# Incentives and Coordination in Project-Driven Supply Chains

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**Abstract**

Collaboration and partnership are the way of life for large complex projects in many industries. While they offer irresistible benefits in market expansion, technological innovation, and cost reduction, they also present a significant challenge in incentives and coordination of the project supply chains. In this paper, we study strategic behaviors of firms under the popular loss-sharing partnership in joint projects by a novel model that integrates the economic theory of teamwork with project management specifics. We provide insights into the impact of collaboration on the project performance. For a general project network with both parallel and sequential tasks where each firm faces a time-cost trade-off, we find an inherent conflict of interests between individual firms and the project. Depending on the cost and network structure, we made a few surprising discoveries, such as, the Prisoners' Dilemma, the Supplier's Dilemma, and the Coauthors' Dilemma; these dilemmas reveal scenarios in which individual firms are motivated to take actions against the best interests of the project and exactly how collaboration can hurt. As remedy, we enhance collaboration by a set of new provisions into a "fair sharing" partnership and prove its effectiveness in aligning individual firms' interests with that of the project.

**Keywords:** Collaboration, partnerships, outsourcing, project management, supply chain coordination, time-cost trade-off, loss sharing, fair sharing.

# 1 Introduction

Over the last three to four decades, advances in technology and the networked economy have led to the evolution of the business models in many project driven industries, from the "one-firm-does-all" approach to a more collaborative one on a global basis. Examples can be found in book publishing, commercial aerospace, and engineering-procurement-construction (EPC) industries. While projects in these industries vary significantly in content and scale, they share the following commonalities: First, they require diverse knowledge and expertise; Second, they demand a significant investment of time and/or capital up front. The significant up front investment mandates market expansion a necessity for success.

The book publishing industry is popularized by books with many coauthors. Using textbooks on operations management as an example, a simple search of the key-word "operations management" on Amazon.com in September 2013 returns 48 textbooks which are the most relevant (definition: (1) production & operations section (2) hardcover (3) four stars & up). Among them, 17 (35.42%) are single authored, 19 (39.58%) have two authors, and the rest have three or more authors. Thus, coauthored books account for a majority (about 65%) of the most relevant textbooks on operations management. Replicating the search on "supply chain management" and "marketing science" returns similar results.

In the commercial aerospace industry, suppliers are playing an increasingly important role in the development of new aircrafts. Recent examples are Boeing 787 Dreamliner, Airbus 380, China Comac C919 and Airbus 350. In particular, the Boeing 787 Dreamliner outsourced 65% of the development work to more than 100 suppliers from 12 countries (see Horng and Bozdogan (2007) and Exostar (2007)). Tier 1 suppliers design and fabricate 11 major subassemblies, Boeing integrates and assembles the airplane. To manage the relationship with the suppliers, Boeing made the suppliers stakeholders of the program by establishing a collaborative partnership (similar to the coauthorship) where the suppliers are responsible for the non-recurring development cost of their tasks and must wait until the completion of the project to get paid (see Xu and Zhao, 2011).

In the EPC industries, the $150 billion international space station (ISS) is a representative example where the design and construction of ISS are spread out to fifteen countries around the world. The elements of ISS are not assembled on the ground but launched from different countries and mated together on orbit. Each country invests a huge amount of money into its elements and takes the responsibility of their maintenance. Five countries are the principals (partners) of ISS due to their significant contributions (see NASA, 2013).

As we can see, collaboration and partnership are everywhere, especially in large complex projects. By definition (Macmillan Dictionary), collaboration is "the action of working with someone to produce or create something". In the project management context, we define collaboration the basic form precisely as follows: the workload of a project, for instance, different tasks, is spread out to multiple players (firms) where each player is fully responsible for the financial needs of its own tasks until the completion of the project and share the revenue (or the credit or the utility) when the project is completed. This definition is consistent to the coauthorship in book publishing, the collaborative partnership of the Boeing 787 Dreamliner program, and the agreement among multiple countries for the International Space Station. For the ease of exposition, we call the financial arrangement of this kind of collaboration, the "loss-sharing partnership", as the loss due to a project delay is shared among all players. We also call the supply chain created by spreading the workload of a project among multiple firms "a project-driven supply chain".

Collaboration and partnership offer significant benefits to projects: First, they allows the project to utilize the best in-class expertise and knowledge. For instance, authors with different expertise can combine their domain knowledge in a single book. Second, a collaborative partnership allows multiple players to share the up front investment and thus make a costly project that is infeasible for any individual player feasible, as in the ISS project. Thirdly, collaboration and partnership are essential to market expansion. As witnessed in the Boeing 787 Dreamliner program, the suppliers are the stakeholders of the program and thus are motivated to sell the plane in their own countries and keep the customers waiting despite the significant delay of the program.

Collaboration (and partnership) is one way to outsource the workload of a project, subcontracting is another. Collaboration (and the "loss-sharing " partnership) differs from subcontracting because in the latter, suppliers get paid when their tasks are completed and certified. Thus in subcontracting, a supplier's interests are tied only to its tasks, whereas in collaboration, its interests are tied to the project. This difference is important because collaboration provides a much stronger incentive than subcontracting to the players to expand the market (so everyone gets more) and keep customers waiting until the final completion of the project (so everyone loses less).

Although the benefits are irresistible, collaboration (and partnership) poses a significant challenge in the incentive and coordination of joint projects (or project-driven supply chains); in the economics terms, collaboration may suffer the externalities. To see this intuitively, let's consider a

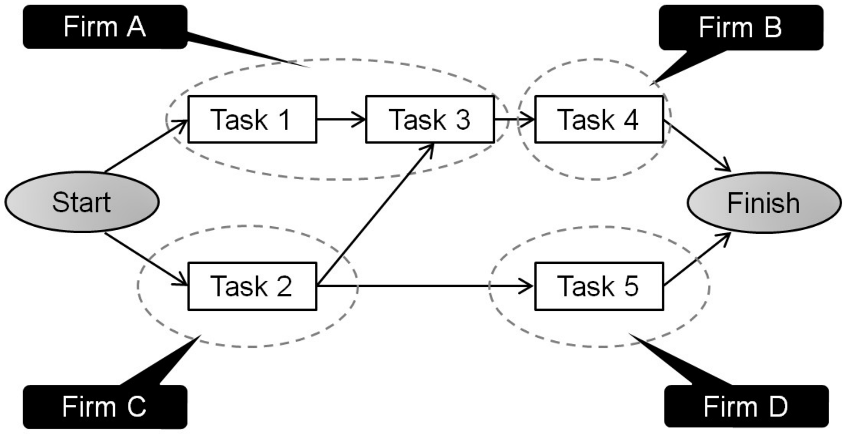


Figure 1: Collaboration in a joint project.

simple example (see Figure 1) where a project has five tasks and four participating companies. It is easily seen that firm B can only start its task after firm A completes its tasks, and has to watch out for firm D's completion time to determine its own task duration. Thus each company's cost and schedule depend not only on its own effort but also on the efforts of other companies working on other parts of the same project. In this way, collaboration introduces gaming issues to project management where the ultimate goal of each firm is to optimize its own benefit even if doing so harms the interests of the project.

Although the economics and supply chain literatures study externalities and gaming issues extensively, they rarely consider project management specifics, e.g., project networks, cost structures and time-cost trade-off. In this paper, we combine the game theoretical models of the economics and supply chain literatures with operational specifics drawn from the project management literature to study strategic gaming behaviors of firms under loss sharing partnership in joint (i.e., collaborative) projects. Our objective is to provide insights into the following issues: (1) What is the performance of the project in time and cost under loss sharing? (2) How do project network and cost structure affect the results? (3) How to design a collaborative partnership that aligns the interests of the firms with that of the project?

To this end, we consider a two-level project network with parallel tasks (e.g., subsystems) in the first level and an integration task (e.g., final assembly) in the second level. Such a project network is quite representative in practice. Each firm faces a time-cost trade-off and must decide its task duration. We study various cost and network structures and characterize the subgame perfect equilibriums either in closed-form or by numerical algorithms. We find that under the loss sharing partnership, there is an inherent mismatch between individual firms' best interests and that of the project. Depending on the cost and network structures, we made a few surprising discoveries, such as (1) the Prisoners' Dilemma: even though keeping the optimal schedule benefits the entire project, it can be in each firm's best interests to delay; (2) the Supplier's Dilemma: if costs are time-dependent, the supplier may have to delay (even at a loss) in order to raise the penalty too high for the manufacturer to delay, to avoid a greater loss; (3) the Coauthors' Dilemma: a firm can expedite its task but cannot expedite the project because if it expedites, other firms will delay. Finally, we present a new "fair sharing" partnership which enhances collaboration the basic form (the loss sharing partnership) by a set of new provisions and prove its capability to align individual firms' financial interests with that of the project.

The paper is organized as follows. In §2, we review the related literature; which is followed by §3 where we introduce our models and methodology. In §4, we study firms' strategic gaming behaviors under loss sharing. In §5, we present the "fair sharing" partnership and prove its effectiveness. We conclude the paper in §6 with a brief summary of our results.

## 2 Literature

This paper is related to the bodies of literature on project management, economics theory of teamwork, development chain management and project/supply chain interfaces. We shall review related results in each area and point out the difference from our work.

**Classic project management literature**. The most well known results in this literature include the critical path method (CPM), project evaluation and review techniques (PERT), time-cost analysis (TCA), and resource constrained project scheduling (RCPS). This literature focuses on the scheduling and planning of project(s) within a single firm and thus the main issue is on optimization. We refer the reader to Nahmias (2008) and Jozefowska and Weglarz (2006) for recent surveys. Our paper draws the project management details, e.g., cost structure, project network and time-cost trade-off, from this literature but analyzes incentives and gaming behaviors under partnerships in a multi-firm joint project by a game theoretic model.

**Classic economics literature of teamwork**. The economics literature of teamwork discusses incentives and contracts in general teamwork settings. This literature is vast, we refer the reader to several seminal papers, e.g., Holmstrom (1982), Demski and Sappington (1984), McAfee and McMillan (1986), and Holmstrom and Milgrom (1991), for principal-agent models and moral hazard games; and Bhattacharyya and Lafontaine (1995), Kim and Wang (1998), and Al-Najjar (1997) for the double moral hazard games. Our paper enriches and expands this literature by integrating the general economics theory with project management specifics.

**Bidding and subcontracting in project management**. This body of literature studies project management issues involving multiple firms, such as project bidding and subcontracting. Elmaghraby (1990) studies project bidding under deterministic and probabilistic activity durations from the contractor's perspective, while Gutierrez and Paul (2000) compares fixed price contracts, cost-plus contracts and menu contracts in project bidding from the project owner's perspective. Paul and Gutierrez (2005) studies how to assign tasks to contractors for projects with parallel or serial tasks. Szmerekovsky (2005) studies the impact of payment schedule on contractors' performance. In this model, the owner selects the payment terms in the first place, the contractor then decides the schedule to maximize its net present value. Aydinliyim and Vairaktarakis (2010) considers a set of manufacturers who outsource certain operations to a single third party by booking its capacity, and the third party identifies a schedule that minimizes the total cost for all manufacturers. Our paper differs from this literature in two ways: first, we consider collaboration and partnerships which are structurally different from subcontracting as shown in §1. Second, all partners considered in this paper have to contribute to the workload and share the outcome, while in the subcontracting literature, the project owner does not work but only supervises the contractors' work.

**Development chain management**. This stream of literature studies issues in the development of new products within a single firm and more recently involving multiple firms. For instance, Bhaskaran and Krishnan (2009) studies a development chain with two firms, a focal firm and a partner firm. Their model considers the cost, time, and quality triangle under three partnerships: revenue sharing, investment sharing and innovation sharing. They show that simple revenue sharing does not work well and leads to underinvestment in quality improvements. Alternatively, the investment sharing and innovation sharing, are better than revenue sharing in collaboration. Our paper contributes to this literature by incorporating project management specifics, such as, project network and time-cost trade-off (concepts developed in the classic project management literature) into the analysis.

