only one opportunity to share information with the manufacturer in one ordering period, then the best timing for sharing information is in the later half of the ordering period. This is true when capacity is large relative to average demand. On the other hand, when capacity is tightly constrained, the manufacturer's cost becomes less sensitive to the time when information is shared.

# Chapter 4

# A multi-stage supply chain: forecast accuracy

In this chapter, we consider a single product distribution system with a single manufacturer, a single distribution center and multiple retailers in infinite time horizon. The retailers place orders periodically and use order-up-to policy to control their inventory. The distribution center serves as a cross docking point and transfers the aggregated orders from the retailers to the manufacturer. Assuming stationary and correlated external demands, we analyze the following two cases: In the first case, which we refer to as *no information sharing*, the manufacturer only receives the aggregated orders from the distribution center. In the second case, which we refer to as *information sharing*, the manufacturer not only receives the aggregated orders, but also the order and demand information of individual retailers. The objective of this chapter is to understand the impact of information sharing on the manufacturer's forecast accuracy.

Recently, a number of papers have explored the benefits of information sharing in distribution systems facing stationary and correlated demand. The results are mixed. As we reviewed in Chapter 1, Lee, So and Tang (2000) show that sharing the retailer's demand information will bring substantial benefits for the manufacturer, and the benefits increase along with demand correlation. On the other hand, Raghunathan (2001) shows that by intelligently analyzing the order history, the manufacturer can retrieve all the demand information, and thus sharing information provides no benefit to the manufacturer when external demand is stationary. Both works focus on a system with a single manufacturer and a single retailer.

Of course, in practice, distribution systems often have multiple retailers, and the aggregated order of all retailers generally cannot be decomposed into orders of individual retailers. Thus, the manufacturer cannot retrieve the exact demand information of all retailers by analyzing the aggregated order history. This raises an important question: Can individual retailer's demand and order information help the manufacturer improve its forecast of future aggregated orders received from the DCs?

To answer this question, we consider a general distribution system with a single manufacturer, a single distribution center and multiple retailers facing correlated external demand. Our objective is to quantify the impact of sharing demand and order information of each individual retailers on the manufacturer's forecast accuracy. We choose to focus on forecast accuracy instead of inventory cost since forecast accuracy is directly related to inventory cost. As Aviv (2001) points out, inventory cost is almost proportional to the forecast error. Thus, reducing forecast error is as important as reducing demand variation.

As far as we are aware of, Aviv (2002) is the only work on forecast accuracy involving a single supplier, multi-retailer system with correlated demand. The focus is on the value of sharing market signals, i.e., unexpected random events such as promotions, part of which maybe observable by different parties, between the supplier and the retailers.

This chapter is organized as follows: we describe the model in detail in Section 4.1, analyze the information sharing model in Section 4.2 and the no information sharing model in Section 4.3. In Section 4.4, we present results from our numerical study. The chapter is concluded in Section 4.5.

## 4.1 The model

Consider a supply chain with a single manufacturer receiving orders from a single distribution center (DC). The DC serves a set $I$ of non-identical retailers. The retailers review their inventory status periodically, and place orders according to an order-up-to inventory policy. The DC does not hold inventory, but serves as a cross-docking point between the manufacturer and the retailers. Transportation lead-time, $L$, between the DC and the manufacturer is assumed to be constant. Since in practice DCs are often close to retailers, we assume that the transportation lead-time between the DC and the retailers is negligible.

Define the time between two consecutive orders to be ordering period. We assume that all the retailers have the same ordering period and order at the same time interval. Without loss of generality, we assume that the transportation lead time $L$ is an integer multiple of an ordering period. Following Lee, So and Tang, we assume that the retailers can return excess inventory for free, and all unsatisfied demand at the retailers are backlogged. Throughout this chapter, we assume no communication among the manufacturer, DC and the retailers within any ordering period, thus the manufacturer cannot update its forecast during any ordering period.

Define the discrete times at which the retailers place orders to be $t_n$, and the external demand faced by retailer $i \in I$ in the $n^{th}$ ordering period (between $t_n$ and $t_{n+1}$) to be $D_n^i$. At the beginning of the $n+1^{th}$ ordering period, $t_{n+1}$, demand $D_n^i$ is realized, the retailer makes a forecast $F_{n+1}^{R,i}$ for future demands and places an order $q_{n+1}^i$ to the DC. Upon receiving the orders, the DC transfers the aggregated order $Q_{n+1} = \sum_{i \in I} q_{n+1}^i$ to the manufacturer. At $t_{n+1}$, the manufacturer receives the order $Q_{n+1}$ from the DC in no time and fills the order from on-hand inventory. If the manufacturer cannot satisfy the entire order, then the DC can receive the missing part from an alternative source (we point out that the same assumption is made by Lee, So and Tang). Finally, the manufacturer makes a forecast $F_{n+1}^M$ for the aggregated retailers' order, $Q_{n+2}$, in next time period.

Our objective is to analyze the benefits of sharing demand information of individual retailers to the manufacturer in infinite time horizon, assuming that external demands at all retailers follow stationary, but retailer dependent, processes.

Notice that in order to satisfy the DC order, $Q_{n+1}$ at $t_{n+1}$, the manufacturer has to make a forecast $F_n^M$ at time $t_n$. The manufacturer's ability of matching demand with supply is determined by how good the forecast $F_n^M$ is compared to $Q_{n+1}$. We realize that the manufacturer may need to forecast the quantity $Q_{n+1}$ well in advance because of production capacity constraints. Here we choose to focus exclusively on the most updated forecast $F_n^M$. Since no information is transferred during an ordering period, one key problem the manufacturer facing at $t_n$ is that she has to forecast the DC's order (the aggregated retailers' orders) at $t_{n+1}$ without knowing the demand $D_n^i$ during the $n^{th}$ time period. This demand information is critical for the retailers to determine their orders $q_{n+1}^i$ and hence $Q_{n+1} = \sum_{i \in I} q_{n+1}^i$ at

$t_{n+1}$. Thus, even the most effective utilization of the demand and order information at each individual store will not enable the manufacturer to forecast the future orders from the retailers without error.

To simply the analysis, we assume that the demand faced by the $i^{th} \in I$ retailer follows an $AR(1)$ process with parameters $a_i, \rho_i$ and $\delta_i, i \in I$ as is assumed in the papers of Lee, Padmanabhan and Whang (1997a, b) and Chen, Drezner, Ryan and Simchi-Levi (1999). Lee, So and Tang (2001) point out that demand is often positively correlated in time, thus, we assume that $\rho_i \geq 0, \forall i \in I$. Finally, the external demands are independent across different retailers.

Focus on the $i^{th}$ retailer, and omit the index $i$ without causing any confusion.

$$D_{n+1} = a + \rho D_n + \varepsilon_{n+1},$$

where $\varepsilon_{n+1} \sim N(0, \delta^2)$ are i.i.d Normal random variables, and $a, \rho, \delta$ are positive constants. It's easy to show that $E(D) = \dfrac{a}{1-\rho}$ and $Var(D) = \dfrac{\delta^2}{1-\rho^2}$. Under the infinite time horizon assumption, the retailer must know exactly the demand process (see also Lee, So and Tang for a similar assumption). Thus, at time $t_{n+1}$ the retailer knows the last period demand, $D_n$, and has to forecast the demand for the time interval $[t_{n+1}, t_{n+1+L}]$ to cover for transportation lead time. The retailer's forecast for period $n+1+l, l = 0, \ldots, L$ made at time $t_{n+1}$ is

$$D'_{n+1+l} = a(1 + \rho + \cdots + \rho^l) + \rho^{l+1} D_n.$$

The retailer has to plan for the next $L+1$ periods, and the forecast of the retailer at time $t_{n+1}$ of total demand in next $L+1$ periods is

$$F^R_{n+1} = D'_{n+1} + \cdots + D'_{n+1+L}.$$

Since the retailer uses a myopic inventory policy, her order up to level is

$$y_{n+1}^R = F_{n+1}^R + s^R,$$

where $s^R$ is the retailer's safety stock, which depends on the demand uncertainty, lead time and the service level provided to external demand by the retailer.

Assuming that $s^R$ does not vary from period to period, the retailer can return excessive stock at no cost, and noticing that all unsatisfied demand at the retailer are backlogged, thus the retailer's order quantity at time $t_{n+1}$ is

$$\begin{aligned} q_{n+1} &= F_{n+1}^R - F_n^R + D_n \\ &= (D_n - D_{n-1})(\rho + \rho^2 + \cdots + \rho^{L+1}) + D_n \\ &= (b+1)D_n - bD_{n-1} \\ &= a(b+1) + (b\rho + \rho - b)D_{n-1} + (b+1)\varepsilon_n \\ &= a + \rho q_n + (b+1)\varepsilon_n - b\varepsilon_{n-1}, \end{aligned} \tag{1}$$

where $b = \rho + \rho^2 + \cdots + \rho^{L+1} = \rho\dfrac{1-\rho^{L+1}}{1-\rho}$. The mean of $q_{n+1}$ is equal to $E(D)$, while the variance of $q_{n+1}$ equals $\sigma^2 = Var(D)(1 + 2b(b+1)(1-\rho))$.

Thus, the aggregated order $Q_{n+1}$ can be written as,

$$\begin{aligned} Q_{n+1} = \textstyle\sum_{i\in I} q_{n+1}^i &= \textstyle\sum_{i\in I} a_i(b_i+1) + \sum_{i\in I}(b_i\rho_i + \rho_i - b_i)D_{n-1}^i + \sum_{i\in I}(b_i+1)\varepsilon_n^i \\ &= \textstyle\sum_{i\in I} a_i + \sum_{i\in I}\rho_i q_n^i + \sum_{i\in I}(b_i+1)\varepsilon_n^i - \sum_{i\in I} b_i\varepsilon_{n-1}^i. \end{aligned}$$

We now analyze the manufacturer's forecast. At time $t_n$, the manufacturer needs to make a forecast $F_n^M$ of the next DC's order, $Q_{n+1}$. We consider two systems depending on whether the manufacturer receives demand information from the retailers. In the case of no information sharing, the manufacturer only receives the aggregated orders from the DC. In the case of information sharing, the manufacturer not only receives the aggregated orders from the DC, but also shares the order and demand information of individual retailers.

## 4.2 Information sharing

Under information sharing, the manufacturer knows the values of $a_i, \rho_i$, for each $i \in I$ and the exact level of demand up-to the demand at the beginning of period $t_n$, $D^i_{n-1}$. Thus, the following Proposition holds

**Proposition 4.1** *Under information sharing, the minimum Mean Square Error (MSE) forecast that the manufacturer can make at time $t_n$, for the aggregated demand $Q_{n+1}$, is*

$$\overline{F}_n^M = \sum_{i \in I} a_i(b_i+1) + \sum_{i \in I}(b_i\rho_i + \rho_i - b_i)D^i_{n-1},$$

*with a mean square error equal to*

$$E((Q_{n+1} - \overline{F}_n^M)^2) = Var(Q_{n+1} - \overline{F}_n^M) = \sum_{i \in I}(b_i+1)^2\delta_i^2.$$

*Proof:* To show that $\overline{F}_n^M$ is the minimum MSE forecast that the manufacturer can make at $t_n$, consider any forecast $F_n^M$,

$$\begin{aligned} E((Q_{n+1} - F_n^M)^2) &= Var(Q_{n+1} - F_n^M) + E^2(Q_{n+1} - F_n^M) \\ &\geq Var(Q_{n+1} - F_n^M) \\ &= Var(\textstyle\sum_{i \in I}(b_i\rho_i + \rho_i - b_i)D^i_{n-1} - F_n^M) + \sum_{i \in I}(b_i+1)^2\delta_i^2. \end{aligned}$$

The second equality comes from the fact that $\varepsilon_n^i$ is independent of $F_n^M$. This is true, since $F_n^M$ depends only on historical data and not on $\varepsilon_n^i$.