**Project management and supply chain interfaces**. This literature studies the management of projects that involve multiple firms from a supply chain perspective and consider project management specifics. It is a fairly new research area but has attracted quite some attentions recently from the operations management community. For instance, Bayiz and Corbett (2005) introduces a principal-multi-agent game to project management by considering projects either with two sequential tasks or with two parallel tasks. They analyze the effectiveness of the fixed-price contracts versus incentive contracts in a subcontracting arrangement. Kwon, Lippman, McCardle, and Tang (2010) analyzes delay payment versus no delay payment in a project management setting where different but parallel tasks are done by different suppliers. They consider a simultaneous game among suppliers while the manufacturer does not contribute to the project but only selects payment regimes. By assuming exponentially distributed task durations, they showed that the delayed payment regime is more preferred by the manufacturer when its revenue is low. In addition, under information symmetry, the delayed payment regime is preferred in the presence of a large number of suppliers. In our paper, the manufacturer contributes to the workload and so the project network has tasks both in parallel and in sequential. This new network entails a more delicate interaction among the suppliers and the manufacturer, and provides a rich ground for new discoveries and insights.

# 3 The Model and Preliminaries

In this section, we introduce the fundamentals of our model. First, we present the project management specifics such as the project cost structure and project network. Second, we provide more details on the loss sharing and fair sharing partnerships. Finally, we present the game theoretical model and our methodology.

**Project Cost Structure**. We can classify project costs into two categories: direct cost and indirect cost. Direct cost includes all costs directly contributing to a task, such as the cost of management, labor, material and shipping. Normally, a longer task duration is coupled with a lower direct cost. Indirect cost includes all costs not directly contributing to tasks but depending on the project duration, such as the overhead (e.g., rent, utilities, benefits), interests and financial costs, delay penalty and order cancellation loss. Normally, a longer project duration is coupled with a higher indirect cost. We refer the reader to Nahmias (2008)) for more details.

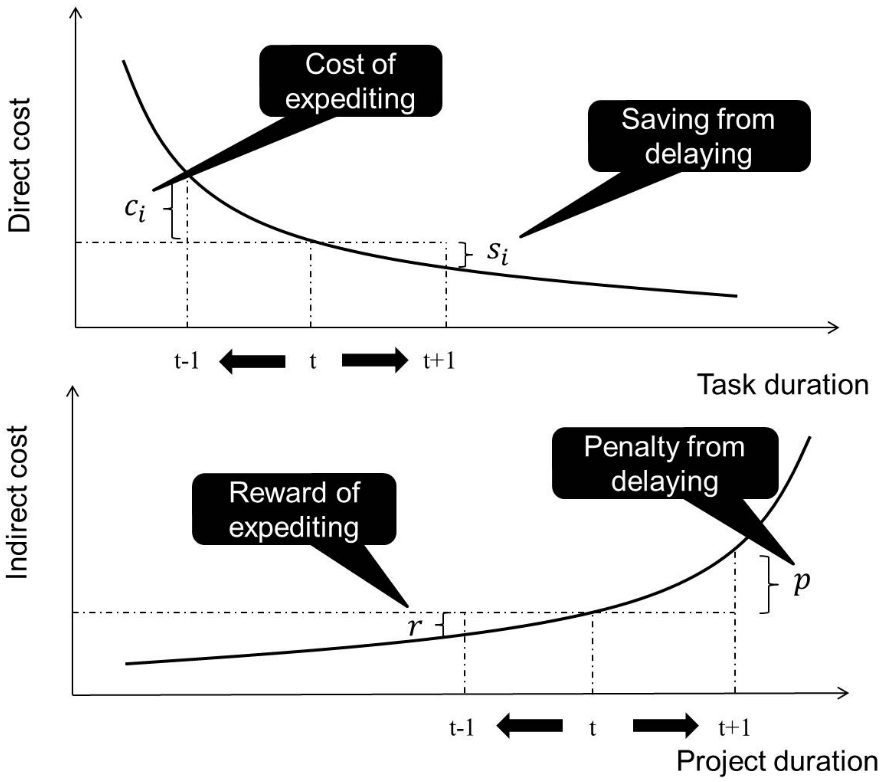


Figure 2: Project Cost Structure.

Consistent to a majority of practical situations, we assume that direct cost is convex and decreasing as task duration increases and indirect cost is convex and increasing as project duration increases (Figure 2, Nahmias (2008)). If task $i$ is delayed by one period, firm $i$ saves $s_i$ in the direct cost. If the project is delayed by one period, it suffers a penalty $p$ in the indirect cost. Conversely, if task $i$ is expedited by one period, firm $i$ incurs a cost $c_i$ for expediting. If the project is completed one period earlier, it receives a reward $r$.

**Project Network**. We consider projects with a network structure shown in Figure 3. It has two levels: At level 1, there are several tasks to be completed simultaneously, similar to the design and fabrication of subsystems in the 787 Dreamliner program, the writing of individual chapters in a coauthored book, and the development of subsystems and components of the International Space Station (ISS). At level 2, there is only one task that is to integrate and assemble all parts completed in level 1, similar to the system integration task in the 787 Dreamliner program, the integration and proofreading of a coauthored book, and the final assembly and testing task of the ISS. Clearly

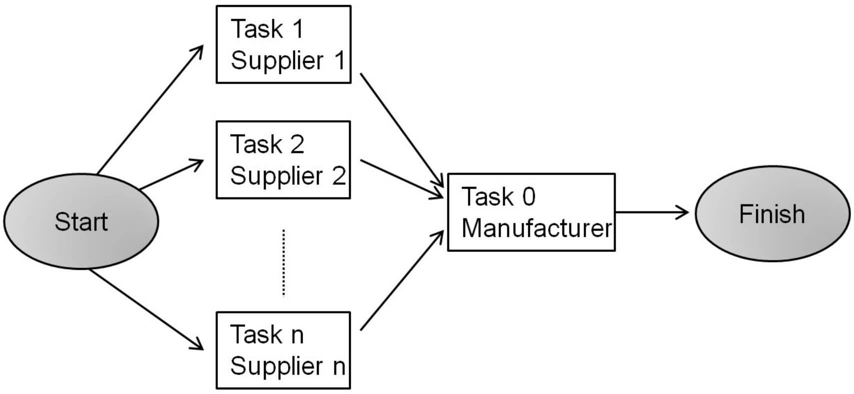


Figure 3: Project Network.

the task at level 2 cannot start until all tasks at level 1 are completed.

Figure 3 shows the general project network, where $n = 1$ denotes the case with only one task at level 1, and thus the project network reduces to two sequential tasks. When $n \geq 2$, there are multiple tasks at level 1, and the project network has an assembly structure. We will discuss these two cases in the paper.

**The Loss Sharing Partnership**. In this partnership, each firm pays for the direct and indirect costs of its own task(s), and get paid when the project is done. We observe that under loss sharing, if a firm delays its task, it saves on its direct cost but everyone (including the delayed firm) suffers an increase (a penalty) in indirect cost if the firm's delay results in a project delay. Thus other firms on time are penalized by this firm's delay, and this delayed firm is not fully responsible for the consequences of its action as the penalty is shared among all firms. While this observation presents a "moral hazard" issue well known in the economics literature of teamwork, it is not known exactly how such an issue may affect the time and cost metrics in a project management setting, which is the focus of this paper.

**The Fair Sharing Partnership**. This partnership works in the same way as loss sharing except that every firm is fully responsible for the consequence of its action. Intuitively, if one firm causes damage to others, it has to compensate others; if it brings benefit to others, it receives compensation from others; we refer the reader to §5 for the exact mechanisms of this partnership.

**Game Theoretical Framework**. We assume that each task in the 2-level project network is

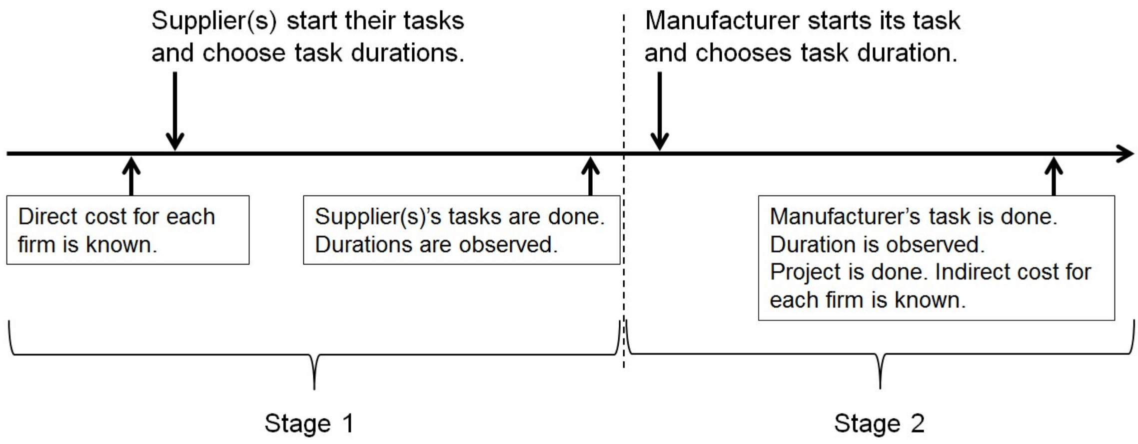


Figure 4: Sequence of Events.

assigned to a different firm. For the ease of exposition, we use "supplier(s)" to name the firm(s) responsible for the tasks at level 1 and "manufacturer" to name the firm responsible for the task at level 2. By the structure of the project network, a two-stage game theoretic model is appropriate for predicting the behaviors of the supplier(s) and the manufacturer in equilibrium. The sequence of events is described as follows (see also Figure 4): At the beginning of stage 1, supplier(s) start their tasks and choose task durations. After all suppliers complete their tasks, stage 1 is concluded. At the beginning of stage 2, the manufacturer starts its task and chooses the task duration. When the manufacturer completes its task, stage 2 is drawn to an end and the project is completed. In this game, the suppliers take the lead by taking actions first (anticipating the manufacturer's response) and the manufacturer follows by responding accordingly. We assume information symmetry thus the direct and indirect cost functions of all players are public knowledge. Under either partnership, each firm aims to maximize its own profit by determining the duration of its own task. We shall derive subgame perfect Nash equilibrium (SPNE) for each case considered below and compare the resulting project performance to the global optimum. If the SPNE is not unique, we shall compare different SPNEs and report the Pareto or strong equilibrium.

**Methodology**. To understand the firms' strategic behaviors under loss sharing and how they may deviate from the optimal decisions under the "one-firm-does-all" (centralized control) model, we assume that the project starts with an "original schedule" and "original task durations" that are optimal under the centralized control. We first analyze one-period models in which each firm can delay or expedite its original task duration by at most one period. Then we relax this constraint to allow the firms to delay or expedite multiple periods. To study the impact of cost structure and project network on the firms' behaviors, we consider both time-independent and time-dependent costs, and both one supplier and multi-supplier cases.

# 4 The Loss Sharing Partnership

In this section, we study firms' strategic behaviors under the loss sharing partnership. We start with the base model in §4.1 which assumes only one supplier and time-independent cost. In this model, each firm can either "keep" the original task duration or "delay" it by one period. In §4.2, we relax the time-independent cost assumption in the base model to allow time-dependent costs, for instance, delay penalty per period may increase as the project delay increases. In §4.3, we consider the base model but allow each firm an additional option of "expediting" its task by one period. In §4.4, we extend the base model to include multiple suppliers, and in the last subsection, §4.5, we consider a general model and develop structural results and algorithms for the equilibrium.

## 4.1 The Base Model – The Prisoners' Dilemma

In this section, we consider the base model (defined by Assumption 1). Our objective is to understand the impact of collaboration and the loss sharing partnership on the project performance in both time and cost.