Finally, it is easily seen that the $\overline{F}_n^M$ is an unbiased estimator of $Q_{n+1}$, and hence

$$E((Q_{n+1} - \overline{F}_n^M)^2) = Var(Q_{n+1} - \overline{F}_n^M).$$

■

## 4.3 No information sharing

At time $t_n$, the manufacturer only has the historical aggregated order information $Q_n, Q_{n-1}, \cdots$. If $\rho_i = \rho$ for all $i \in I$, then we can treat all retailers as one, and utilize the result of Raghunathan for the corresponding single manufacturer and single retailer model. In this case, Raghunathan observes that information provides no value to the manufacturer since the manufacturer can use the order history to retrieve all the demand information and hence reduce forecast error.

In our model, of course, the situation is more complex since the $\rho_i$ are retailer dependent. In this case, it is not possible for the manufacturer to decompose the aggregated order information $Q_n, Q_{n-1}, \cdots$ into the individual orders placed by each retailer, $q_n^i$, $i \in I$. Thus, Raghunathan's result does not apply in our model. Notice that $Q_{n+1}$ is the sum of stationary ARMA processes, hence $Q_{n+1}$ is also stationary.

$$\begin{aligned} E(Q) &= \textstyle\sum_{i \in I} \frac{a_i}{1-\rho_i} \\ Var(Q) &= \textstyle\sum_{i \in I} \sigma_i^2 \\ Cov(Q_n, Q_{n-1}) &= \textstyle\sum_{i \in I} \lambda_i \\ Cov(Q_n, Q_{n-k}) &= \textstyle\sum_{i \in I} \rho_i^{k-1} \lambda_i, k > 1, \end{aligned}$$

where $\lambda_i = Cov(q_n^i, q_{n-1}^i) = \rho_i \sigma_i^2 - b_i(b_i+1)\delta_i^2 = Var(D^i)(\rho_i - b_i(b_i+1)(1-\rho_i)^2)$.

The question is how to generate the best forecast for $Q_{n+1}$ using only the aggregated order history $Q_n, Q_{n-1}, \cdots$. We are not aware of any theory that identifies the "best" forecast for the sum of auto-regressive processes. Of course, one method that can be used is the minimum Mean Square Error (MSE) estimation, a method for which substantial theory has been developed. The following Proposition is well known, see e.g. Bertsekas (1995), Proposition E.1.

**Proposition 4.2** *The minimum MSE estimation of the aggregated demand $Q_{n+1}$ based on $Q_n, Q_{n-1}, \cdots, Q_{n-K}$ is given by $E(Q_{n+1}|Q_n, Q_{n-1}, \cdots, Q_{n-K})$.*

Because $Q_{n+1}$ is the sum of independent Normal random variables, $\varepsilon_n^i$, it follows multivariate Normal distribution. Which implies that the minimum MSE estimation, $E(Q_{n+1}|Q_n, Q_{n-1}, \cdots, Q_{n-K})$, is a linear function of $Q_n, Q_{n-1}, \cdots, Q_{n-K}$ (Bertsekas 1995, Proposition E.2).

In what follows, we discuss two approaches for estimating $Q_{n+1}$. In the first approach, we apply the linear minimum MSE estimation to order history. Since we are interested in techniques that can be implemented in practice, we also analyze a simple heuristic based on moving average.

Linear minimum MSE estimation

In this case, the manufacturer generates the forecast $F_n^M$ (his forecast of $Q_{n+1}$) based on $\mathbf{Q_K} = \{Q_n, Q_{n-1}, \cdots, Q_{n-K}\}$. Using Bertsekas (1995) Proposition E.3, the minimum MSE estimation, $F_n^M$, of $Q_{n+1}$ is given by $E(Q) + \sum_{k=0}^{K} \alpha_k (Q_{n-k} - E(Q))$. The objective is to find the optimal $\alpha_{\mathbf{K}} = \{\alpha_0, \alpha_1, \cdots, \alpha_K\}$ so that $E((Q_{n+1} - F_n^M)^2)$ is minimized.

Following a standard linear minimum MSE approach, we observe that

$$
\begin{aligned}
E((Q_{n+1} - F_n^M)^2) \;&=\; E((Q_{n+1} - E(Q))^2) + E([\textstyle\sum_{k=0}^{K} \alpha_k (Q_{n-k} - E(Q))]^2) - \\
&\quad -2E((Q_{n+1} - E(Q))[\textstyle\sum_{k=0}^{K} \alpha_k (Q_{n-k} - E(Q))]),
\end{aligned}
$$

where

$$E([\sum_{k=0}^{K} \alpha_k (Q_{n-k} - E(Q))]^2) = \alpha_{\mathbf{K}} A \alpha_{\mathbf{K}}^{'},$$

$$A = \begin{bmatrix} Var(Q_n) & Cov(Q_n, Q_{n-1}) & \cdots & Cov(Q_n, Q_{n-K}) \\ Cov(Q_{n-1}, Q_n) & Var(Q_{n-1}) & \cdots & Cov(Q_{n-1}, Q_{n-K}) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(Q_{n-K}, Q_n) & \cdots & \cdots & Var(Q_{n-K}) \end{bmatrix}$$

and

$$E((Q_{n+1} - E(Q))[\sum_{k=0}^{K} \alpha_k (Q_{n-k} - E(Q))]) = \alpha_{\mathbf{K}} B,$$

where

$$B = \begin{bmatrix} Cov(Q_{n+1}, Q_n) \\ \vdots \\ Cov(Q_{n+1}, Q_{n-K}) \end{bmatrix}$$

It is easy to see that $E((Q_{n+1} - F_n^M)^2)$ is quadratic with respect to $\alpha_{\mathbf{K}}$, and $A$ is symmetric and positive definite (this is true, since not all variances of external demand processes equal zero, and the time correlations of all demand processes are less than 1). Thus $E((Q_{n+1} - F_n^M)^2)$ is jointly convex in $\alpha_{\mathbf{K}}$, and the optimal (minimum MSE) $\alpha_{\mathbf{K}}$ satisfies the following linear equations,

$$A\alpha_{\mathbf{K}}^{'} = B.$$

Using the optimal $\alpha_{\mathbf{K}}$, the minimum mean square error becomes

$$E((Q_{n+1} - F_n^M)^2) = E((Q_{n+1} - E(Q))^2) - \alpha_{\mathbf{K}} B. \tag{2}$$

Notice, that the only information that the manufacturer needs to know in order to find the optimal $\alpha_{\mathbf{K}}$ is the aggregated order history $Q_n, Q_{n-1}, \cdots$. Also observe, see equation (2), that the minimum mean square error is a non-increasing function of $K$.

Classical linear minimum MSE estimation theory (Bertsekas 1995, Corollary E.3.1 and E.3.2) gives the following characteristics of linear minimum MSE estimation.

**Proposition 4.3** *If the model is linear and all noises are additive, then the minimum MSE estimation $F_n^M$ of the aggregated demand, $Q_{n+1}$, based on $\mathbf{Q_K}$ is unbiased, and $Q_{n+1} - F_n^M$ is uncorrelated with $Q_n, Q_{n-1}, \cdots, Q_{n-K}$ and $F_n^M$.*

The proposition thus implies that the forecast proposed by Raghunathan for the single retailer case, which is indeed based on a linear combination of historical orders, is not a minimum MSE estimation. This is true, since, in his model, the $Q_{n+1} - F_n^M$ is correlated with $F_n^M$.

To measure the impact of information sharing on the manufacturer's forecast accuracy, we use the ratio of the mean square error when the manufacturer receives information on retailer demand to the mean square error with no information sharing.

$$\frac{E((Q_{n+1} - \overline{F}_n^M)^2)}{E((Q_{n+1} - F_n^M)^2)} = \frac{Var(Q_{n+1} - \overline{F}_n^M)}{Var(Q_{n+1} - F_n^M)}.$$

We refer to this quantity as the **forecast error ratio**. The forecast error ratio is a function of the following parameters: $|I|$ the number of retailers, $L$ the transportation lead-time between the DC and the manufacturer, and $\rho_i, \delta_i/\delta_0, \forall i \in I$, where retailer 0 is a default retailer. $a_i, \forall i \in I$ and $\delta_0$ have no impact on the forecast error ratio since $a_i$ does not show up in the ratio and we can divide both the nominator and the dominator by $\delta_0$.

Our objective now is to characterize the forecast error ratio as $K \to +\infty$. For this purpose we need to characterize the minimum mean square error as $K$ tends to infinity. In what follows, we utilize a result first introduced by A. N. Kolmogorov (1941) to quantify the asymptotic minimum mean square error.

We start with the special case of a single retailer, proving that as $K \to \infty$, the forecast error ratio is equal to one. Thus, in this case information sharing does not reduce the mean square error of the manufacturer forecast. Then, we characterize the asymptotic mean square error in the multi-retailer case.

Consider a single retailer, and define the following notation:

$$\begin{aligned} mse_\infty &= \lim_{K\to\infty} E((q_{n+1} - F_n^M)^2) \\ c_0 &= Var(q_n) = \sigma^2 \\ c_1 &= Cov(q_n, q_{n-1}) = \rho\sigma^2 - b(b+1)\sigma^2 = \lambda \\ c_k &= Cov(q_n, q_{n-k}) = \rho^{k-1}\lambda, \forall k \geq 2. \end{aligned}$$

**Proposition 4.4** *For the special case of a single retailer, the linear minimum MSE estimation $F_n^M$ based on $q_n, q_{n-1}, \cdots$ has an asymptotic minimum mean square error equal to $(b+1)^2\delta^2$ as $K \to \infty$. Thus, the forecast error ratio equals 1.*

*Proof:* From Kolmogorov (1941),

$$\begin{aligned} mse_\infty &= e^P \\ P &= \tfrac{1}{\pi}\textstyle\int_0^\pi ln(\omega(x))dx \\ \omega(x) &= \frac{d\Omega(x)}{dx} \\ \Omega(x) &= c_0 x + 2\textstyle\sum_{k=1}^\infty \frac{c_k}{k} sin(kx). \end{aligned}$$

Since $c_k$ is decreasing to zero and $\sum_{k=1}^\infty sin(kx)/k$ is bounded, by Dirichlet's test, $\Omega(x)$ is a convergent power series with convergent radius $|\rho| < 1$. We can take derivatives term by term for power series within its convergent radius, hence,

$$\omega(x) = c_0 + 2\sum_{k=1}^\infty c_k cos(kx).$$

Since $c_k = \rho^{k-1}\lambda$ and $\sum_{k=1}^\infty \rho^k cos(kx) = \dfrac{\rho(cos(x) - \rho)}{1 - 2\rho cos(x) + \rho^2}$, we obtain

$$\omega(x) = \sigma^2 + 2\lambda\frac{cos(x) - \rho}{1 - 2\rho cos(x) + \rho^2}.$$

Now

$$\begin{aligned} P &= \tfrac{1}{\pi}\textstyle\int_0^\pi ln(\omega(x))dx \\ &= \tfrac{1}{\pi}\textstyle\int_0^\pi ln(\alpha + \beta cos(x))dx - \tfrac{1}{\pi}\int_0^\pi ln(1 - 2\rho cos(x) + \rho^2)dx \\ &= ln(\frac{\alpha + \sqrt{\alpha^2 - \beta^2}}{2}), \end{aligned}$$

where

$$\begin{aligned}\alpha &= \sigma^2(1+\rho^2) - 2\lambda\rho \\ &= \delta^2 + 2b(b+1)\delta^2 \\ \beta &= 2\lambda - 2\rho\sigma^2 \\ &= -2b(b+1)\delta^2.\end{aligned}$$

Finally, $mse_\infty = e^P = \dfrac{\alpha + \sqrt{\alpha^2 - \beta^2}}{2} = (b+1)^2\delta^2$. From Proposition 4.1, it is easily seen that the forecast error ratio equals 1. ∎

The Proposition thus implies that in the single retailer case, asymptotically, the mean square error in the no information sharing case is equal to the mean square error in the case with information sharing. As a result, in this case information does not increase the manufacturer's forecast accuracy.

Proposition 4.4 can be extended to the case of multiple retailers.

**Proposition 4.5** *In the case of multi-retailer, the linear minimum MSE estimation $F_n^M$, based on $Q_n, Q_{n-1}, \cdots$, has an asymptotic minimum mean square error equal to*

$$\begin{aligned}mse_\infty &= \lim_{K\to\infty} E((Q_{n+1} - F_n^M)^2) = e^P \\ P &= \frac{1}{\pi}\int_0^\pi ln(\textstyle\sum_{i\in I}\omega_i(x))dx \\ \omega_i(x) &= \sigma_i^2 + 2\lambda_i \frac{cos(x) - \rho_i}{1 - 2\rho_i cos(x) + \rho_i^2}, \forall i \in I.\end{aligned}$$

*Proof:* Applying Kolmogorov's results to the multi-retailer case,

$$\begin{aligned}\Omega(x) &= (\textstyle\sum_{i\in I} c_0^i)x + 2\sum_{k=1}^\infty \frac{\sum_{i\in I} c_k^i}{k} sin(kx) \\ &= \textstyle\sum_{i\in I}(c_0^i x + 2\sum_{k=1}^\infty \frac{c_k^i}{k} sin(kx)) \\ &= \textstyle\sum_{i\in I}\Omega_i(x),\end{aligned}$$

where $\forall i \in I$,

$$\begin{aligned}c_0^i &= \sigma_i^2 \\ c_k^i &= \rho_i^{k-1}\lambda_i \\ \Omega_i(x) &= c_0^i x + 2\textstyle\sum_{k=1}^\infty \frac{c_k^i}{k} sin(kx).\end{aligned}$$

The second equality in the derivation of $\Omega(x)$ is due to the rule of the sum of convergent series. ∎

To compare the asymptotic (in $K$) mean square error in the case of no information sharing to that of information sharing, we need to determine the quantity $\int_0^\pi ln(r_0 + r_1 cos(x) + \cdots + r_n cos^n(x))dx$, which is analytically difficult. Thus, in the computational study reported in the next section, we use numerical integration methods to determine $mse_\infty$ when the number of retailer is relatively small, e.g. $|I| \leq 50$.

In practices, the number of retail stores in a distribution system can be very large, e.g. Estee Lauder Companies, Inc. serves more than 2000 retail stores in the domestic market alone. Thus, it is interesting and important to characterize the forecast error ratio as the number of retail outlets, $|I|$, tends to infinity.

**Proposition 4.6** *Let $\rho_i$, $i = 1, 2, \cdots, |I|$, be a sequence of independent and identical random variables having a distribution $f(\rho)$ with support $[0, 1-\epsilon]$. Similarly, let $\delta_i$, $i = 1, 2, \cdots, |I|$, be a sequence of independent and identical random variables having a distribution $g(\delta)$ with support $[\delta(1), \delta(2)]$, and assume that $\delta$ and $\rho$ are independent of each other. Finally, assume the probability distribution functions have sufficiently many finite moments. As $|I| \to \infty$, we have,*

1. *The forecast error ratio converges to $S/\exp(\frac{1}{\pi}\int_0^\pi ln(r+u(x))dx)$ almost surely. Where*

$$
\begin{aligned}
S &= \int_0^{1-\epsilon}(b+1)^2 f(\rho)d\rho \\
r &= \int_0^{1-\epsilon}\frac{1+\rho-2\rho^{L+2}-2\rho^{L+3}+2\rho^{2L+4}}{(1-\rho)(1-\rho^2)}f(\rho)d\rho \\
u(x) &= \int_0^{1-\epsilon}\frac{2\rho^{L+2}(1+\rho-\rho^{L+2})}{1-\rho^2}\frac{cos(x)-\rho}{1-2\rho cos(x)+\rho^2}f(\rho)d\rho,
\end{aligned}
$$

2. *The forecast error ratio is only a function of L and $f(\cdot)$ but not a function of $g(\cdot)$.*

3. *As L $\to \infty$, the forecast error ratio tends to one for all probability distribution of $\rho$.*

*Proof:* See Section 5.7 for details. ∎

The third result depends on the $AR(1)$ assumption of the demand process. In Equation 1, the coefficient of $D_{n-1}$, $b\rho+\rho-b=\rho^{L+2}$. Thus, the demand information of individual retailers becomes less important as $L \to \infty$.

To illustrate the results of the proposition, we determine the forecast error ratio for $\rho \sim Uniform[0, 1-\epsilon]$. In this case, $S = 1+(1-\epsilon)+(1-\epsilon)^2+\cdots+(1-\epsilon)^{L+1}+\frac{L+1}{L+3}(1-\epsilon)^{L+2}+\frac{L}{L+4}(1-\epsilon)^{L+3}+\cdots+\frac{1}{2L+3}(1-\epsilon)^{2L+2}$. We also determine the forecast error ratio for $\rho \sim Normal(0.5, 0.04)$. The integration of $\int_0^{\pi} ln[r+u(x)]dx$ is quite complicated even for uniformly distributed $\rho$, thus numerical integration methods are used to calculate the forecast error ratio.

Figure 14 shows the forecast error ratio as a function of $L$ when $|I| \to \infty$ and infinite historical data is included in the forecast. We observe that

- Information sharing can always improve the manufacturer's forecasting accuracy, e.g. $L=0$, information sharing allows the manufacturer to reduce forecast mean square error by nearly 6.3% when $\rho \sim Uniform[0, 0.99]$.

- As $L$ increases, the value of information sharing decreases.

Moving average

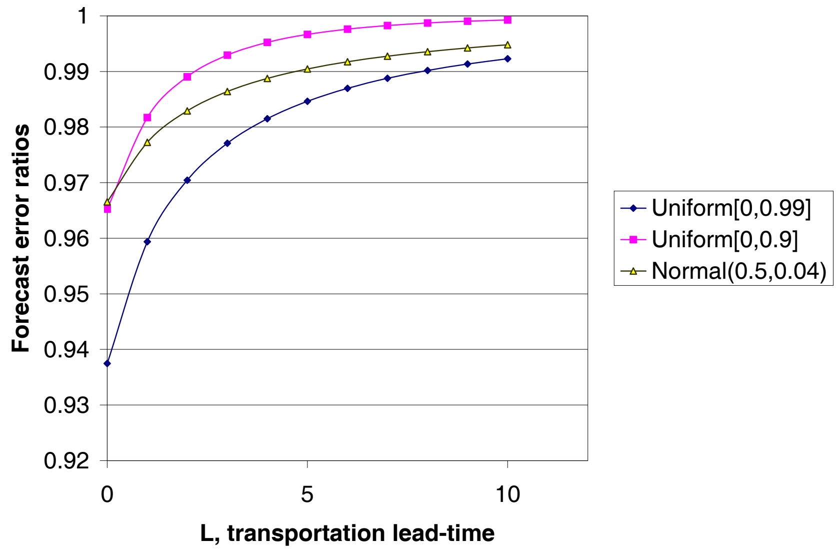


Figure 14: The impact of the transportation lead-time when $|I| \to \infty$

The moving average forecast is based on order history, $\mathbf{Q_K}$, and can be written as $F_n^M = \alpha_{\mathbf{K}} \mathbf{Q}'_{\mathbf{K}}$ with $\alpha_{\mathbf{K}} = \frac{1}{K+1}\mathbf{e_K}$, where $\mathbf{e_K} = \{1, 1, \cdots, 1\}$ is a row vector with $K+1$ dimensions. Thus, the mean square error of moving average is

$$E((Q_{n+1} - F_n^M)^2) = Var(Q_{n+1} - F_n^M) = Var(Q_{n+1}) + \frac{1}{(K+1)^2}\mathbf{e_K} A \mathbf{e}'_{\mathbf{K}} - \frac{2}{K+1}\mathbf{e_K} B,$$

The first equality comes from the fact that the moving average estimator is unbiased. It's easy to see that as $K$ increases, the moving average's mean square error may increase. Finally, for $K \to \infty$, we have $E((Q_{n+1} - F_n^M)^2) = Var(Q)$.

## 4.4 Computational results

In this section, we use computational analysis to study the impact of information sharing on the manufacturer's forecast accuracy when the number of retailers is relatively small, e.g. less than 50.

We conduct the following two computational studies. First, we compute the forecast error ratio for the linear minimum MSE estimation as the number of historical data included in the forecast, $K \to \infty$. In this study, we vary the following parameters: $|I|$, the number of retailers, and $L$, the transportation lead-time. In particular, $|I| \in \{2, 3, \cdots, 50\}$ and $L \in \{1, 2, \cdots, 20\}$. For each combination of $|I|$ and $L$, we randomly choose $\rho_i \in [0, 1)$ and $\delta_i/\delta_0 \in [1, 3]$ following the uniform distribution. For each combination of $|I|$ and $L$, we compute the forecast error ratio for 40 randomly generated data instances. For each instance, we select randomly the values $\rho_i$ and $\delta_i/\delta_0$. Finally, using the 40 instances, we determine the average of the forecast error ratio and its 95% confidence interval.

Second, we compute the forecast error ratio for both the minimum MSE estimation and moving average when $K$ is finite. In our study we vary the following parameters: $|I| \in \{10, 20, 50\}$, $L \in \{5, 10\}$ and $K \in [1, 75]$, the number of historical data included in the forecast. To analyze the impact of $\rho_i$ and $\delta_i/\delta_0$, we randomly generate $\rho_i \in [0, 1)$ and $\delta_i/\delta_0 \in [1, 3]$ using the uniform distribution. For every combination of $K$, $|I|$ and $L$ we compute the forecast error ratio for 10 randomly generated data instances. Each instance selects randomly the values $\rho_i$ and $\delta_i/\delta_0$. Finally, using the ten instances, we determine the average of the forecast error ratio and its 95% confidence interval.

Figure 15 and 16 illustrate the average and 95% lower and upper bounds of the

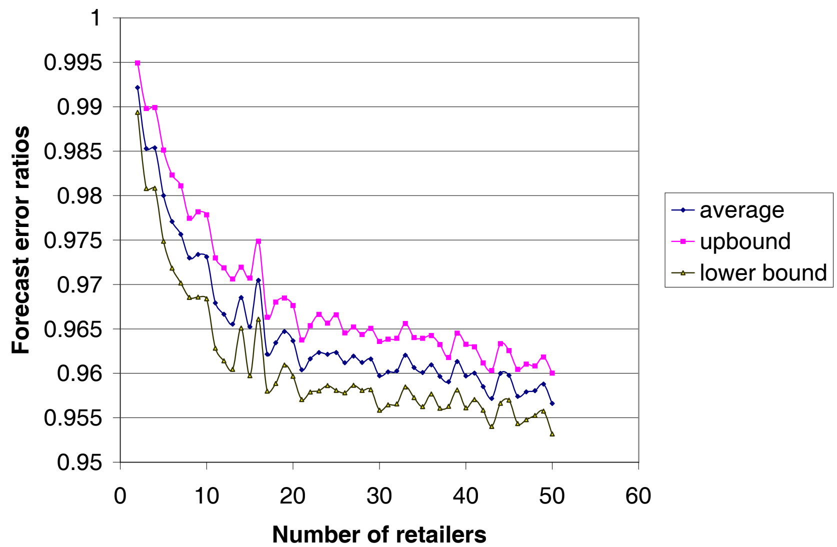


Figure 15: The impact of the number of retailers ($K$ tends to infinity)

forecast error ratio for the linear minimum MSE estimation as $K \to \infty$. Figure 15 shows the impact of the number of retailers when $L = 2$, while Figure 16 shows the impact of the transportation lead-time when $|I| = 40$. These figures demonstrate that

- Information sharing allows the manufacturer to reduce forecast error.
- As the number of retailer increases, the benefits of information sharing tends to increase.
- As the transportation lead-time increases, the benefits of information sharing tends to decrease.