**Assumption 1** *At level 1 of the project network, there is only one task. Each task cannot be expedited but can be delayed by at most one period. If the project is delayed, it is subject to a penalty which is time independent.*

In this model, the supplier and manufacturer only have two options (actions) available: "keep" (keeping the original task duration) or "delay" (delaying it by one period). We use K for "keep" and D for "delay" for simplicity. We assume that firm $i$ is responsible for task $i$ for $i = 0, 1$ where firm 1 (or 0) refers to the supplier (or manufacturer, respectively). The action set, [supplier's action, manufacturer's action], is {[K, K], [D, D], [K, D], [D, K]}. When task $i$ is delayed, firm $i$ receives a saving of $s_i$ in terms of its direct cost. When the project is delayed, a penalty of $p$ per period in terms of the indirect cost is shared by the firms, where firm $i$ pays $p_i$ and $p_0 + p_1 = p$.

Recall that, by assumption, the project starts with an original schedule that is optimal under the centralized control. In other words, the action set [K, K] has a pay-off higher than those under

[D, K], [K, D] and [D, D] for the project as a whole. To this end, we need the following necessary condition,

**Condition 1** *Global Optimum - Base Model:* $s_1 < p$, $s_0 < p$.

We can easily verify Condition 1 as follows: at [K, K], there is neither a saving nor a penalty for the project, and thus the pay-off of the project relative to the original schedule is zero. At [D, K], task 1 is delayed by one period but task 0 is kept at its original duration. Thus, we receive a saving of $s_1$ from task 1 but must pay a penalty of $p$ because the project is delayed by one period. The pay-off of the project is $s_1 - p$ and thus $s_1 < p$ is a necessary condition for [K, K] to outperform [D, K] from the project's perspective. Repeating a similar logic to [K, D] and [D, D] leads to Condition 1.

Now we are ready to study the firms' strategic behaviors under the loss sharing partnership and their impact on project performance. Before introducing the general theory, we first present an example (see Figure 5) to illustrate the key idea and insight. In this example, task 1 has an original duration of 9 weeks, which can be delayed to 10 weeks with a saving of $s_1 = \$900$. Task 0 has an original duration of 5 weeks which can be delayed to 6 weeks with a saving of $s_0 = \$1200$. The project is due in 14 weeks; each week of delay incurs a penalty of $p = \$1600$ for the project. Clearly, Condition 1 is satisfied in the example, and so it is in the project's best interests to keep the original schedule.

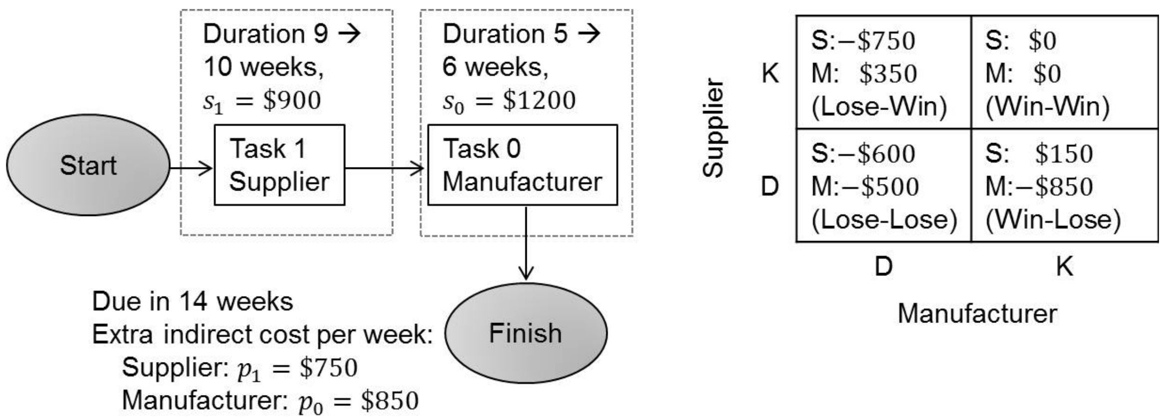


Figure 5: An example of the base Model and its pay-off matrix. (K:keep, D:delay)

Under the lost sharing partnership, we assume that upon each week of the project's delay, the supplier's share of the penalty is $p_1 = \$750$ and the manufacturer's share is $p_0 = \$850$. To see what the supplier and the manufacturer would do in their own best interests (i.e., the equilibrium), we consider the following four scenarios:

- **Win-Lose**: firm 1 (the supplier) delays but firm 0 (the manufacturer) keeps its original task duration. In this scenario, firm 1 saves \$900 but must pay \$750 with a net gain of \$150. However, firm 0 must pay \$850 for firm 1's delay. The firms' pay-offs (relative to the original schedule) are $(\pi_1, \pi_0) = (150, -850)$ and the project's pay-off is $-\$700$.

- **Lose-Win**: firm 1 keeps its original task duration but firm 0 delays. In this scenario, firm 0 saves \$1200 but must pay \$850 with a net gain of \$350. However, firm 1 must pay \$750 for the delay caused by firm 0. The firms' pay-offs are $(-750, 350)$ and the project's pay-off is $-\$400$.

- **Lose-Lose**: both firms delay. In this scenario, the project is delayed by two weeks and the firms' pay-offs are $(-600, -500)$. This is the worst scenario for the project as a whole with a total loss of \$1100.

- **Win-Win**: both firms keep their original task duration. This is the best scenario for the project where both the firms and the project lose nothing with a pay-off of zero (relative to the original schedule).

Figure 5 summarizes the action sets and the corresponding pay-off matrix. We can see that no matter what the supplier does, the manufacturer's optimal strategy is always to "delay". In other words, "delay" is the dominant strategy for the manufacturer. Similarly, the supplier's best strategy is also to "delay" regardless of the manufacturer's response. Thus, although the "Win-Win" scenario has the best outcome for the project, it is unstable – each firm will find every excuse to delay. The "Lose-Lose" scenario, although having the worst outcome for the project, is the subgame perfect Nash equilibrium (SPNE), as in a typical Prisoners' Dilemma.

We now present the general theory for the base model. Note that in this game, the supplier leads and the manufacturer follows (see §3). If the project is finished on time, there is no penalty. For every period of the project delay, the supplier pays a penalty of $p_1$ and the manufacturer pays the rest which is $p_0$. The firm whichever delays obtains a saving from the direct cost of its own task. For example, if the supplier delays but the manufacturer keeps the original duration of its task, the supplier saves $s_1$ from its direct cost which brings its pay-off to be $s_1 - p_1$, and the manufacturer bears a pure penalty of $p_0$. Figure 6 shows the extensive form of the game in the base model.

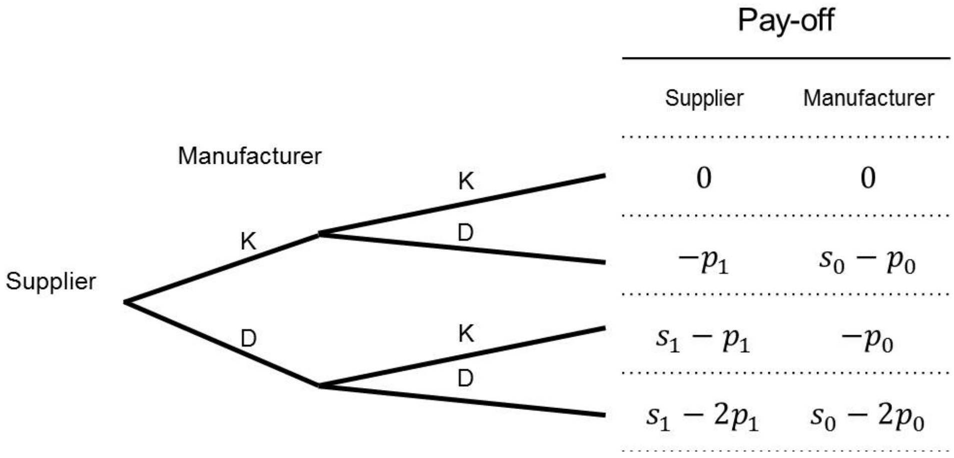


Figure 6: The extensive form of the game in the base model.

We derive the following results on the dominant strategies and equilibrium (all proofs of this paper are presented in the Appendix unless otherwise mentioned).

**Lemma 1 (Dominant Strategy)**: *Under Condition 1, when $s_i < p_i$, "keep" is the dominant strategy for firm $i$, $i = 0, 1$; when $s_i > p_i$, "delay" is the dominant strategy for firm $i$, $i = 0, 1$.*

For simplicity, we use "S" ("M") to denote the supplier (the manufacturer, respectively).

**Theorem 1 (Equilibrium)**: *For the base model, under Condition 1, the subgame perfect Nash equilibrium (SPNE) is given by,*

| *Case* | *Condition on S* | *Condition on M* | *Optimal strategy for S* | *M's best response* |
|---|---|---|---|---|
| *1* | $s_1 < p_1$ | $s_0 < p_0$ | *K* | *K* |
| *2* | $s_1 > p_1$ | $s_0 < p_0$ | *D* | *K* |
| *3* | $s_1 < p_1$ | $s_0 > p_0$ | *K* | *D* |
| *4* | $s_1 > p_1$ | $s_0 > p_0$ | *D* | *D* |

Based on these results, we present the following key insight for the base model under the loss sharing partnership:

**The Prisoners' Dilemma**: *In the base model, for a schedule to be optimal, we need $s_1 < p, s_0 < p$ (Condition 1). For the optimal schedule to be the SPNE under loss sharing, a much stronger condition is required, that is, $s_1 < p_1$ and $s_0 < p_0$ where $p_1 + p_0 = p$. Thus, if $s_1 > p_1$ and $s_0 > p_0$*

*but $s_1 < p$ and $s_0 < p$, then it is in each firm's best interests to delay although being on time benefits the entire project.*

### 4.2 The Base Model with Time-dependent Costs – The Supplier's Dilemma

In this section, we relax the "time-independent cost" assumption in the base model to study the impact of time-dependent penalty costs on the results, e.g., the dominant strategies, the Prisoners' Dilemma. We define the model by Assumption 2.

**Assumption 2** *Assumption 1 holds here except that project delay penalties are time dependent.*

Let $p^1$ (or $p^2$) be the penalty for the 1st (the 2nd, respectively) period of project delay; and let $p_i^1$ and $p_i^2$) be the corresponding penalties shared by firm $i$, where $p_1^1 + p_0^1 = p^1$ and $p_1^2 + p_0^2 = p^2$. The assumption of starting with the optimal schedule and the assumptions of convex and increasing cost functions (see §3) mandate,

**Condition 2** *(1) Global Optimum - Time-Dependent: $s_1 < p^1$, $s_0 < p^1$. (2) Monotonicity - Time-Dependent: $p^1 < p^2$, $p_1^1 < p_1^2$, $p_0^1 < p_0^2$.*

To see the impact of time-dependent penalty costs, we slightly modify the example in §4.1 (shown in Figure 5). In this modified example, everything remains the same except that (1) the saving per week for task 1 is reduced to $s_1 = \$600$ from \$900; (2) the second period delay penalty of the project, $p^2$, is increased to \$2500 from \$1600, where the supplier bears $p_1^2 = \$1100$ and the manufacturer bears $p_0^2 = \$1400$. Figure 7 depicts the modified example. Clearly, Condition 2 is satisfied in this example, and it is in the project's best interests to keep the original schedule.

We consider the following four scenarios under the loss sharing partnership,

- **"Win"-Lose**: firm 1 (the supplier) delays but firm 0 (the manufacturer) keeps its original task duration. In this scenario, firm 1 saves \$600 but must pay \$750 with a net loss of \$150, while firm 0 must pay \$850. The firms' pay-offs (relative to the original schedule) are $(\pi_1, \pi_0) = (-150, -850)$ and the project's pay-off is $-\$1000$.

- **Lose-Win**: firm 1 keeps its original task duration but firm 0 delays. This scenario is identical to the "Lose-Win" scenario of the example in §4.1 with the firms' pay-offs being $(-750, 350)$ and the project's pay-off being $-\$400$.

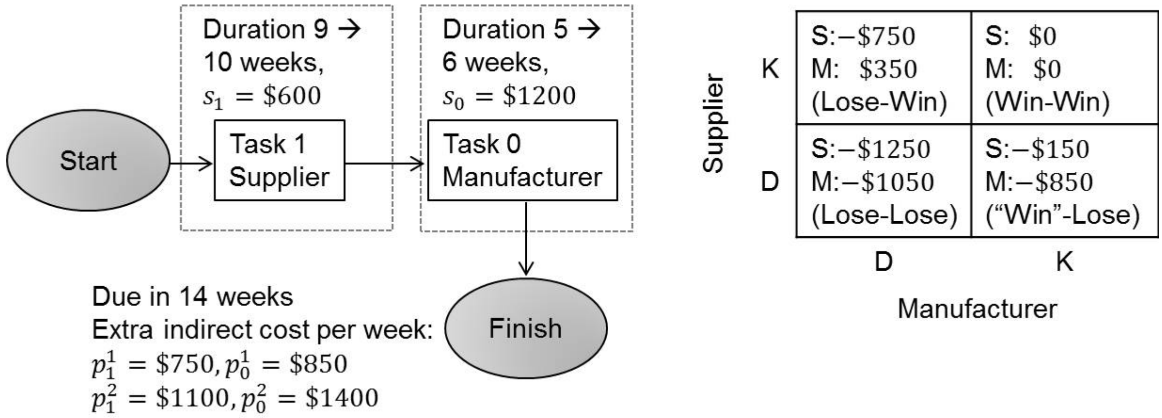


Figure 7: An example for the base model with time-dependent costs and its pay-off matrix. (K:keep, D:delay)

- **Lose-Lose**: both firms delays. In this scenario, the project is delayed by two weeks and the firms' pay-offs are $(-1250, -1050)$. This is the worst scenario for the project as a whole with a total loss of \$2300.
- **Win-Win**: both firms keep. The firms' pay-offs are $(0, 0)$.

Figure 7 summarizes the action set and the pay-off matrix. Clearly, if the supplier (firm 1) keeps its original task duration, the manufacturer's best response is to "delay" because its saving exceeds its penalty of the **1st** period project delay. However, if the supplier delays, the manufacturer's best response is to "keep" its original task duration because now its penalty of the **2nd** period project delay exceeds its saving. Thus the supplier has to delay (even at a loss) to raise the penalty so high that the manufacturer would have to keep, to avoid a greater loss. We call such a phenomenon the "Supplier's Dilemma". It is easy to verify that the SPNE in this example is [D, K].

We now analyze the base model with time-dependent costs in general. We note that the only difference between this model and the base model in §4.1 is that when both firms delay, the delay penalty is $p_i^1 + p_i^2$ for firm $i$. Figure 8 shows the extensive form of the game between the supplier and the manufacturer.

We can derive the following results on the dominant strategies and equilibrium.

**Lemma 2 (Dominant Strategy)**: *In the base model with time-dependent costs, under Condition 2, when $s_0 < p_0^1$, "Keep" is the dominant strategy for the manufacturer; when $s_0 > p_0^2$, "Delay" is the dominant strategy for the manufacturer.*

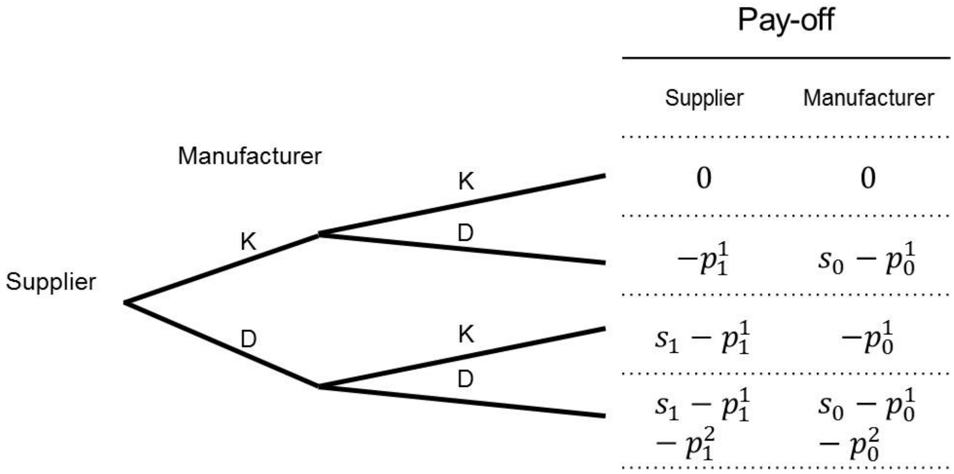


Figure 8: The extensive form of the game in the base model with time-dependent costs.

**Theorem 2 (Equilibrium)**: *For the base model with time-dependent costs, under Condition 2, the subgame perfect Nash equilibrium is given by:*

| *Case* | *Condition on S* | *Condition on M* | *Optimal strategy for S* | *M's best response* |
|---|---|---|---|---|
| *1* | $s_1 < p_1^1$ | $s_0 < p_0^1$ | $K$ | $K$ |
| *2* | $s_1 > p_1^1$ | $s_0 < p_0^1$ | $D$ | $K$ |
| *3* | | $p_0^1 < s_0 < p_0^2$ | $D$ | $K$ |
| *4* | $s_1 < p_1^2$ | $s_0 > p_0^2$ | $K$ | $D$ |
| *5* | $s_1 > p_1^2$ | $s_0 > p_0^2$ | $D$ | $D$ |

Theorem 2 is similar to Theorem 1 except for one new case (3rd case in Theorem 2): when $p_0^1 < s_0 < p_0^2$ (also illustrated in the example), the manufacturer's best strategy depends on the supplier's action. If the supplier keeps its original task duration, the manufacturer will delay; otherwise, the manufacturer will keep its original task duration. Thus, in this case, the supplier must take the manufacturer's response into account in making its own decision.

Based on these results, we present the following key insight for the base model with time-dependent costs under the loss sharing partnership:

**The Supplier's Dilemma**: *if $p_0^1 < s_0 < p_0^2$, the supplier has to delay (even at a loss) to raise the penalty too high for the manufacturer to delay, to avoid a greater loss.*

### 4.3 The Base Model with Expediting and Reward – The Coauthors' Dilemma

In this section, we relax the base model by allowing each firm an additional option: expediting by one period (see Assumption 3). With the new action of "expediting", the project could be completed earlier than the original schedule. The question is, will this happen in equilibrium under loss sharing?

**Assumption 3** *Assumption 1 holds here except that each task can be expedited by at most one period, and there is a reward per period if the project is expedited.*

We use "E" to denote "expediting". Let $c_0$ (or $c_1$) be the cost of expediting (i.e., the additional direct cost) for task 0 (or 1, respectively). Let $r$ be the reward for the project per period expedited, and $r_0$ and $r_1$ be rewards received by the firms where $r_1 + r_0 = r$. When a firm expedites, the pay-off functions are different from previous sections where firms cannot expedite. Specifically, if the supplier expedites, the action set [E, K] yields $-c_1 + r_1$ for the supplier and $r_0$ for the manufacturer, [E, D] yields $-c_1$ for the supplier and $s_0$ for the manufacturer, and [E, E] yields $-c_1 + 2r_1$ for the supplier and $-c_0 + 2r_0$ for the manufacturer. If the manufacturer expedites, the pay-off functions could be derived in a similar way.

As in all previous sections, we assume that the project starts with an original schedule that is optimal under the centralized control. To this end, Condition 3 (Global Optimum) provides a necessary condition. For instance, [E, K] should yield less profit for the entire project than [K, K], which requires $-c_1 + r_1 + r_0 < 0$, and [E, D] should yield less profit for the project than [K, K], which requires $s_0 < c_1$. Condition 3 (Monotonicity) comes from the assumption of convex and increasing indirect cost and convex and decreasing direct cost (see §3). Condition 3 (Loss Sharing) indicates that the monotonicity condition on the project's reward and penalty also applies to each firm's share of the reward and penalty.

**Condition 3** *(1) Global Optimum - Expediting:* $s_1 < p$, $s_0 < p$; $r < c_1$, $r < c_0$; $s_1 < c_0$, $s_0 < c_1$. *(2) Monotonicity - Expediting:* $r < p$; $s_1 < c_1$, $s_0 < c_0$. *(3) Loss Sharing - Expediting:* $r_1 < p_1$, $r_0 < p_0$.

The extensive form of the game is shown in Figure 9. For instance, if the supplier expedites while the manufacturer keeps its original task duration, the supplier gets an award of $r_1$ but must pay an expediting cost of $c_1$; the manufacturer gets an award of $r_0$ without any cost.

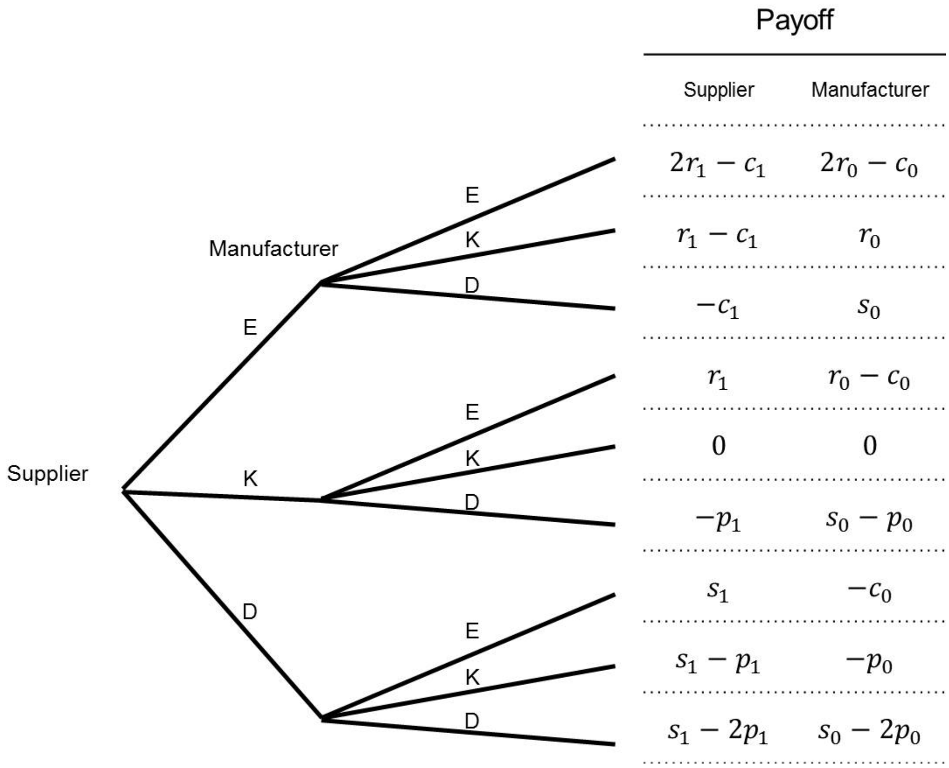


Figure 9: The extensive form of the game in the base model with expediting and reward.

We can derive the following results on the dominant strategies and equilibrium.

**Lemma 3 (Dominant Strategy)**: *In the base model with expediting and reward, under Condition 3, when $s_i > p_i$, "delay" is the dominant strategy for firm $i$, $i = 1, 0$; when $s_i < r_i < p_i < c_i$, "keep" is the dominant strategy for firm $i$, $i = 1, 0$.*

Lemma 3 differs from Lemma 1 on the conditions for "keep" because we must consider not only "delay" but also "expediting" in this model.

**Theorem 3 (Equilibrium)**: *For the base model with expediting and reward, under Condition 3, the subgame perfect Nash equilibrium is given by,*

| *Case* | *Condition on S* | *Condition on M* | *Optimal strategy for S* | *M's best response* |
|---|---|---|---|---|
| *1* | | $c_0 < p_0$ | *D* | *E* |
| *2* | $s_1 < p_1$ | $s_0 < p_0 < c_0$ | *K* | *K* |
| *3* | $s_1 > p_1$ | $s_0 < p_0 < c_0$ | *D* | *K* |
| *4* | $c_1 < p_1$ | $s_0 > p_0$ | *E* | *D* |
| *5* | $s_1 < p_1 < c_1$ | $s_0 > p_0$ | *K* | *D* |
| *6* | $s_1 > p_1$ | $s_0 > p_0$ | *D* | *D* |

Theorem 3 is similar to Theorem 1 except for the 1st and 4th cases that involve expediting and have equilibriums of [D, E] and [E, D]. We shall first explain the intuition behind these two new cases and then discuss the other cases.