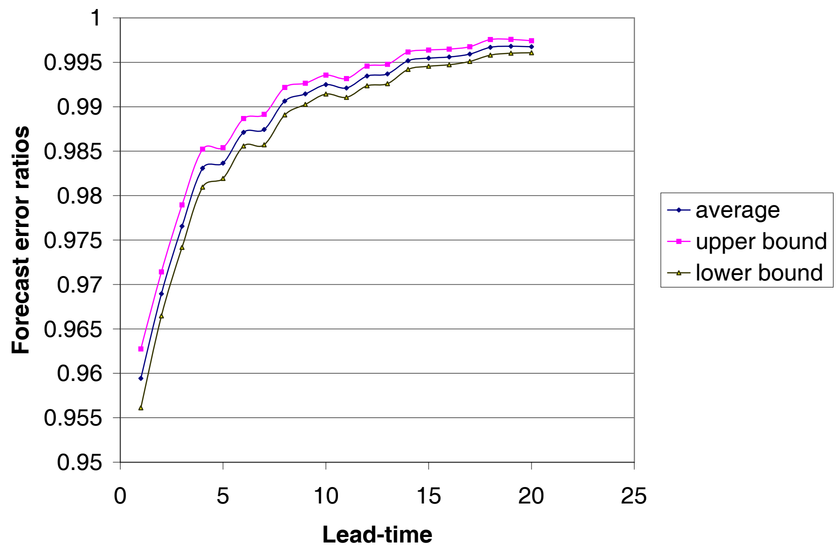


Figure 16: The impact of transportation lead-time ($K$ tends to infinity)

- Overall, the benefits of information sharing is not high, i.e., if $|I| \leq 50$, the forecast error ratio is larger than 95% in all of our test examples.

In Figure 17 we analyze the impact of the number of historical orders used by the manufacturer to create the forecast. The Figure shows the average of the forecast error ratio for both the minimum MSE estimation (solid lines) and moving average (dashed lines) when $L = 5$. In each case, we analyze the forecast error ratio for systems with 10, 20 and 50 retailers. Figure 18 illustrates the 95% confidence interval of the forecast error ratio for the case with 50 retailers. These figures demonstrate that

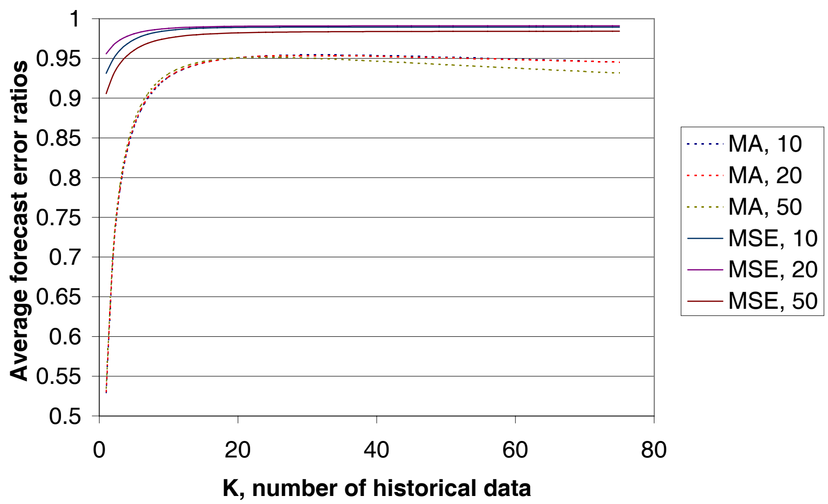


Figure 17: The averaged ratio of forecast errors

- As the number of historical data, $K$, increases, the forecast accuracy of both the minimum MSE estimation and the moving average increases for small values of $K$. On the other hand, for large values of $K$, the forecast accuracy of the minimum MSE estimation continues to increase while the forecast accuracy of the moving average may not always increase.

- Using the minimum MSE estimation, the mean square error of the manufacturer's forecast quickly converges to the asymptotic mean square error.

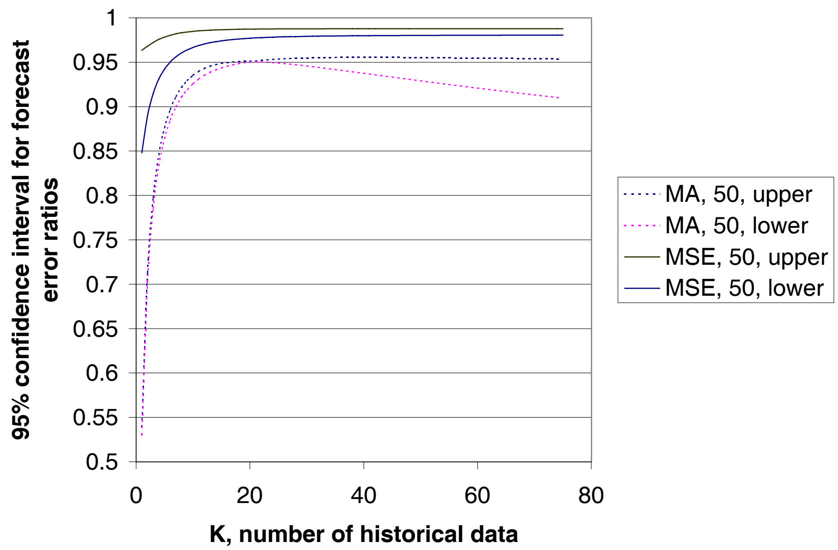


Figure 18: The 95% confidence interval of the ratio of forecast errors

- By simply using moving average forecast, a manufacturer can generate a forecast that has a slightly higher mean square error relative to the forecast generated when the manufacturer receives demand information from each retailer. For instance, when $K$ is chosen appropriately, the forecast error ratio for moving average is larger than 90% with 95% confidence.

## 4.5 Discussion and conclusion

In this chapter we considered a simple supply chain with a single manufacturer, a single distribution center (DC) and multiple retailers. We analyzed the impact of information sharing on the manufacturer's forecast accuracy.

For this purpose, we compare two systems, a supply chain with information sharing and a supply chain without information sharing. When there is no information sharing the manufacturer's forecast is based on the historical data of aggregated orders received from the DC. In a system with information sharing, the manufacturer's forecast is based on the historical data of external customer's demand as well as orders from each retailer.

Our analytic and computational results reveal the following insights:

1. For a single retailer, utilizing infinite order history (e.g., minimum MSE forecast) can generate the same impact on forecast accuracy as information sharing. Indeed, the same insight is provided in the work of Raghunathan.

2. For multiple non-identical retailers, information sharing has an impact on forecast accuracy even if the manufacturer utilizes infinite order history. More specifically, information sharing provides relatively large benefit when the transportation lead times between the manufacturer and retailers are short.

3. In general the shorter the transportation lead time and the larger the number of retailers, the higher the impact of information sharing on forecast accuracy.

4. Moving average forecast can perform quite well when the number of retailer is relatively small and the transportation lead-time is relatively long, e.g. $L \geq 5$.

Finally, we would like to point out that similar questions arise in other systems. For instance, consider a multi-product assembly system where each product is assembled from many common components. The question in this case, is whether individual product's demand information can help the component supplier better forecast future orders? It will be interesting to try and generalize the results obtained in this chapter so that we can also analyze assembly systems.

# Chapter 5

# Proofs and technical details

## 5.1 Proof of Lemma 2.1

*Proof:* : The proof of parts (a) and (b) are identical to the one in Federgruen & Zipkin (1986b). Here we focus on the proof of part (c).

Let's first consider the last ordering period in the planning horizon, and rewrite the dynamic programming formulation

$$\begin{array}{rcl} U_n^{'}(x) & = & -(c+h_{n-1})x + V_n^{'}(x) \\ V_n^{'}(x) & = & \min_{x \le y \le x+C}\{J_n^{'}(y)\} \\ J_n^{'}(y) & = & \left\{ \begin{array}{ll} cy + h_{n-1}y + \beta U_{n-1}^{'}(y) & n = 2, ..., N \\ cy + EL(y, \sum D) + \beta E(U_{n-1}^{'}(y - \sum D)) & n = 1. \end{array} \right. \end{array}$$

as

$$\begin{array}{rcl} J_n^{'}(y) & = & \varphi_n^{'}(y) + \beta U_{n-1}^{'}(y - D) \\ & = & r_n^{'}(y) + \beta V_{n-1}^{'}(y - D) \\ & = & r_n^{'}(y) + \beta \min_{y \le y^{'} \le y+C}\{J_{n-1}^{'}(y^{'})\}, \end{array}$$

where $\varphi_n^{'}(y)$ is defined as

$$\varphi_n^{'}(y) = \left\{ \begin{array}{ll} cy + h_{n-1}y, & 2 \le n \le N \\ cy + E(L(y, \sum D)), & n = 1. \end{array} \right.$$

Since salvage cost $U_0(\cdot) \equiv V_0(\cdot) \equiv 0$, so

$$r_n^{'}(y) = \left\{ \begin{array}{ll} (1-\beta)cy + (h_{n-1} - \beta h_{n-2})y, & 2 \le n \le N \\ cy + E(L(y, \sum D)), & n = 1. \end{array} \right.$$

The minimal total cost for this ordering period $J_N^{'}(y_N)$ can be obtained by using the equation for $J_n^{'}(y)$ recursively,

$$\begin{aligned} J_N^{'}(y_N) &= r_N^{'}(y_N) + \beta \min_{y_N \le y_{N-1} \le y_N + C}\{r_{N-1}^{'}(y_{N-1}) \\ &\quad + \cdots + \beta \min_{y_3 \le y_2 \le y_3 + C}\{r_2^{'}(y_2) + \beta \min_{y_2 \le y_1 \le y_2 + C}\{r_1^{'}(y_1)\}\} \cdots\}. \end{aligned} \tag{1}$$

Notice that for all $n = N, N-1, \cdots, 1$, if $y+\Delta_n$ minimizes $r_n^{'}(y^{'})$ in interval $[y, y+C]$, then

$$\begin{aligned} |\min_{y \le y^{'} \le y+C}\{r_n^{'}(y^{'})\} - r_n^{'}(y)| &= |r_n^{'}(y+\Delta_n) - r_n^{'}(y)| \\ &\propto |\Delta_n| \\ &\le C, \end{aligned}$$

since $r_n^{'}(y)$ is at most proportional to a linear function of $y$. Thus, we can show that the absolute difference between equation (1) and $\sum_{n=1}^{N} r_n^{'}(y_N)$ is bounded by a finite number, which is independent of $y$. Finally, after expanding $\sum_{n=1}^{N} r_n^{'}(y)$ and canceling some terms, we have $\sum_{n=1}^{N} r_n^{'}(y) = cy + h_{N-1}y + \beta^{N-1}E(L(y, \sum D)) +$ *constant*. Hence, in order for $J_n(y) \to +\infty$ for all $n$ when $y \to -\infty$, we need $\beta^{N-1}\pi > c + h_{N-1}$.