- **1st case, $c_0 < p_0$, [D, E] is the equilibrium**: In this case, the manufacturer faces a delay penalty that is greater than its expediting cost, and so it would do anything to prevent the project from being delayed. Taking advantage of the manufacturer's weakness, the supplier could delay regardless of its own cost structure, and earn a net saving without any penalty. Thus, even if the manufacturer expedites its task, the project will not be expedited because the supplier will delay.

  **An example in the book publishing industry**: Let's consider a coauthor and a lead author working sequentially on a textbook. The coauthor writes parts of the book and must pass on the manuscripts to the lead author to integrate and complete. The lead author is responsible for the delivery and is very concerned about the deadline. Thus the lead author will do anything possible to finish the book on time. Knowing this, the coauthor will delay as much as what the lead author can catch up without a penalty.

- **4th case, $c_1 < p_1$ and $p_0 < s_0$, [E, D] is the equilibrium**: In this case, "delay" is the dominant strategy for the manufacturer (by Lemma 3). In addition, the supplier faces a delay penalty that is greater than its expediting cost, and so the supplier will have to expedite to prevent the project from being delayed.

  **An example in the academic thesis completion**: Let's consider a PhD student and his/her advisor. The student shall write the PhD thesis and handle it over to the advisor to read and approve. The student needs to graduate and will do anything possible to complete his/her thesis on time. The advisor, on the other hand, is already established and much less concerned. Knowing the advisor to be bottleneck, the student has to work extra hard in the hope of getting the thesis done on time.

- **2nd case,** $s_1 < p_1$ **and** $s_0 < p_0 < c_0$**, [K, K] is the equilibrium**: In this case, the supplier cannot be better off by delaying, so it either keeps or expedites its task. If the supplier keeps, the manufacturer will also keep because either delaying or expediting will make itself worse off. If the supplier expedites, the manufacturer may delay or keep: delaying renders the supplier a pure expediting cost while keeping provides the supplier a reward, $r_1$, but still insufficient to cover its expediting cost because $r_1 < c_1$ by Condition 3 (Global Optimum). So the supplier would choose to keep.

- **3rd case,** $s_1 > p_1$ **and** $s_0 < p_0 < c_0$**, [D, K] is the equilibrium**: In this case, "delay" is the dominant strategy for the supplier (by Lemma 3). The manufacturer will choose to keep because either delaying or expediting makes itself worse off.

- **5th case,** $s_1 < p_1 < c_1$ **and** $s_0 > p_0$**, [K, D] is the equilibrium**: "delay" is the dominant strategy for the manufacturer (by Lemma 3). The supplier's saving from "delay" is less than the delay penalty, which, in turn, is less than its expediting cost. This fact makes "keep" the best strategy for the supplier.

- **6th case,** $s_1 > p_1$ **and** $s_0 > p_0$**, [D, D] is the equilibrium**: "delay" is the dominant strategy for both firms.

Theorem 3 implies that in the base model with expediting and reward, the project will never be expedited in the equilibrium under the loss sharing partnership as compared to the optimal schedule. We summarize the results in this section by the following dilemma:

**The Coauthors' Dilemma**: *A firm can expedite its task but cannot expedite the project because if it expedites, the other will delay; if it delays, the other may or may not expedite.*

## 4.4 The Base Model with Multiple Suppliers – The Worst Supplier Dominance

In this section, we extend the base model to include two suppliers at level 1 to study the impact of the project network. The analysis of a N-supplier system is similar. The model is defined in Assumption 4 where suppliers play a simultaneous game among themselves anticipating the manufacturer's response to their aggregated actions.

**Assumption 4** *Assumption 1 holds here except that level 1 has two tasks each conducted by a unique supplier, and the manufacturer can only start its task after both suppliers complete their work.*

We denote supplier 1 (2)'s saving in the direct cost from delay to be $s_1$ ($s_2$) per period. The project penalty shared by the supplier 1 (or 2) is $p_1$ (or $p_2$ respectively) where $p_1 + p_2 + p_0 = p$. A necessary condition for the original schedule to be optimal under the centralized control is provided as follows,

**Condition 4** *Global Optimum - Two Suppliers:* $s_1 + s_2 < p$, $s_0 < p$.

Without the loss of generality, we assume that the original durations of tasks 1 and 2 are identical (otherwise, the system reduces to the base model as we can ignore the supplier with a shorter duration). The same assumption applies to systems with more than two suppliers which will be discussed later in the paper.

The extensive form of the game is shown in Figure 10.

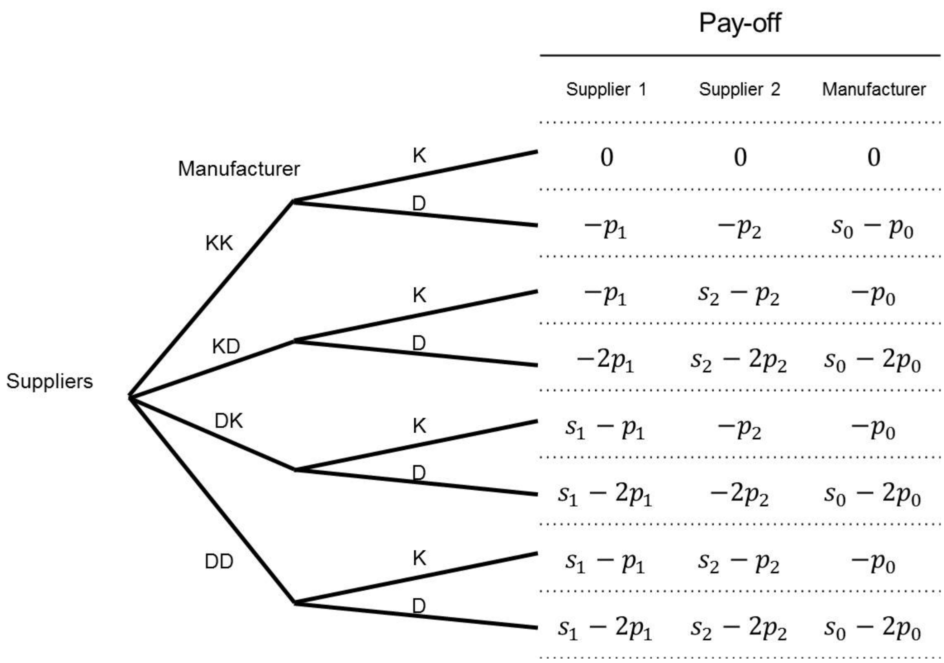


Figure 10: The extensive form of the game in the base model with multiple suppliers.

We have the following results on the dominant strategies and equilibrium.

**Lemma 4 (Dominant Strategy)**: *In the base model with two suppliers, under Condition 4, when $s_0 < p_0$, "keep" is the dominant strategy for the manufacturer; when $s_0 > p_0$, "delay" is the dominant strategy for the manufacturer. When $s_i > p_i$, "delay" is the dominant strategy for supplier $i$.*

Lemma 4 differs from Lemma 1 because of the assembly-like structure at level 1 – there is no unilateral condition for a supplier to keep the original duration of its task as the level 1's on time performance depends on both suppliers' actions.

**Theorem 4 (Equilibrium)**: *For the base model with two suppliers, under Condition 4, the subgame perfect Nash equilibrium is given by,*

| *Case* | *Condition on S* | *Condition on M* | *Optimal strategy for S1, S2* | *M's best response* |
|---|---|---|---|---|
| *1* | $s_1 < p_1$ *and* $s_2 < p_2$ | $s_0 < p_0$ | *K, K* | *K* |
| *2* | $s_1 > p_1$ *or* $s_2 > p_2$ | $s_0 < p_0$ | *D, D* | *K* |
| *3* | $s_1 < p_1$ *and* $s_2 < p_2$ | $s_0 > p_0$ | *K, K* | *D* |
| *4* | $s_1 > p_1$ *or* $s_2 > p_2$ | $s_0 > p_0$ | *D, D* | *D* |

**Remarks**: With two suppliers, the SPNE is no longer unique due to the simultaneous game played among the suppliers in level 1. For instance, when $s_0 < p_0$, the manufacturer keeps its original task duration, and the pay-off matrix for suppliers 1 and 2 is given by:

| 1\2 | K | D |
|---|---|---|
| K | $0, 0$ | $-p_1, s_2 - p_2$ |
| D | $s_1 - p_1, -p_2$ | $s_1 - p_1, s_2 - p_2$ |

Clearly, if $s_1 < p_1$ and $s_2 < p_2$, both [K, K] and [D, D] are SPNE. We only report [K, K] here because it is Pareto optimal but [D, D] is not.

Theorem 4 illustrates the impact of the project network on the equilibrium and project performance, that is, the project is more likely to be delayed with multiple suppliers. For the original schedule to be the SPNE, we require $s_1 < p_1$ and $s_2 < p_2$ (i.e., penalty exceeds saving for both suppliers) and $s_0 < p_0$. If the saving exceeds penalty for any supplier, all suppliers will have to delay in equilibrium. This observation gives rise to the following key insight:

**The Worst Supplier Dominance**: if one supplier delays, the other supplier(s) have to follow.

## 4.5 The General Model

In previous sections, we reveal many managerial insights from the base model and its extensions. In this section, we put all the extensions together into a general model where we also allow each firm to delay or expedite its task by multiple periods (see Assumption 5). The question is, do the results obtained from the special cases in previous sections (§4.1-4.4), especially the Coauthors' Dilemma, still hold in the general model? And how to compute the project schedule in equilibrium?

**Assumption 5** *The system has multiple suppliers and one manufacturer; each task can be either expedited or delayed by multiple periods; the cost structure, including penalty, reward, saving and expediting costs, are time dependent.*

We first consider the system with a single supplier. For the ease of exposition, we define the strategy pair as $(x_1, x_0)$ where $x_1$ (or $x_0$) is an integer and its absolute value represents the number of periods expedited or delayed by the supplier (the manufacturer, respectively) relative to the original schedule. A negative integer means expediting, a positive integer means delaying, and zero means keeping the original task duration.

In this game, the supplier is the first mover and takes an action $x_1$. Let's define the manufacturer's best response (to the supplier's action) to be $x_0^*(x_1)$. The project duration will therefore be changed by $x_1 + x_0^*(x_1)$. We use superscripts on $s_i$, $c_i$, $r$ and $p$ to index the associated periods. For example, if task $i$ is delayed by two periods, then the total saving should be $s_i^1 + s_i^2$ where $s_i^1$ ($s_i^2$) is the saving from the 1st (2nd) period of delay. if task $i$ is expedited by two periods, then $c_i^1$ ($c_i^2$) is the cost for the 1st (2nd) period of expediting. Lastly, we define $\pi_1(x_1, x_0)$ ($\pi_0(x_1, x_0)$) to be the pay-off function for the supplier (the manufacturer, respectively).

For this system, Condition 5 (Global Optimum) is necessary for the original schedule to be optimal under the centralized control; Condition 5 (Monotonicity) comes from the convex increasing indirect cost and convex decreasing direct cost; finally, Condition 5 (Loss Sharing) indicates that the monotonicity condition on project reward and penalty also applies to each player's reward and penalty.

**Condition 5** *(1) Global Optimum - General:* $\sum_{i=0}^{n} \pi_i(x_1, \cdots, x_n, x_0) \leq 0$ *for any* $x_i$, $i = 0, 1, \ldots, n$; *(2) Monotonicity - General:* $r^k > r^{k+1}$, $p^k < p^{k+1}$, $s_i^k > s_i^{k+1}$, $c_i^k < c_i^{k+1}$ *for any positive integer* $k$

*and any $i = 0, 1, \ldots, n$, and $r^1 < p^1$, $s_i^1 < c_i^1$ for any $i = 0, 1, \ldots, n$; (3) Loss Sharing - General: $r_i^k > r_i^{k+1}$, $p_i^k < p_i^{k+1}$ and $r_i^1 < p_i^1$ for $i = 0, 1, \ldots, n$.*

We first characterize the pay-off function for the manufacturer for a given action of the supplier.

**Lemma 5** *Given $x_1 = a$, $\pi_0(a, x_0)$ is a uni-modal function of $x_0$.*

Lemma 5 indicates that the manufacturer has a unique best response to each of the supplier's actions. The following lemma shows some monotonicity properties of the manufacturer's best response function.

**Theorem 5 (Monotonicity Property)***: As $x_1$ increases, $x_0^*(x_1)$ is non-increasing but $x_1 + x_0^*(x_1)$ is non-decreasing.*

Lemma 5 implies that if the supplier delays more, the manufacturer will delay less, but the project will be delayed for a longer time.

In the case that the task duration is sufficiently long and so $x_1$ is effectively unbounded from below, the following theorem specifies a limit by which the supplier would expedite its task.

**Theorem 6 (Expedition Limit)***: There exists a $x_L = \max\{x_1 | x_1 + x_0^*(x_1) = 0\} > -\infty$ such that if $x_1 \leq x_L$, the supplier will be better off if it increases $x_1$ to $x_L$.*

Combining Theorems 5-6, we arrive at the following key insight,

**Corollary 1 (The General Coauthor's Dilemma)***: No matter by how much each firm expedites its task, the project will never be expedited in equilibrium under the loss sharing partnership.*

For the system with multiple suppliers, we define $x_s = \max\{x_1, \ldots, x_n\}$. We can show that Theorems 5-6 hold if we replace $x_1$ by $x_s$.

To numerically compute the equilibrium (the SPNE), we design an algorithm which enumerates $x_1$ between $x_L$ and a pre-specified maximum allowable project delay, to find the optimal $x_1^*$ for the supplier. Here is the key idea: we start by setting $x_1 = 0$. First, we search the region of $x_1 < 0$ until $x_1$ reaches $x_L$ (if $x_L < 0$); second, we search the region of $x_1 > 0$ until we reach the maximum allowable project delay. We keep updating the best $\pi_1$ found to date and the corresponding $x_1$ and $x_0$, denoted by $(\pi_1^{\max}, x_1^*, x_0^*)$, until the enumeration is completed.

Let $U$ be the maximum allowable project delay, the implementation details of this algorithm are described as follows:

**Algorithm**

- Step 1 - initialization: set $x_1 \leftarrow 0$. If $s_0^1 < p_0^1$, $x_0^*(0) \leftarrow 0$ otherwise $x_0^*(0)$ equals to $i$ that satisfies $s_0^i > p_0^i$ and $s_0^{i+1} < p_0^{i+1}$. Initialize $\{\pi_1^{\max}, x_1^*, x_0^*\}$ with $\{\pi_1(0, x_0^*(0)), 0, x_0^*(0)\}$. Let $k \leftarrow x_0^*(0)$.

- Step 2 - search the region of $x_1 < 0$: $x_1 \leftarrow x_1 - 1$. Find $x_0^*(x_1)$ by comparing $\pi_0(x_1, k)$ and $\pi_0(x_1, k+1)$: if the former is greater, $k$ remains; otherwise $k \leftarrow k+1$. Compute $\pi_1(x_1, k)$, and update $\{\pi_1^{\max}, x_1^*, x_0^*\}$ with $\{\pi_1(x_1, k), x_1, k\}$ if $\pi_1^{\max} < \pi_1(x_1, k)$. If $x_1 + k > 0$, repeat Step 2, otherwise reset $x_1 \leftarrow 0$, $k \leftarrow x_0^*(x_1)$ and go to Step 3.

- Step 3 - search the region of $x_1 > 0$: if $x_1 + k \leq U$, find $x_0^*(x_1')$ by comparing $\pi_0(x_1', k)$ and $\pi_0(x_1', k-1)$: if the former is greater, $k$ remains the same; otherwise $k \leftarrow k-1$. Compute $\pi_1(x_1, k)$, and update $\{\pi_1^{\max}, x_1^*, x_0^*\}$ with $\{\pi_1(x_1, k), x_1, k\}$ if $\pi_1^{\max} < \pi_1(x_1, k)$. If $x_1 + k > U$, stop and output the current $\{\pi_1^{\max}, x_1^*, x_0^*\}$.

# 5 The Fair Sharing Partnership

In this section, we present some provisions to enhance collaboration the basic form (i.e., collaboration under the loss sharing partnership); we call the resulting new partnership "fair sharing". The fair sharing partnership is designed to have each partner fully responsible for the consequence of its actions. In principle, if one firm causes damage to other firms, it has to compensate the others. Conversely, if one firm brings benefits to other firms, it shall receive compensations from the others. Our objective of this section is to specify the detailed sharing scheme in the fair sharing partnership for various project networks and cost structures so as to align each partner's best interest with that of the project. We shall first revisit the base model (see §4.1) in §5.1 to illustrate the key ideas, and then present a complete solution for the general model (see §4.5) in §5.2.

## 5.1 The Base Model Revisited

In this section, we specify the "fair sharing" partnership for the base model (defined by Assumption 1 in §4.1) according to the following principle: if firm $i$ delays, it not only suffers its own share of the project delay penalty $p_i$, but also must reimburse firm $j$ ($j \neq i$) her share of the penalty $p_j$ due to firm $i$'s delay. In this way, each firm is fully responsible for the penalty incurred by its delay. Specifically,

**The Fair Sharing Scheme (The Base Model)**: *if both firms keep their original task duration, no payment is transferred. If only the supplier delays its task, the supplier not only suffers a penalty of $p_1$, but also pays the manufacturer $p_0$ to compensate her loss due to the supplier's delay. Similarly, if only the manufacturer delays its task, the manufacturer suffers a penalty of $p_0$ and must pay the supplier $p_1$, that is, the supplier's loss due to the manufacturer's delay. If both firms delay, each will compensate the other for the loss caused by its delay, that is, the supplier pays $p_0$ to the manufacturer and the manufacturer pays back the supplier $p_1$. In any event, if a firm delays, it will pay the full penalty $p$.*

The pay-off matrix is shown in Figure 11. It is obvious that the action set [K, K] is the SPNE under Condition 1 in §4.1. Thus fair sharing is capable of aligning individual firms' interests with that of the project in the base model.

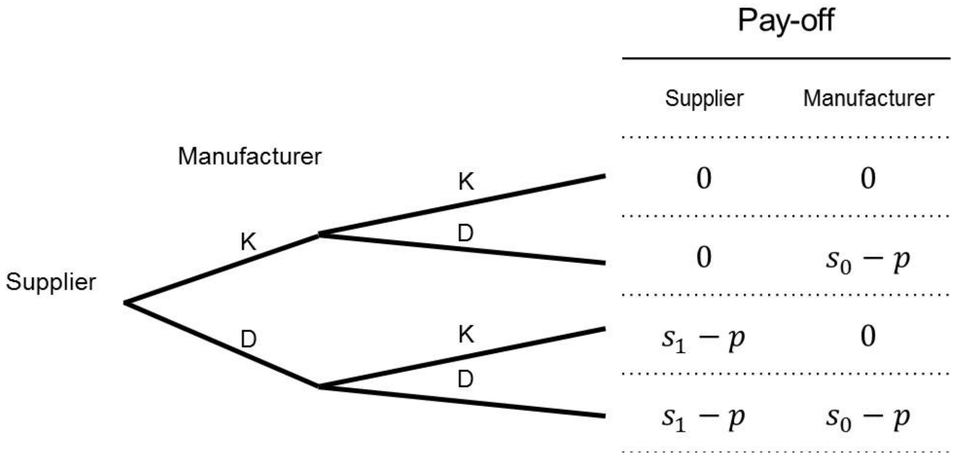


Figure 11: Extensive form of the game in base model under fair sharing.

### Extension to Two Suppliers

The system with multiple suppliers complicates the fair sharing partnership. Let's consider the base model with two suppliers (defined by Assumption 4 in §4.4) and modify the above sharing scheme as follows.

**The Fair Sharing Scheme (The Base Model With Two Suppliers)**: *if the manufacturer delays, it pays p which is the delay penalty of the project. Likewise, if one of the suppliers delays while the other keeps its original task duration, the delayed supplier pays p. If both suppliers delay, they split the penalty according to a rationing rule* ($\beta_1 > 0$, $\beta_2 > 0$) *where* $\beta_1 + \beta_2 = 1$ *and supplier 1 (2) pays* $\beta_1 p$ ($\beta_2 p$).

An analysis of the extensive form of the game reveals,

**Theorem 7** *Consider the base model with two suppliers. Under the fair sharing partnership and Condition 4, the SPNE is to keep the original schedule (which is optimal under the centralized control) for any* $(\beta_1, \beta_2)$ *as long as* $\beta_1 > 0$, $\beta_2 > 0$ *and* $\beta_1 + \beta_2 = 1$.

Note that Theorem 7 holds regardless of the value of $\beta_i, i = 1, 0$. Thus, the fair sharing partnership leaves the firms a flexibility in negotiating the contract.

## 5.2 The General Model Revisited

In this section, we present the details of the fair sharing partnership for the general model (defined by Assumption 5 in §4.5) and prove its effectiveness. Note that fair sharing can be seen as a way to redistribute the incremental indirect cost of the project (either reward or penalty) due to schedule changes among the firms. We denote this incremental indirect cost by $B$. Under fair sharing, $B$ is distributed to levels 1 and 2 firms. Suppose that level 1 firms (the suppliers) get $A_1$ and the level 2 firm (the manufacturer) gets $A_2$, then $A_1 + A_2 = B$. For the ease of exposition, we also define $x_s = \max\{x_1, x_2, \ldots, x_N\}$ where $x_s$ represents the change of level 1 completion date as compared to the original schedule. Using this notation, we specify the fair sharing scheme for the general model in two steps.

**The Fair Sharing Scheme (The General Model)**:

- **Step 1**: *we decide the payment transferred between the two levels by allocating B to levels 1 and 2. If level 1 completion date is expedited by k periods* ($x_s = -k$), *level 1 firms shall be compensated by the rewards (i.e., savings in the indirect cost) for the project for the first k periods, that is,* $A_1 = r^1 + r^2 + \ldots + r^k$. *If level 1 completion date is delayed by k periods* ($x_s = k$), *level 1 firms shall pay the penalty (i.e., the additional indirect cost) for the project for the first k periods, that is,* $A_1 = p^1 + p^2 + \ldots + p^k$. *After the suppliers' allocation* $A_1$ *is determined, the manufacturer's allocation* $A_2 = B - A_1$ *accordingly.*

- **Step 2**: *we decide the payment transferred within level 1 firms by allocating $A_1$ among the suppliers. If level 1 completion date is expedited by k periods ($x_s = -k$), then each supplier must have expedited its task by at least k periods. $A_1$ is the reward and should be shared among all the suppliers. If level 1 completion date is delayed by k periods ($x_s = k$), then each supplier delays its task by at most k periods. $A_1$ is now the penalty and should be shared on a period-by-period basis among all delayed suppliers. For those suppliers who didn't delay in this case, they neither receive any reward nor share any penalty. More details are provided below.*

To see how Step 1 works, we provide an example:

- **Case 1**: If level 1 is expedited by 5 weeks but level 2 is delayed by 2 weeks, the project is therefore expedited by 3 weeks. Level 1 firms should be rewarded by $r^5, r^4, \ldots, r^1$, among which $r^3, r^2, r^1$ come from the project's earlier completion, but the rewards $r^5$ and $r^4$ are not materialized due to the delay at level 2, and so must be paid by the firm (the manufacturer) at level 2.

- **Case 2**: If level 1 is delayed by 5 weeks but level 2 is expedited by 2 weeks, the project is therefore delayed by 3 weeks. Level 1 firms must pay the penalties $p^1, p^2, \ldots, p^5$. However, $p^4$ and $p^5$ are not materialized by the level 2 firm's expedition, and so must be paid to the level 2 firm.

The general pay-off function of the manufacturer (the level 2 firm) is shown in Table 1.