The proof for other ordering periods is similar. ∎

## 5.2 Proof of Lemma 2.3

Proof of Part (c) of Lemma 2.3.

In this Appendix, we show that if $\beta^{N-1}\pi > c + h_{N-1}$, then $J_n(y) \to +\infty$ for all $n = N, N-1, \cdots, 1$ when $y \to -\infty$. The case when $y \to +\infty$ is straight forward. For this purpose, let's consider the last ordering period in the planning horizon. From equation (4)

$$\begin{aligned} J_n(y) &= \varphi_n(y) + \beta E(U_{n-1}(y-D)) \\ &= r_n(y) + \beta E(V_{n-1}(y-D)) \\ &= r_n(y) + \beta E(\min_{y-D \le y^{'} \le y-D+C}\{J_{n-1}(y^{'})\}), \end{aligned}$$

where $n = N, \cdots, 1$, salvage cost $U_0(\cdot) \equiv V_0(\cdot) \equiv 0$, and

$$r_n(y) = \begin{cases} (1-\beta)cy + (h_{n-1} - \beta h_{n-2})y + \beta(c + h_{n-2})\mu, & 2 \leq n \leq N \\ cy + E(L(y, D)), & n = 1. \end{cases}$$

The minimal total cost for this ordering period $J_N(y_N)$ can be obtained by using the equation for $J_n(y)$ recursively,

$$\begin{array}{rcl} J_N(y_N) & = & r_N(y_N) + \beta E(\min_{y_N - D \leq y_{N-1} \leq y_N - D + C}\{r_{N-1}(y_{N-1}) \\ & & + \cdots + \beta E(\min_{y_2 - D \leq y_1 \leq y_2 - D + C}\{r_1(y_1)\}) \cdots \}). \end{array} \tag{2}$$

Similar to Appendix A, we notice that for all $n = N, N-1, \cdots, 1$,

$$|E(\min_{y-D \leq y' \leq y-D+C}\{r_n(y')\}) - r_n(y)| = |E(r_n(y - \Delta_n(D))) - r_n(y)|,$$

with $D - C \leq \Delta_n(D) \leq D$, and

$$\begin{array}{rcl} |E(r_n(y - \Delta_n(D))) - r_n(y)| & = & |\int_{-\infty}^{+\infty}[r_n(y - \Delta_n(D)) - r_n(y)]f_D(D)dD| \\ & \propto & |\int_{-\infty}^{+\infty}\Delta_n(D)f_D(D)dD| \\ & \leq & \max\{|\mu - C|, |\mu|\}, \end{array}$$

since $r_n(y)$ is at most proportional to a linear function of $y$. Thus, we can show that the absolute difference between equation (2) and $\sum_{n=1}^{N} r_n(y_N)$ is bounded by a finite number, which is independent of $y$. Finally, after expanding $\sum_{n=1}^{N} r_n(y_N)$ and canceling some terms, we have $\sum_{n=1}^{N} r_n(y) = cy + h_{N-1}y + \beta^{N-1}E(L(y, D)) + constant$. Hence, in order to have $J_n(y) \to +\infty$ for all $n$ when $y \to -\infty$, we need $\beta^{N-1}\pi > c + h_{N-1}$.

The proof for other ordering periods is similar. ∎

Observe that the condition $\beta^{N-1}\pi > c + h_{N-1}$ implies that the discounted penalty cost has to be larger than production cost plus inventory holding cost for $N-1$ information periods. Intuitively, this condition requires that the manufacturer starts production even in the first information period of any ordering period if her initial inventory position at the beginning of the ordering period is sufficiently low.

## 5.3 Proofs of Chapter 3

**The proof of Proposition 3.1:**

*Proof:* First consider the shortfall process $\{s_{mN+i}, m = 0, 1, \cdots\}$ for $i = 1, \cdots, N$. We start our proof by assuming i.i.d. demand $D$ with mean $ED$. Without loss of generality, we assume initial state $x_0 \leq 0$ since states larger than zero are transient. Since

$$Pr\{D = i\} \begin{cases} \geq 0, & \forall i \geq 0 \\ = 0, & otherwise, \end{cases}$$

the shortfall process $\{s_n, n = 0, 1, \cdots\}$ has state space $S = \{0, 1, \cdots\}$ and transition function: $s_n = (s_{n-1} + D - C)^+$. Thus, the transition matrix is

$$P = \begin{bmatrix} Pr\{D \leq C\} & Pr\{D = C+1\} & Pr\{D = C+2\} & \cdots \\ Pr\{D \leq C-1\} & Pr\{D = C\} & Pr\{D = C+1\} & \cdots \\ Pr\{D \leq C-2\} & Pr\{D = C-1\} & Pr\{D = C\} & \cdots \\ \cdots & & & \\ Pr\{D = 0\} & Pr\{D = 1\} & Pr\{D = 2\} & \cdots \\ 0 & Pr\{D = 0\} & Pr\{D = 1\} & \cdots \\ \cdots & & & \end{bmatrix}$$

Let $d(i) = E(s_{n+1} - s_n | s_n = i) = \sum_{j \in S} (j - i) P_{ij}$. If $i \geq C$,

$$\begin{aligned} d(i) &= \textstyle\sum_{j=0}^{\infty} [(j + i - C) - i] Pr\{D = j\} \\ &= \textstyle\sum_{j=0}^{\infty} (j - C) Pr\{D = j\} \\ &= ED - C. \end{aligned}$$

If $i < C$,

$$\begin{aligned} d(i) &= -iPr\{D \leq C - i\} + \textstyle\sum_{j=1}^{\infty} (j - i) Pr\{D = C + j - i\} \\ &= (C - i) Pr\{D < C - i\} + \textstyle\sum_{j=0}^{\infty} (C + j - i) Pr\{D = C + j - i\} - C \\ &\leq (C - i) Pr\{D < C - i\} + ED - C \\ &< \infty. \end{aligned}$$

So from Pakes's Lemma (Kulkarni 1995), the shortfall process of order-up-to zero policy with i.i.d demand is positive recurrent if $ED < C$.

These results can be easily extended to periodical demand $D_1, D_2, \cdots, D_N$ for finite $N$ and under order-up-to zero policy. Assume demand in different periods are independent of each other. The transition matrices for demand $D_l$,

$$P_{ij}^l = \begin{cases} Pr\{D_l = j - i + C\}, & i \geq C \\ Pr\{D_l = j - i + C\}, & 0 \leq i < C, j \neq 0 \\ Pr\{D_l \leq -i + C\}, & 0 \leq i < C, j = 0, \end{cases}$$

for $l = 1, 2, \cdots, N$. To simplify the notation, define

$$P_{ij}^{1 \to N} = \sum_{k_1 \in S, \cdots, k_{N-1} \in S} P_{ik_1}^1 P_{k_1 k_2}^2 \cdots P_{k_{N-2} k_{N-1}}^{N-1} P_{k_{N-1} j}^N,$$

the transition matrix between $1^{th}$ information period in one ordering period and the $1^{th}$ information period in the next ordering period.

Thus, if we start from $1^{th}$ information period in one ordering period, and proceed to the $1^{th}$ information period in the next ordering period,

$$\begin{aligned} E(s_{t+N} - s_t | s_t = i) &= \textstyle\sum_{j \in S} j P_{ij}^{1 \to N} - i \\ &= \textstyle\sum_{j \in S} (j - i) P_{ij}^{1 \to N} \\ &= \textstyle\sum_{j \in S} (j - k_{N-1} + k_{N-1} - k_{N-2} + \cdots + k_1 - i) \times P_{ij}^{1 \to N} \\ &= \textstyle\sum_{k_{N-1} \in S} P_{ik_{N-1}}^{1 \to N-1} \sum_{j \in S} (j - k_{N-1}) P_{k_{N-1} j}^N + \\ &\quad + \textstyle\sum_{k_{N-2} \in S} P_{ik_{N-2}}^{1 \to N-2} \sum_{k_{N-1} \in S} (k_{N-1} - k_{N-2}) P_{k_{N-2} k_{N-1}}^{N-1} \times \\ &\quad \times \textstyle\sum_{j \in S} P_{k_{N-1} j}^N + \cdots + \sum_{k_1 \in S} (k_1 - i) P_{ik_1}^1 \sum_{j \in S} P_{k_1 j}^{2 \to N}. \end{aligned}$$

We can change the order of summation because of Fubini's Theorem (Karr 1993). For the first term of the summation, if $i \geq NC$, then $k_{N-1} \geq C$ because the shortfall can decrease by at most $C$ in each information period. Together with $\sum_{k_{N-1} \in S} P_{ik_{N-1}}^{1 \to N-1} = 1, \forall i \geq NC$, this implies that the first term is equal to $ED_N - C$. For the second term, notice $\sum_{j \in S} P_{k_{N-1} j}^N = 1, \forall k_{N-1}$ and $\sum_{k_{N-2} \in S} P_{ik_{N-2}}^{1 \to N-2} = 1, \forall i$, similar to the argument of the first term, if $i \geq (N-1)C$, then $k_{N-2} \geq C$, which implies that the second term is equal to $ED_{N-1} - C$. We can keep doing this until the last term. For the last term, notice $\sum_{j \in S} P_{k_1 j}^{2 \to N} = 1$ for all $k_1$, thus the last term is equal to $ED_1 - C$ if $i \geq C$. So we proved that if $i \geq NC$, $E(s_{t+N} - s_t | s_t = i) = \sum_{i=1}^{N} ED_i - NC$. In addition, since $E(s_{t+N} - s_t | s_t = i) < \infty$ for $0 \leq i < NC$ (otherwise there must be a $D_l$ with infinite mean since $s_{t+1} = (s_t + D - C)^+$), from Pakes's Lemma, we prove the positive recurrence for order-up-to zero policy.

The relationship between $y_n$ and $s_n$ is $s_n = -y_n$, and we also have $x_{n+1} = \min\{0, x_n + C\} - D_n$. The proof of positive recurrence for $x_n$ and $y_n$ are similar. ∎

For the reader's convenience, we present a complete proof of Lemma 3.1 and 3.2 in this appendix. We prove Lemma 3.1 using the following two steps: first we show, Proposition 5.1, that the long-run average cost is finite (a similar proof is given in Bertsekas 1987). Then, for Markov chains with certain structure, we show, Proposition 5.2, that finite long-run average cost implies finite steady-state average cost. To prove Lemma 3.2, we just need to show that the generalization of Foster's criterion is satisfied in the information sharing model analyzed in this chapter.

**Proposition 5.1** *Consider Markov chain $\{X_n, n = 0, 1, \cdots\}$, if there exists a function $V(\cdot) : S \to [0, \infty)$, and a constant $\eta$ such that*

$$E\{V(X_{n+1}) - V(X_n) | X_n = x\} \leq -r(x) + \eta, \forall x \in S,$$

*where $r(\cdot)$ is bounded from below. Then for initial state $x_0$ with $V(x_0) < \infty$, the stochastic process $X_n$ with single-period cost function $r(x)$ has finite long-run average cost.*

*Proof:* Re-arrange the inequality,

$$\eta + V(x) \geq r(x) + E(V(X_{n+1}) | X_n = x), \forall x \in S,$$

To simplify notation, we define $(TV)(x) := r(x) + E(V(X_{n+1})|X_n = x)$. Thus, since the inequality holds for every $x$, we can drop the reference to $x$ and have

$$\eta + V \geq TV.$$

From the monotone property of $T$ (Bertsekas 1987), clearly we have

$$2\eta + V \geq \eta + TV = T(\eta + V) \geq T(TV) := T^2V.$$

Continuing in this fashion $N$ times we get,

$$N\eta + V \geq T^N V,$$

where $T^N V := T(T^{N-1}V)$. Dividing by $N$, and letting $N \to +\infty$, we have

$$\lim_{N\to+\infty} \frac{1}{N} T^N V(x_0) \leq \lim_{N\to+\infty} (\eta + \frac{1}{N} V(x_0)) = \eta,$$

where the equality is justified since $V(x_0) < \infty$.