To see how the reward or penalty is shared among the suppliers in Step 2, we show the pay-off functions of the suppliers in Table 2. In principle, each supplier is only responsible for the penalty of the periods delayed by itself, and it will not be rewarded if its expedition is not effective – does not lead to an expedition of level 1 completion date.

The three cases of Table 2 can be explained as follows:

- **Case 1:** $x_s < 0$. All suppliers share the expediting rewards of $|x_s|$ periods. The pay-off for supplier $i$ is $\pi_i = -\sum_{j=1}^{|x_i|} c_i^j + \alpha_i \sum_{j=1}^{|x_s|} r^j$, where $\alpha_i > 0$ for $i = 1, \ldots, N$ and $\sum_{i=1}^{N} \alpha_i = 1$. Here $\alpha_i$ is supplier $i$'s ration of the reward.

- **Case 2:** $x_s = 0$. If supplier $i$ keeps its original task duration, its pay-off is $\pi_i = 0$; if supplier $i$ expedites, its pay-off is $\pi_i = -\sum_{j=1}^{|x_i|} c_i^j$.

| Level 1 | The manufacturer | Pay-off of the manufacturer |
|---|---|---|
| E: $x_s < 0$ | E: $x_0 < 0$ | $\sum_{i=\lvert x_s\rvert+1}^{\lvert x_s\rvert+\lvert x_0\rvert} r^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$ |
| | K: $x_0 = 0$ | 0 |
| | D: $x_0 > 0$ | $-\sum_{i=\lvert x_s+x_0\rvert+1}^{\lvert x_s\rvert} r^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$, if $x_s + x_0 \leq 0$ |
| | | $-\sum_{i=1}^{\lvert x_s\rvert} r^i - \sum_{i=1}^{\lvert x_s+x_0\rvert} p^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$, if $x_s + x_0 > 0$ |
| K: $x_s = 0$ | E: $x_0 < 0$ | $\sum_{i=1}^{\lvert x_0\rvert} r^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$ |
| | K: $x_0 = 0$ | 0 |
| | D: $x_0 > 0$ | $-\sum_{i=1}^{\lvert x_0\rvert} p^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$ |
| D: $x_s > 0$ | E: $x_0 < 0$ | $\sum_{i=1}^{\lvert x_s\rvert} p^i + \sum_{i=1}^{\lvert x_s+x_0\rvert} r^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$, if $x_s + x_0 < 0$ |
| | | $\sum_{i=\lvert x_s+x_0\rvert+1}^{\lvert x_s\rvert} p^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$, if $x_s + x_0 \geq 0$ |
| | K: $x_0 = 0$ | 0 |
| | D: $x_0 > 0$ | $-\sum_{i=\lvert x_s\rvert+1}^{\lvert x_s\rvert+\lvert x_0\rvert} p^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$ |

Table 1: The pay-off function of the manufacturer under fair sharing in the general model.

| Level 1 | Supplier $i$ | Pay-off of supplier $i$ |
|---|---|---|
| E: $x_s < 0$ | E: $x_i < 0$ | $-\sum_{j=1}^{\lvert x_i\rvert} c_i^j + \alpha_i \sum_{j=1}^{\lvert x_s\rvert} r^j$ |
| K: $x_s = 0$ | E: $x_i < 0$ | $-\sum_{j=1}^{\lvert x_i\rvert} c_i^j$ |
| | K: $x_i = 0$ | 0 |
| D: $x_1 > 0$ | E: $x_i < 0$ | $-\sum_{j=1}^{\lvert x_i\rvert} c_i^j$ |
| | K: $x_i = 0$ | 0 |
| | D: $x_i > 0$ | $\sum_{j=1}^{\lvert x_i\rvert} s_i^j - \sum_{j=1}^{\lvert x_s\rvert} \beta_i^j p^j$ |

Table 2: The pay-off function of suppliers under fair sharing in the general model. Note: (1) $\alpha_i > 0$ and $\sum_{i=1}^{N} \alpha_i = 1$. (2) $\beta_i^j = 0$ if $j > x_i$, otherwise, $\beta_i^j > 0$. (3) $\sum_{i=1}^{N} \beta_i^j = 1$ for all $j = 1, 2, \ldots, |x_s|$.

- **Case 3:** $x_s > 0$. If supplier $i$ expedites, its pay-off is $\pi_i = -\sum_{j=1}^{|x_i|} c_i^j$; if it keeps, its pay-off is $\pi_i = 0$; if it delays, its pay-off is $\pi_i = \sum_{j=1}^{|x_i|} s_i^j - \sum_{j=1}^{|x_s|} \beta_i^j p^j$ where $\beta_i^j$ is supplier $i$'s ration for the penalty of the $j^{th}$ period delayed. If $j > x_i$ (that is, this supplier does not contribute to the $j^{th}$ period of delay), $\beta_i^j = 0$; otherwise $\beta_i^j > 0$ and $\beta_i^j$ satisfies $\sum_{i=1}^{N} \beta_i^j = 1$ for all $j = 1, 2, \ldots, |x_s|$.

Under this sharing scheme, we have the following result.

**Theorem 8** *In the general under the fair sharing scheme, "keep" for all firms is the unique SPNE.*

Theorem 8 implies that fair sharing is capable of aligning individual firms' interests with that of the project in the general model.

**Extension: Starting from A Suboptimal Schedule**

So far, we proved the effectiveness of the fair sharing partnership by assuming that the project starts from an original schedule that is optimal under the centralized control. An interesting question is, what happens if we relax this assumption and so the project starts from a suboptimal (or any) schedule?

When starting from an arbitrary schedule, the schedule in equilibrium may differ from the starting schedule under fair sharing. To see this, let's consider an example with a single supplier. Let $p^1 = 220, p^2 = 300, p^3 = 400, \ldots$, $s_i^1 = 250, s_i^2 = 200, \ldots$, and $s_0^1 = 280, s_0^2 = 200, \ldots$. Given such costs, the original schedule is clearly not optimal. In fact, keeping task 1's original duration but delaying task 2's duration by 1 week is the optimal schedule under the centralized control. It is easily to verify that the subgame perfect Nash equilibrium is $x_1 = 1$ and $x_0 = 0$ under fair sharing. Thus, when the project starts from an arbitrary schedule, such a schedule may not be the equilibrium schedule under fair sharing.

Interestingly, the equilibrium schedule will not worsen the project performance under fair sharing relative to the starting schedule. To see this, let's consider the suppliers first. By Table 2, a supplier could always choose "keep" in order to get a zero pay-off regardless of the actions of other suppliers and the manufacturer. This is true because fair sharing ensures that each partner is fully responsible for consequences of its actions and so a partner who choose to keep won't be penalized by damages caused by others. By a similar logic, the manufacturer can secure a zero pay-off by choosing "keep" even in such a second mover situation (see Table 1) regardless of the suppliers' actions. Suppose in equilibrium, a partner's optimal action is not "keep", then this partner must get a positive pay-off because otherwise, it can always choose "keep" to avoid a negative pay-off.

**Proposition 1** *In the general model under fair sharing, if the project starts from an arbitrary schedule, all firms can not be worse off in their pay-offs in equilibrium.*

# 6 Conclusions

In this paper, we consider collaborative partnerships in a two-level project management setting where the workload of the project is spread out to multiple firms (partners). We study the strategic behaviors of the firms under the loss sharing partnership in these joint projects by combing the economics/supply chain gaming models with project management specifics. This paper highlights the negative impact of collaboration and the loss sharing partnership on the project performance in both time and cost by discovering exactly why and how they can hurt. We find an inherent mismatch between individual firms' best interests and that of the project. Depending on the project network and cost structure, a firm may be motivated to delay even if doing so harms the entire project (the Prisoners' Dilemma); a firm may have to delay (even at a loss) just to prevent the others from delaying, to avoid a much greater loss (the Suppliers' Dilemma); and no matter by how much a firm expedites its task, it cannot expedite the project because other firms will delay (the Coauthors' Dilemma). To resolve the incentive issue, we enhance the loss sharing partnership by a set of provisions with the principle of each firm being fully responsible for the consequences of its action. We present the exact form of the fair sharing partnership and prove its effectiveness in aligning the interests of individual firms with that of the project.

Going forward beyond the scope of this paper, research on economics/supply chain and project management interfaces promises to be fruitful to both practitioners and academicians because of the high impact on practice, and the potential of exciting theoretical discoveries and insights by integrating two rich bodies of literature. The potential in coordinating the project-driven supply chains (or joint projects) has recently been recognized both in academia and in industry. While there is ample work to be done, we suggest the following future research directions:

1. *Empirical Studies*: The recent slips of the 787 Dreamliner and Airbus 380 have drawn the attention of both practitioners and academicians on how to ensure successful innovation by collaboration. While theoretical models can be built to aid the development of the next mega project, empirical studies should also be done to discover what really happened in these programs.

2. *Uncertain Task Durations*: While deterministic task durations greatly simplify the analysis and thus allow us to establish clean results on incentives and gaming behaviors in joint projects, it is of great interest to allow randomness in task durations and to potentially integrate the economics/supply chain incentive theory with project evaluation and review technique (PERT).

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## Appendix

### Proof of Lemma 1

For the supplier with $s_1 < p_1$, if the manufacturer chooses "keep", then $0 > s_1 - p_1$ and so the supplier will choose "keep"; if the manufacturer chooses "delay", then $-p_1 > s_1 - 2p_1$ so that the supplier will choose "keep" as well. Thus, the supplier has a dominant strategy of "keep" when $s_1 < p_1$. Similarly, we can prove that when $s_1 > p_1$, "delay" is the dominant strategy for the supplier.

For the manufacturer with $s_0 < p_0$, if the supplier chooses "keep", then $0 > s_0 - p_0$ and so the manufacturer will choose "keep"; if the supplier chooses "delay", then $-p_0 > s_0 - 2p_0$ so that the manufacturer will choose "keep" as well. Thus, the manufacturer has a dominant strategy of "keep" when $s_0 < p_0$. Similarly, we can prove that when $s_0 > p_0$, "delay" is the dominant strategy for the manufacturer. □

### Proof of Theorem 1

This theorem is a straightforward result of Lemma 1. □

### Proof of Lemma 2

For the manufacturer with $s_0 < p_0^1$, if the supplier chooses "keep", then $0 > s_0 - p_0^1$ and so the manufacturer will choose "keep"; if the supplier chooses "delay", then $-p_0^1 > s_0 - p_0^1 - p_0^2$ and so the manufacturer will choose "keep" as well. Thus, the manufacturer has a dominant strategy of "keep" when $s_0 < p_0^1$. Similarly, we can prove that when $s_0 > p_0^2$, "delay" is the dominant strategy for the manufacturer. □

### Proof of Theorem 2

Lemma 2 implies,

- when $s_1 > p_1^1$ and $s_0 < p_0^1$, the supplier has a dominant strategy of "delay" and the manufacturer has a dominant strategy of "keep".
- when $s_1 > p_1^2$ and $s_0 > p_0^2$, the supplier has a dominant strategy of "delay" and the manufacturer has a dominant strategy of "delay".

When $s_1 < p_1^1$ and $s_0 < p_0^1$, if the supplier chooses "keep", then the manufacturer will choose "keep" as $0 > s_0 - p_0^1$; if the supplier chooses "delay", then the manufacturer will choose "keep" as $-p_0^1 > s_0 - p_0^1 - p_0^2$. The former strategy gives the supplier a higher pay-off (0) than the latter strategy ($s_1 - p_1^1$) and thus the supplier will choose "keep" and then the manufacturer will choose "keep".

When $p_0^1 < s_0 < p_0^2$, if the supplier chooses "keep", then the manufacturer will choose "delay" as $s_0 > p_0^1$; if the supplier chooses "delay", then the manufacturer will choose "keep" as $s_0 < p_0^2$. The latter strategy gives the supplier a higher pay-off ($-p_1^1$) than the former strategy ($s_1 - p_1^1$) and thus the supplier will choose "delay" and then the manufacturer will choose "keep".