Finally, notice that the left-hand side of above inequality is the long-run average cost of $\{X_n, n = 0, 1, \cdots\}$ with a single-period cost function $r(\cdot)$, starting with initial state $x_0$ and terminating with cost $V(\cdot)$. Since $r(\cdot)$ is bounded from below, we have

$$\lim_{N\to+\infty} \frac{1}{N} T^N V(x_0) > -\infty.$$

∎

Now we want to show that finite long-run average cost implies finite steady-state average cost for ergodic Markov chain.

**Proposition 5.2** *Consider an ergodic Markov chain* $\{X_n, n = 0, 1, \cdots\}$ *and let* $r(\cdot)$ *be the single period cost function. If* $r(\cdot)$ *is a continuous function and bounded from below, then finite long-run average cost implies finite steady-state average cost.*

*Proof:* Since $X_n$ is ergodic, thus $X_n \xrightarrow{d} X$, as $n \to +\infty$, where $\xrightarrow{d}$ means convergence in distribution. This implies $r(X_n) \xrightarrow{d} r(X)$, as $n \to +\infty$, since $r(X)$ is a continuous function (Karr 1993). Without loss of generality, we may assume that $r(\cdot)$ is positive since by the assumption $r(\cdot)$ is bounded from below and hence we can always add a constant to change it to a non-negative function.

Billingsley (1999) Theorem 3.4 shows that for a random sequence $Z_n$, if $Z_n \xrightarrow{d} Z$, then $E(|Z|) \leq lim_{n\to+\infty} \inf E(|Z_n|)$. Hence,

$$\begin{aligned} E(r(X)) &\leq lim_{n\to+\infty} \inf E(r(X_n)) \\ &\leq lim_{N\to+\infty} \frac{1}{N} \textstyle\sum_{n=0}^{N-1} E(r(X_n)) < +\infty. \end{aligned}$$

To justify the last inequality, first notice that $v_{inf} = lim_{n\to+\infty} \inf E(r(X_n)) < \infty$, otherwise long-run average cost will not be finite. Second, for any sequence $n_k \to \infty$ so that $E(r(X_{n_k})) \to v_{inf}$, $v_{inf} = lim_{N\to+\infty} \frac{1}{N} \sum_{n_k=0}^{N-1} E(r(X_{n_k}))$ (Kemeny, Snell, Knapp 1966).

∎

Lemma 3.1 is a direct result of these two Propositions.

**Proof of Lemma 3.2:**

*Proof:* To prove the **first part** of the lemma, it is sufficient to show the existence of a function $V$ satisfying the requirement of Proposition 5.1.

We start by analyzing **i.i.d. demand** $D$ with mean $ED$. Let $V(x) = q_\rho(x + C)^{\rho+1}$ and $q_\rho = \frac{1}{(\rho+1)(C-ED)}$. Clearly, $V(\cdot)$ maps the state space of the shortfall $S = \{0, 1, 2, \cdots\}$ to $R^+$.

If $i \geq C$,

$$\begin{aligned} E(V(s_{n+1}) - V(s_n) | s_n = i) &= q_\rho \textstyle\sum_{j=0}^{\infty} [(j+i)^{\rho+1} - (i+C)^{\rho+1}] Pr\{D = j\} \\ &= q_\rho \textstyle\sum_{k=0}^{\rho+1} \begin{pmatrix} \rho+1 \\ k \end{pmatrix} (m_k - C^k) i^{\rho+1-k}, \end{aligned}$$

where $m_k = \sum_{j=0}^{\infty} j^k Pr\{D = j\}$, is the $k^{th}$ moment of demand. Further expanding the equation, we obtain

$$E(V(s_{n+1})-V(s_n)|s_n = i) = -i^\rho + q_\rho[\begin{pmatrix} \rho+1 \\ 2 \end{pmatrix}(m_2-C^2)i^{\rho-1}+\cdots+(m_{\rho+1}-C^{\rho+1})].$$

If $i < C$,

$$\begin{aligned}
& E(V(s_{n+1}) - V(s_n)|s_n = i) \\
= \; & q_\rho[(C^{\rho+1} - (i+C)^{\rho+1})Pr\{D < C-i\}+ \\
& +\textstyle\sum_{j=C-i}^{\infty}[(j+i)^{\rho+1} - (i+C)^{\rho+1}]Pr\{D = j\}] \\
= \; & q_\rho[C^{\rho+1}Pr\{D < C-i\} + \textstyle\sum_{j=C-i}^{\infty}(j+i)^{\rho+1}Pr\{D = j\} - (i+C)^{\rho+1}] \\
= \; & q_\rho[\textstyle\sum_{j=0}^{C-i-1}[C^{\rho+1} - (j+i)^{\rho+1}]Pr\{D = j\}+ \\
& +\textstyle\sum_{j=0}^{\infty}[(j+i)^{\rho+1} - (i+C)^{\rho+1}]Pr\{D = j\}] \\
= \; & q_\rho[\textstyle\sum_{j=0}^{C-i-1}[C^{\rho+1} - (j+i)^{\rho+1}]Pr\{D = j\} \\
& +q_\rho[\begin{pmatrix} \rho+1 \\ 2 \end{pmatrix}(m_2 - C^2)i^{\rho-1} + \cdots + (m_{\rho+1} - C^{\rho+1})].
\end{aligned}$$

To summarize, in both cases,

$$\begin{aligned}
E(V(s_{n+1}) - V(s_n)|s_n = i) \;=\; & q_\rho g_\rho(C,i) - i^\rho + q_\rho[\begin{pmatrix} \rho+1 \\ 2 \end{pmatrix}(m_2 - C^2)i^{\rho-1}+ \\
& +\cdots + (m_{\rho+1} - C^{\rho+1})],
\end{aligned}$$

where

$$g_\rho(C,i) = \begin{cases} 0, & i \geq C, \\ \sum_{j=0}^{C-i-1}[C^{\rho+1} - (j+i)^{\rho+1}]Pr\{D = j\}, & 0 \leq i < C. \end{cases}$$

Define the single-period cost function

$$r_\rho(x) = x^\rho - q_\rho[\begin{pmatrix} \rho+1 \\ 2 \end{pmatrix}(m_2 - C^2)x^{\rho-1} + \cdots + \begin{pmatrix} \rho+1 \\ \rho \end{pmatrix}(m_\rho - C^\rho)x],$$

and since $m_k < +\infty$ for all positive integer $k \leq \rho + 1$, then from Lemma 3.1 and Proposition 3.1, steady-state average cost is finite for the shortfall $s_n$ with single-period cost function $r_\rho(x)$.

In fact, if the single period cost function is $r_l(x), \forall 0 < l < \rho$, where

$$r_l(x) = x^l - q_l[\begin{pmatrix} l+1 \\ 2 \end{pmatrix}(m_2 - C^2)x^{l-1} + \cdots + \begin{pmatrix} l+1 \\ l \end{pmatrix}(m_l - C^l)x],$$

the same analysis shows that the corresponding steady-state average cost is finite for the shortfall $s_n$.

Finally, our objective is to show that if the single period cost function is $x^l$, then the steady state average cost of the shortfall is finite. For this purpose we use induction on $l$. The case $l = 1$ is obvious since we already know that for $r_1(x) = x$ the steady-state average cost is finite. By induction on $l$ and the fact that steady-state average cost of the shortfall is finite for $r_l(x)$, $0 < l < \rho$, we get our result.

Now we extend the result to independent demand with periodically **varying** distribution $D_1, D_2, \cdots, D_N$. Define $\sum D = \sum_{n=1}^{N} D_n$, $V(x) = Q_\rho(x + NC)^{\rho+1}$ and $Q_\rho = \frac{1}{(\rho+1)(NC-\sum_{n=1}^{N} ED_n)}$. We consider Markov chains $\{s_{mN+i}, m = 0, 1, \cdots\}$ for $i = 1, 2, \cdots, N$. Following the same proof as for i.i.d demand,

$$\begin{aligned} E(V(s_{n+N}) - V(s_n)|s_n = i) &= Q_\rho G_\rho(C, i) + Q_\rho[\textstyle\sum_{j=0}^{\infty}[(j+i)^{\rho+1} - (i + NC)^{\rho+1}] \\ &\quad \times Pr\{\textstyle\sum D = j\}] \\ &= Q_\rho G_\rho(C, i) - i^\rho + Q_\rho[\begin{pmatrix} \rho+1 \\ 2 \end{pmatrix}(M_2 - (NC)^2)\times \\ &\quad \times i^{\rho-1} + \cdots + (M_{\rho+1} - (NC)^{\rho+1})], \end{aligned}$$

where

$$G_\rho(C, i)\begin{cases} = 0, & i \geq NC \\ < +\infty, & 0 \leq i < NC, \end{cases}$$

and $M_k = \sum_{j=0}^{\infty} j^k Pr\{\sum D = j\}$ is the $k^{th}$ moment of total demand during one ordering period.

If the shortfall process is aperiodic, then apply the same logic as for i.i.d. demand to obtain the desired result.

If the shortfall process $\{s_{mN+i}, m = 0, 1, \cdots\}$ is periodic, then the period $d$ must be finite since $\{s_{mN+i}, m = 0, 1, \cdots\}$ generates a single irreducible and positive recurrent class (Proposition 3.1). Thus, the $d$ subsequences of shortfall process are aperiodic Markov chains for each of which we can apply our result (Kulkarni 1995, Theorem 3.16).

We now prove the **second part** of the Lemma. Since $s_t$ is non-negative, the Monotone Convergence Theorem implies that

$$E(\sum_{t=1}^{\infty} \beta^t |s_t|^n) = \sum_{t=1}^{\infty} \beta^t E|s_t|^n.$$

Because of the first part of this lemma, and $0 < \beta < 1$, this summation is for a power series with positive and bounded coefficients, so it is finite.

Since the inventory position processes $y_t = -s_t$ and $x_{t+1} = y_t - D_t$, it's easy to show that the same arguments apply for $y_t, x_t$. ∎

**Proof of Proposition 3.7:**

*Proof:* Consider time discounted factor $\beta$ such that $0 < \epsilon \leq \beta < 1$, we first prove that there exists $M > 0$ and independent of $\beta$ so that $lim_{m \to \infty} U^{\beta}_{mN+i}(0)$ is bounded from above by $\frac{M}{1-\beta}$. Then we show that the optimal order-up-to levels $y^*_i, i = 1, 2, \dots, N$ are bounded both from above and below by a constant independent of $\beta$.

Using order-up-to 0 policy, referred to as $\sigma_0$, and starting with 0 inventory position, define the expected total discounted cost using policy $\sigma_0$ to be

$$U^{\beta}_{mN+i}(0, \sigma_0) \quad = \quad E\{\textstyle\sum_{k=0}^{mN+i-1} \beta^k g_{mN+i-k}(x_{mN+i-k}, \sigma_0(x_{mN+i-k}))|x_{mN+i} = 0\},$$

where $E(\cdot)$ is the expectation with respect to all demand in periods from $mN+i$ (the initial period) to 1 (the last period). Of course, we can interchange expectation and summation when $m \to +\infty$, due to the fact that the cost is non-negative and the Monotone Convergence Theorem. Since the steady-state average cost of $\sigma_0$ is finite (see Lemma 3.2), so there must exist a finite $M > 0$ independent of $\beta$ and initial period $i$, so that $\forall m$ and $\forall k = mN+i, \cdots, 1$, $E\{g_k(x_k, \sigma_0(x_k))|x_{mN+i} = 0\} \leq M$.

Hence,

$$\begin{array}{rcl} lim_{m\to\infty} U^{\beta}_{mN+i}(0) & \leq & lim_{m\to\infty} U^{\beta}_{mN+i}(0,\sigma_0) \\ & = & lim_{m\to\infty} \sum_{k=0}^{mN+i-1} \beta^k \times \\ & & \times E\{g_{mN+i-k}(x_{mN+i-k}, \sigma_0(x_{mN+i-k}))|x_{mN+i}=0\} \\ & \leq & lim_{m\to\infty} \sum_{k=0}^{mN+i-1} \beta^k M \\ & = & \frac{M}{1-\beta} \end{array}$$

To complete the proof, we need to show that the optimal order-up-to levels $y_i^*, i = 1, 2, \ldots, N$ are uniformly bounded from above and below for all $0 < \epsilon \leq \beta < 1$. Observe $J_i^{\beta}(y_i^*) \leq \frac{M}{1-\beta} + b, \forall i$, where $b$ is a constant, since $\phi_i(0) + J_i^{\beta}(y_i^*) \leq U_i^{\beta}(0) \leq \frac{M}{1-\beta}$.

Consider $J_N^{\beta}(y)$ first,

$$\begin{array}{rcl} \frac{M}{1-\beta} + b & \geq & J_N^{\beta}(y_N^*) = w_N(y_N^*) + \beta E\{Min_{y_{N-1}\in B^N_{y_N^*}} J^{\beta}_{N-1}(y_{N-1})\} \\ & \geq & W_N(y_N^*) + \beta^N J_N^{\beta}(y_N^*) \\ & \geq & \sum_{j=0}^{\infty} \beta^{jN} W_N(y_N^*) = \frac{W_N(y_N^*)}{1-\beta^N}, \end{array}$$

where $W_i(y)$ are defined in Section 3.3.1. Thus, $W_N(y_N^*) \leq M(1+\beta+\cdots+\beta^{N-1}) + (1-\beta^N)b$. Since $W_N(y)$ is convex and $W_N(y) \to +\infty$ as $|y| \to \infty$, then $y_N^*$ is uniformly bounded from both above and below for all $0 < \epsilon \leq \beta < 1$. Same procedure can be applied to any $i \neq N$, where $W_i(y) \to +\infty$ as $|y| \to \infty$ if $\beta^{i-1}\pi > c + h_{i-1}$. ∎

## 5.4 IPA in the case of information sharing

Following the notation of Section 3.1, let $x_{mN+i}$ be the inventory level at the beginning of information period $mN+i$, where $i = 1, \cdots, N$, and $N$ is the number of information periods in one ordering period. Recall that 1 indices the first information period and $N$ indices the last in any ordering period, and $a_i$ is the order-up-to levels for $i^{th}$ information period, $i = 1, \cdots, N$. Let $\overline{a} = \{a_1, \cdots, a_N\}$. For simplicity, we only consider i.i.d. demand. The dynamics for $x_{mN+i}$ is

$$x_{mN+i} = \begin{cases} x_{mN+i-1} - D, & x_{mN+i-1} \geq a_{i-1} \\ a_{i-1} - D, & x_{mN+i-1} < a_{i-1} \text{ and } x_{mN+i-1} + C \geq a_{i-1} \\ x_{mN+i-1} + C - D, & x_{mN+i-1} + C < a_{i-1}, \end{cases}$$

the amount produced in period $mN + i - 1$ is $y_{mN+i-1} - x_{mN+i-1}$, where

$$y_{mN+i-1} = \begin{cases} x_{mN+i-1}, & x_{mN+i-1} \geq a_{i-1} \\ a_{i-1}, & x_{mN+i-1} < a_{i-1} \text{ and } x_{mN+i-1} + C \geq a_{i-1} \\ x_{mN+i-1} + C, & x_{mN+i-1} + C < a_{i-1}. \end{cases}$$

Finally, the cost function is

$$cost = \begin{cases} h_N x^+_{mN+i} + \pi(-x_{mN+i})^+, & i = 1 \\ h_{N-i+1}(y_{mN+i-1} - x_{mN+i-1}), & otherwise. \end{cases}$$

For any $j = 1, \cdots, N$, the sample path derivatives with respect to $a_j$ can be computed in the following way,

$$\frac{\partial x_{mN+i}}{\partial a_j} = \frac{\partial y_{mN+i-1}}{\partial a_j} = \begin{cases} 1_{(j=i-1)}, & x_{mN+i-1} < a_{i-1} \text{ and } x_{mN+i-1} + C \geq a_{i-1} \\ \frac{\partial x_{mN+i-1}}{\partial a_j}, & otherwise, \end{cases}$$

and

$$\frac{\partial cost}{\partial a_j} = \begin{cases} h_N \frac{\partial x_{mN+i}}{\partial a_j}, & i = 1 \text{ and } x_{mN+1} \geq 0 \\ -\pi \frac{\partial x_{mN+i}}{\partial a_j}, & i = 1 \text{ and } x_{mN+1} < 0 \\ h_{N-i+1}(\frac{\partial y_{mN+i-1}}{\partial a_j} - \frac{\partial x_{mN+i-1}}{\partial a_j}), & otherwise, \end{cases}$$

where $1_{(x=y)}$ is an indicator function which equals to one when $x = y$. The following standard iteration is used to find the optimal order-up-to levels. Let $\Gamma^k = diag(\gamma^k_1, \gamma^k_2, \ldots, \gamma^k_N)$ be the stepsize matrix,

$$\overline{a}^{k+1} = \overline{a}^k - \Gamma^k \nabla' cost(\overline{a}^k),$$

where $\overline{a}^k$ represents the order-up-to levels in $k$th iteration, and $\nabla' cost$ is the estimation of the gradient $\nabla cost = \{\frac{\partial cost}{\partial a_1}, \ldots, \frac{\partial cost}{\partial a_N}\}^T$. The initial condition is $x_1 = 0$ and $\nabla x_1 = \overline{0}$. In the simulation, it is important to make sure that each order-up-to level $a_i$, $i = 1, 2, \ldots, N$ is positive Harris recurrent (see, Meyn and Tweedie 1993), otherwise $\frac{\partial cost}{\partial a_i} = 0$. Assume that $Prob\{D = 0\} > 0$ and demand can reach its maximum value, $\max\{D\}$, with a positive probability, sufficient conditions for positive Harris recurrence in this model are that $\sum_{i=1}^{N} ED_i < NC$ and $\overline{a}$ satisfies the following property (see Theorem 6, Kapuscinsky and Tayur 1998),

$$a_{i-1} - \max\{D\} \leq a_i \leq a_{i-1} + C, i = 1, \cdots, N.$$

In the simulation, we follow Glasserman & Tayur (1995) in their selection of the step-sizes, transient deletion, run-length and stopping criterion. We will use the following method to reduce step-sizes: we pick the initial step sizes to be 0.2 for all order-up-to levels, and run the simulation for a fixed number of iterations. We then halved the step size for any order-up-to level if the partial derivative of cost with respect to this order-up-to level changes sign. Once the step size is reduced, we will keep it constant until after a certain number of iterations. The minimum value used for a step-size is 0.002.

In the simulation, we find that there is little difference if transient deletion is more than 500 and run-length is more than 20000. Finally, in order to terminate simulation in finite time, we set the following stopping criterion: either iteration exceeds a given number (e.g., 600), or $(\nabla' cost \cdot \nabla' cost)^{1/2} < 0.01$. The initial order-up-to levels are chosen to be zero.

To verify that the simulation results are indeed close to true optimal values, we performed the following test. We simulated the system for discrete values of the order-up-to levels starting from 10% below the estimated optimal order-up-to level to 10% above that level.

## 5.5 IPA in the case of no information sharing

Let $x_{mN+i}$ be the inventory level at the beginning of the information period $mN+i$. For $i = 2, \cdots, N$, we have:

$$x_{mN+i} = \begin{cases} x_{mN+i-1}, & x_{mN+i-1} \geq a_{i-1} \\ a_{i-1}, & x_{mN+i-1} < a_{i-1} \text{ and } x_{mN+i-1} + C \geq a_{i-1} \\ x_{mN+i-1} + C, & x_{mN+i-1} + C < a_{i-1}, \end{cases}$$

and for $i = 1$,

$$x_{mN+1} = \begin{cases} x_{mN} - \sum D, & x_{mN} \geq a_N \\ a_N - \sum D, & x_{mN} < a_N \text{ and } x_{mN} + C \geq a_N \\ x_{mN} + C - \sum D, & x_{mN} + C < a_N, \end{cases}$$

where $\sum D$ is the total demand in one ordering period. The cost functions are

$$cost = \begin{cases} h_N x^+_{mN+i} + \pi(-x_{mN+i})^+, & i = 1 \\ h_{N-i+1}(x_{mN+i} - x_{mN+i-1}), & otherwise. \end{cases}$$

The sample path derivatives of inventory levels and cost functions are computed in the following way,

$$\frac{\partial x_{mN+i}}{\partial a_j} = \begin{cases} 1_{(j=i-1)}, & x_{mN+i-1} < a_i \text{ and } x_{mN+i-1} + C \geq a_i \\ \frac{\partial x_{mN+i-1}}{\partial a_j}, & otherwise \end{cases}$$

and

$$\frac{\partial cost}{\partial a_j} = \begin{cases} h_N \frac{\partial x_{mN+i}}{\partial a_j}, & i = 1 \text{ and } x_{mN+1} \geq 0 \\ -\pi \frac{\partial x_{mN+i}}{\partial a_j}, & i = 1 \text{ and } x_{mN+1} < 0 \\ h_{N-i+1}(\frac{\partial x_{mN+i}}{\partial a_j} - \frac{\partial x_{mN+i-1}}{\partial a_j}), & otherwise. \end{cases}$$

In the model of no information sharing, we find that convergence can be speeded up by adjusting the order-up-to levels in each iteration using the constraints $a_{i+1} \geq a_i, i = 1, \cdots, N-1$.

## 5.6 IPA and the timing of information sharing

Since sharing information at the end of $N^{th}$ interval is the same as no information sharing, we will only consider situations in which information is shared at the end of $k^{th}$ interval, $k = 1, 2, \cdots, N-1$.

$$x_{mN+i} = \begin{cases} x_{mN+i-1} - D^i, & x_{mN+i-1} \geq a_{i-1} \\ a_i - D^i, & x_{mN+i-1} < a_{i-1} \text{ and } x_{mN+i-1} + C \geq a_{i-1} \\ x_{mN+i-1} + C - D^i, & x_{mN+i-1} + C < a_{i-1}, \end{cases}$$

where $D^i, i = 1, \ldots, N$ is defined as

$$D^i = \begin{cases} 0, & i \neq k+1, 1 \\ \sum_{j=1}^{k} D_j, & i = k+1 \\ \sum_{j=k+1}^{N} D_j, & i = 1. \end{cases}$$

The production in period $mN + k$ is $y_{mN+k} - x_{mN+k}$, where

$$y_{mN+k} = \begin{cases} x_{mN+k}, & x_{mN+k} \geq a_k \\ a_k, & x_{mN+k} < a_k \text{ and } x_{mN+k} + C \geq a_k \\ x_{mN+k} + C, & x_{mN+k} + C < a_k. \end{cases}$$

The cost functions are

$$cost = \begin{cases} h_N x_{mN+i}^+ + \pi(-x_{mN+i})^+, & i = 1 \\ h_{N-i+1}(y_{mN+i-1} - x_{mN+i-1}), & i = k+1 \\ h_{N-i+1}(x_{mN+i} - x_{mN+i-1}), & otherwise. \end{cases}$$

The sample derivatives are

$$\frac{\partial x_{mN+i}}{\partial a_j} = \begin{cases} 1(j = i-1), & x_{mN+i-1} < a_{i-1} \text{ and } x_{mN+i-1} + C \geq a_{i-1} \\ \frac{\partial x_{mN+i-1}}{\partial a_j}, & otherwise, \end{cases}$$

and

$$\frac{\partial cost}{\partial a_j} = \begin{cases} h_N \frac{\partial x_{mN+i}}{\partial a_j}, & i = 1 \text{ and } x_{mN+1} \geq 0 \\ -\pi \frac{\partial x_{mN+i}}{\partial a_j}, & i = 1 \text{ and } x_{mN+1} < 0 \\ h_{N-i+1}(\frac{\partial x_{mN+i}}{\partial a_j} - \frac{\partial x_{mN+i-1}}{\partial a_j}), & otherwise. \end{cases}$$

To speed up the convergence, we adjust the order-up-to levels at each iteration according to the same idea used in the model of no information sharing: suppose information is shared at $k \neq 1, N-1$, we set $a_i \leq a_{i+1}, i = 1, \cdots, k-1, k+1, \cdots, N$; if $k = 1$, we set $a_i \leq a_{i+1}, i = 2, \cdots, N$; and if $k = N-1$, we set $a_i \leq a_{i+1}, i = 1, \cdots, N-2$.

## 5.7 Proofs of Chapter 4

We first cite the following lemma by Rust (1997) and Andrews (1992) without proof, then we present a proof of Proposition 4.6.

**Lemma 5.1** *(***Uniform Strong Law of Large Numbers***) Let $x_1, x_2, \cdots$ be i.i.d. random variables choosing value from domain $X$ ($X$ is a Borel space). Let $g(x, \theta) = h(x, \theta) - E_x(h(x, \theta))$ be a measurable function of $x$ for all $\theta \in \Theta$, and a continuous function of $\theta$ for almost all $x \in X$, where $E_x$ is the expectation with respect to $x$. Assume*

1. *$\Theta$ is compact.*
2. *$\frac{1}{N} \sum_{n=1}^{N} g(x_n, \theta) \to 0$ almost surely for all $\theta \in \Theta$.*
3. *$|g(x, \theta)| \leq d(x)$ for some function $d$ satisfying $E_x(d(x)) < \infty$.*

*Then, we have as $N \to \infty$, $\sup_{\theta \in \Theta} |\frac{1}{N} \sum_{n=1}^{N} g(x_n, \theta)| \to 0$, almost surely.*

Proof of Proposition 4.6:

*Proof:* We first prove (1) and (2).

Let's consider the case of no information sharing and when the manufacturer uses linear least square estimation based on $Q_n, Q_{n-1}, \cdots$. Proposition 4.5 gives

$$\begin{aligned} mse_\infty &= e^P \\ P &= \frac{1}{\pi}\int_0^\pi ln(\sum_{i\in I}\omega_i(x))dx \\ &= \frac{1}{\pi}\int_0^\pi ln[|I|(\frac{1}{|I|}\sum_{i\in I}\omega_i(x))]dx \\ &= ln|I| + \frac{1}{\pi}\int_0^\pi ln[\frac{1}{|I|}\sum_{i\in I}\omega_i(x)]dx. \end{aligned}$$

Then, consider the case of information sharing. From Proposition 4.1 we have

$$\begin{aligned} mse &= e^{P'} \\ P' &= ln[\sum_{i\in I}(b_i+1)^2\delta_i^2] \\ &= ln|I| + ln[\frac{1}{|I|}\sum_{i\in I}(b_i+1)^2\delta_i^2]. \end{aligned}$$

Thus, the forecast error ratio equals

$$e^{P'-P} = \exp(ln[\frac{1}{|I|}\sum_{i\in I}(b_i+1)^2\delta_i^2] - \frac{1}{\pi}\int_0^\pi ln[\frac{1}{|I|}\sum_{i\in I}\omega_i(x)]dx).$$

Following the Strong Law of Large Numbers (Karr 1993 Theorem 5.33),

$$\begin{aligned} \lim_{|I|\to\infty}\frac{1}{|I|}\sum_{i\in I}(b_i+1)^2\delta_i^2 &= \int_{\rho,\delta}(b+1)^2\delta^2 f(\rho)g(\delta)d\rho d\delta, a.s. \\ \lim_{|I|\to\infty} ln[\frac{1}{|I|}\sum_{i\in I}(b_i+1)^2\delta_i^2] &= ln[\int_{\rho,\delta}(b+1)^2\delta^2 f(\rho)g(\delta)d\rho d\delta], a.s. \end{aligned}$$

The second equality comes from Karr 1993 Theorem 5.23 (continuous mappings). Similarly, for all $x \in [0,\pi]$,

$$\lim_{|I|\to\infty}\frac{1}{|I|}\sum_{i\in I}\omega_i(x) = \int_{\rho,\delta}\omega(x)f(\rho)g(\delta)d\rho d\delta, a.s.$$

Notice that $\omega(x) = \sigma^2 + 2\lambda\dfrac{cos(x)-\rho}{1-2\rho cos(x)+\rho^2} = \dfrac{\delta^2(1+2b(b+1)-2b(b+1)cos(x))}{1-2\rho cos(x)+\rho^2}$. It's easy to see

$$0 < \frac{\delta^2}{(1+\rho)^2} < \omega(x) < \frac{\delta^2(1+4b(b+1))}{(1-\rho)^2} < \infty,$$

for $\rho \in [0, 1-\epsilon]$ and $\delta > 0$.

We now verify the conditions of lemma 5.1 for $\omega(x)$: $[0, \pi]$ is compact. $\omega(x) - E_{\rho,\delta}(\omega(x))$ is continuous in $x$ for all $\rho, \delta$, and it satisfies the third condition of lemma 5.1 if we choose $d(\rho, \delta) = \dfrac{\delta^2(1 + 4b(b+1))}{(1-\rho)^2}$. From the Strong Law of Large Numbers, it is easily seen that $|\frac{1}{|I|} \sum_{i \in I}(\omega_i(x) - E_{\rho,\delta}(\omega(x)))| \to 0$ almost surely given $\forall x \in [0, \pi]$. Thus, the lemma 5.1 implies $\sup_{x \in [0,\pi]} |\frac{1}{|I|} \sum_{i \in I}(\omega_i(x) - E_{\rho,\delta}(\omega(x)))| \to 0$, a.s.

By definition, the Uniform Strong Law of Large Numbers implies

$$P\{(\overline{\rho}, \overline{\delta}) : | \lim_{|I| \to \infty} \frac{1}{|I|} \sum_{i \in I} \omega_i(x) = E_{\rho,\delta}(\omega(x)), \forall x \in [0, \pi]\} = 1,$$

where $(\overline{\rho}, \overline{\delta})$ is an element in the sample space. Define

$$W = \{(\overline{\rho}, \overline{\delta}) : | \lim_{|I| \to \infty} \frac{1}{|I|} \sum_{i \in I} \omega_i(x) = E_{\rho,\delta}(\omega(x)), \forall x \in [0, \pi]\}.$$

For any $(\overline{\rho}, \overline{\delta}) \in W$, we have

$$\begin{aligned} \lim_{|I| \to \infty} \frac{1}{|I|} \textstyle\sum_{i \in I} \omega_i(x) &= E_{\rho,\delta}(\omega(x)), \forall x \in [0, \pi] \\ \lim_{|I| \to \infty} ln[\frac{1}{|I|} \textstyle\sum_{i \in I} \omega_i(x)] &= ln[E_{\rho,\delta}(\omega(x))], \forall x \in [0, \pi]. \end{aligned}$$

The second equation comes from the fact that $ln(\cdot)$ is continuous and finite. Because $ln[\frac{1}{|I|} \sum_{i \in I} \omega_i(x)]$ is uniformly bounded for all $x \in [0, \pi]$ and $\rho, \delta$, by Dominated convergence theory, we have

$$\int_0^\pi ln[\frac{1}{|I|} \sum_{i \in I} \omega_i(x)]dx \to \int_0^\pi ln[E_{\rho,\delta}(\omega(x))]dx, \forall (\overline{\rho}, \overline{\delta}) \in W.$$

Which implies

$$\lim_{|I| \to \infty} \frac{1}{\pi} \int_0^\pi ln[\frac{1}{|I|} \sum_{i \in I} \omega_i(x)]dx = \frac{1}{\pi} \int_0^\pi ln[\int_{\rho,\delta} \omega(x) f(\rho) g(\delta) d\rho d\delta]dx, a.s.$$

Thus, the forecast error ratio converges to

$$\exp(ln[\int_{\rho,\delta}(b+1)^2\delta^2 f(\rho)g(\delta)d\rho d\delta] - \frac{1}{\pi}\int_0^{\pi} ln[\int_{\rho,\delta}\omega(x)f(\rho)g(\delta)d\rho d\delta]dx)$$

almost surely.

Observe that

$$\int_{\rho,\delta}(b+1)^2\delta^2 f(\rho)g(\delta)d\rho d\delta = \int_{\delta(1)}^{\delta(2)}\delta^2 g(\delta)d\delta \times S,$$

$$\begin{aligned}\textstyle\int_{\rho,\delta}\omega(x)f(\rho)g(\delta)d\rho d\delta &= \textstyle\int_{\delta(1)}^{\delta(2)}\delta^2 g(\delta)d\delta \times \int_0^{1-\epsilon}(\sigma^2 + 2\lambda\frac{cos(x)-\rho}{1-2\rho cos(x)+\rho^2})f(\rho)d\rho \\ &= \textstyle\int_{\delta(1)}^{\delta(2)}\delta^2 g(\delta)d\delta \times [r+u(x)],\end{aligned}$$

where

$$\begin{aligned} S &= \textstyle\int_0^{1-\epsilon}(b+1)^2 f(\rho)d\rho \\ r &= \textstyle\int_0^{1-\epsilon}\frac{1+\rho-2\rho^{L+2}-2\rho^{L+3}+2\rho^{2L+4}}{(1-\rho)(1-\rho^2)}f(\rho)d\rho \\ u(x) &= \textstyle\int_0^{1-\epsilon}\frac{2\rho^{L+2}(1+\rho-\rho^{L+2})}{1-\rho^2}\frac{cos(x)-\rho}{1-2\rho cos(x)+\rho^2}f(\rho)d\rho. \end{aligned}$$

Canceling the terms associated with $\delta$, the forecast error ratio converges to

$$S/\exp(\frac{1}{\pi}\int_0^{\pi} ln[r+u(x)]dx), a.s.$$

Second, we prove (3). As $L \to \infty$, $b+1 \to \frac{1}{1-\rho}$ thus

$$\begin{aligned} S &\to \textstyle\int_0^{1-\epsilon}\frac{1}{(1-\rho)^2}f(\rho)d\rho \\ r &\to \textstyle\int_0^{1-\epsilon}\frac{1+\rho}{(1-\rho)(1-\rho^2)}f(\rho)d\rho \\ &= \textstyle\int_0^{1-\epsilon}\frac{1}{(1-\rho)^2}f(\rho)d\rho \\ u(x) &\to 0. \end{aligned}$$

Finally, $\exp(\frac{1}{\pi}\int_0^{\pi} ln(r)dx) = r = \int_0^{1-\epsilon}\frac{1}{(1-\rho)^2}f(\rho)d\rho$. As desired. ∎

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