When $s_1 < p_1^2$ and $s_0 > p_0^2$, the manufacturer has the dominant strategy of "delay". Since $-p_1^1 > s_1 - p_1^1 - p_1^2$, the supplier will choose "keep". □

**Proof of Lemma 3**

When $s_0 > p_0$, we know that $s_0 > r_0$ and $r_0 < c_0$ from Condition 3. If the supplier chooses "expediting" or "keep", the manufacturer always gets the highest pay-off if it delays. If the supplier chooses "delay", because $p_0 < s_0 < c_0$, "delay" yields the highest pay-off for the manufacturer. Thus, the manufacturer has a dominant strategy of "delay" in this scenario. Similarly, we can prove that the supplier has a dominant strategy of "delay" when $s_1 > p_1$. By a similar analysis, we could prove that when $s_i < r_i < p_i < c_i$, "keep" is the dominant strategy for firm $i$, $i = 1, 0$. □

**Proof of Theorem 3**

All potential actions are listed below:

| S | M | S's Pay-off | Conditions | | M's Best Response | M's Pay-off |
|---|---|---|---|---|---|---|
| | E | $2r_0 - c_0$ | | | | |
| E | K | $r_0$ | if $r_0 > s_0$ | | K | $r_1 - c_1$ |
| | D | $s_0$ | if $r_0 < s_0$ | | D | $-c_1$ |
| | E | $r_0 - c_0$ | | | | |
| K | K | 0 | if $p_0 > s_0$ | | K | 0 |
| | D | $s_0 - p_0$ | if $p_0 < s_0$ | | D | $-p_1$ |
| | E | $-c_0$ | if $p_0 > s_0$ | if $p_0 > c_0$ | E | $s_1$ |
| D | K | $-p_0$ | if $p_0 > s_0$ | if $p_0 < c_0$ | K | $s_1 - p_1$ |
| | D | $s_0 - 2p_0$ | if $p_0 < s_0$ | | D | $s_1 - 2p_1$ |

- When $p_0 < s_0$, "delay" is the dominant strategy for the manufacturer by Lemma 3. The supplier's pay-off is $-c_1$ with "expediting", $-p_1$ with "keep", and $s_1 - 2p_1$ with "delay". We consider three cases:

  - (a) When $p_1 > c_1$, the supplier's optimal strategy is "expediting" because $c_1 > s_1$ by Condition 3(2) and so $-c_1$ is the largest payoff.
  - (b) When $s_1 < p_1 < c_1$, the supplier's optimal strategy is "keep".
  - (c) When $p_1 > c_1$, the supplier's optimal strategy is "delay".

- When $s_0 < p_0 < c_0$ and $r_0 > s_0$, "keep" is the dominant strategy for the manufacturer by Lemma 3. The supplier's pay-off is $r_1 - c_1$ with "expediting", 0 with "keep", and $s_1 - p_1$ with "delay". We consider two cases:

  - (a) When $p_1 > s_1$, the supplier's optimal strategy is "keep" because $r_1 < c_1$ by Condition 3(1).
  - (b) When $p_1 < s_1$, the supplier's optimal strategy is "delay" because $r_1 < c_1$.

- When $s_0 < p_0 < c_0$ and $r_0 < s_0$, there is no dominant strategy for the manufacturer. If the supplier chooses "expediting", the manufacturer will choose "delay". If the supplier chooses "keep" or "delay", the manufacturer will choose "keep". Thus, the supplier's pay-off is $-c_1$ with "expediting", 0 with "keep", and $s_1 - p_1$ with "delay".

  - (a) When $p_1 > s_1$, the supplier's optimal strategy is "keep".
  - (b) When $p_1 < s_1$, the supplier's optimal strategy is "delay".

- When $p_0 > c_0$ and $r_0 > s_0$, by $c_0 > s_0$ (Condition 3(2)) we obtain $p_0 > s_0$. If the supplier chooses "expediting", the manufacturer will choose "keep". If the supplier chooses "keep", the manufacturer will choose "keep". If the supplier chooses "delay", the manufacturer will choose "expediting". (Note: the manufacturer will do whatever it could to prevent project delay.) Given the manufacturer's optimal response, the supplier's pay-off is $r_1 - c_1$ with "expediting", 0 with "keep", and $s_1$ with "delay". Since $r_1 < c_1$ by Condition 3(1), the supplier's optimal strategy is "delay".

- When $p_0 > c_0$ and $r_0 < s_0$, by $c_0 > s_0$ (Condition 3(2)) we obtain $p_0 > s_0$. If the supplier chooses "expediting", the manufacturer will choose "delay". If the supplier chooses "keep", the manufacturer will choose "keep". If the supplier chooses "delay", the manufacturer will choose "expediting". (Note: the manufacturer will do whatever he could to prevent delay.) Given the manufacturer's optimal response, the supplier's pay-off is $-c_1$ with "expediting", 0 with "keep", and $s_1$ with "delay". Clearly, the supplier's optimal strategy is "delay".

Summarizing all cases, we have proved the theorem. □

**Proof of Lemma 4**

By Lemma 1, the first two results are immediate, that is, when $s_0 < p_0$, "keep" is the dominant strategy for the manufacturer; when $s_0 > p_0$, "delay" is the dominant strategy for the manufacturer.

When $s_1 > p_1$, an enumerating over all options of supplier 2 and the manufacturer finds that supplier 1 archives the highest pay-off when it delays. □

**Proof of Theorem 4**

By Lemma 4, as long as one of the suppliers has a dominant strategy of "delay", the other has to delay as well. Otherwise, it suffers a pure penalty. Combining the dominant strategies leads to the theorem. □

**Proof of Lemma 5**

When $a < 0$, we consider three cases:

(1) If $x_0 \le 0$, $\pi_0(a, x_0) = r_0^1 + \ldots + r_0^{|a|+|x_0|} - c_0^1 - \ldots - c_0^{|x_0|}$.

(2) If $0 < x_0 \le |a|$, $\pi_0(a, x_0) = r_0^1 + \ldots + r_0^{|a|-x_0} + s_0^1 + \ldots + s_0^{x_0}$.

(3) If $x_0 > |a|$, $\pi_0(a, x_0) = s_0^1 + \ldots + s_0^{x_0} - p_0^1 - \ldots - p_0^{a+x_0}$.

In case (1), when $x_0 \in (-\infty, 0)$, $\pi_0(a, x_0)$ is an increasing function in $x_0$ because $r_0^{|x_0|} < r_0^1 < c_0^1 < c_0^{|x_0|}$ by Condition 5. At $x_0 = 0$, $\pi_0(a, 0) > \pi_0(a, -1)$ because $r_0^1 < c_0^1$. Thus, $\pi_0(a, x_0)$ is a monotonically increasing function of $x_0$ on $x_0 \in (-\infty, 0]$. Note that $\pi_0(a, 0) = r_0^1 + \ldots + r_0^{|a|}$. It is easy to show that when $x_0 \to +\infty$, $\pi_0(a, x_0) \to -\infty$. There always exists $x_0 = \hat{x}_0 \in [0, +\infty)$ that maximizes $\pi_0(a, x_0)$.

We now show that $\pi_0(a, x_0)$ is monotonically increasing in $(-\infty, \hat{x}_0]$ and monotonically decreasing in $[\hat{x}_0, +\infty)$. We discuss three scenarios:

1°, if $\hat{x}_0 = 0$, we have $\pi_0(a, 0) > \pi_0(a, 1)$, indicating that $r_0^{|a|} > s_0^1$. Since both $\{s_0^t\}$ and $\{r_0^t\}$ are decreasing series in $t$, we have $r_0^{|a|-1} > r_0^{|a|} > s_0^1 > s_0^2 \Rightarrow \pi_0(a, 1) > \pi_0(a, 2)$. By induction, we could prove that $\pi_0(a, x_0)$ is decreasing in $[0, +\infty)$.

$2^\circ$, if $0 < \hat{x}_0 < |a|$, we have $\pi_0(a, \hat{x}_0) > \pi_0(a, \hat{x}_0 - 1)$ and $\pi_0(a, \hat{x}_0) > \pi_0(a, \hat{x}_0 + 1)$, indicating that $s_0^{\hat{x}_0} > r_0^{|a|-(\hat{x}_0-1)}$ and $r_0^{|a|-\hat{x}_0} > s_0^{\hat{x}_0+1}$. Furthermore, as both $\{s_0^t\}$ and $\{r_0^t\}$ are decreasing series in $t$, we have $s_0^{\hat{x}_0-1} > s_0^{\hat{x}_0} > r_0^{|a|-(\hat{x}_0-1)} > r_0^{|a|-(\hat{x}_0-2)}$ and $r_0^{|a|-\hat{x}_0} > s_0^{\hat{x}_0+1} > s_0^{\hat{x}_0+2}$, which lead to $\pi_0(a, \hat{x}_0 - 1) > \pi_0(a, \hat{x}_0 - 2)$ and $\pi_0(a, \hat{x}_0 + 1) > \pi_0(a, \hat{x}_0 + 2)$. We first consider the left side of $\hat{x}_0$ and show $\pi_0(a, x_0)$ is monotonically increasing in $[0, \hat{x}_0]$ by induction. The induction assumption is $\pi_0(a, x_0') > \pi_0(a, x_0' - 1)$ where $x_0' \in (0, \hat{x}_0)$. We have $\pi_0(a, x_0') > \pi_0(a, x_0' - 1) \Rightarrow s_0^{x_0'} > r_0^{|a|-x_0'+1} \Rightarrow s_0^{x_0'-1} > s_0^{x_0'} > r_0^{|a|-x_0'+1} > r_0^{|a|-x_0'+2} \Rightarrow \pi_0(a, x_0' - 1) > \pi_0(a, x_0' - 2)$. In addition, when $x_0' = 1$, we could show that $\pi_0(a, 1) > \pi_0(a, 0)$. Thus, $\pi_0(a, x_0)$ is monotonically increasing in $[0, \hat{x}_0]$. Similarly, we could prove that $\pi_0(a, x_0)$ is monotonically decreasing in $[\hat{x}_0, +\infty)$. Recall that $\pi_0(a, x_0)$ is a monotonically increasing function of $x_0$ on $x_0 \in (-\infty, 0]$, therefore $\pi_0(a, x_0)$ is a concave unimodal function with the peak $x_0 = \hat{x}_0$ when $0 < \hat{x}_0 < |a|$.

$3^\circ$, if $\hat{x}_0 \geq |a|$, by a similar analysis, it is easy to prove that $\pi_0(a, x_0)$ is a concave unimodal function with the peak $x_0 = \hat{x}_0$.

In summary, $\pi_0(a, x_0)$ is a unimodal function of $x_0$ when $a < 0$.

The proof for the case of $a \geq 0$ is similar and thus omitted. In conclusion, given $x_1 = a$, $\pi_0(a, x_0)$ is a uni-modal function of $x_0$. $\square$

**Proof of Theorem 5**

We first show that when $x_1 \to -\infty$, $x_0^*(x_1) > 0$ and $x_1 + x_0^*(x_1) < 0$.

- When $x_1 < 0$, the supplier expedites; the manufacturer will never expedite because a negative $x_0$ yields $r_0^{|x_1+x_0|} < r_0^1 < c_0^1$. Consider the manufacturer's response in three scenarios: (1) $x_0 < |x_1|$, (2) $x_0 = |x_1|$, $\pi_0(x_1, x_0) = s_0^1 + \ldots + s_0^{|x_1|}$. (3) $x_0 > |x_1|$, $\pi_0(x_1, x_0) = s_0^1 + \ldots + s_0^{|x_0|} - p_0^1 - \ldots - p_0^{x_1+x_0}$.

- Scenario (1) yields the highest pay-off for the manufacturer when $x_1 \to -\infty$. Explanation: In scenario (3), when $x_1 \to -\infty$, $x_0 \to +\infty$ and thus $s_0^{x_0} \to 0$ and $p_0^{x_1+x_0} \to +\infty$. It is clear that scenario (2) yields a higher pay-off than scenario (3). Next, let $x_0 = |x_1| - 1$. $\pi_0(x_1, |x_1| - 1) = s_0^1 + \ldots + s_0^{|x_1|-1} + r_0^1$. When $x_1 \to -\infty$, $s_0^{|x_1|} < r_0^1$ and thus $\pi_0(x_1, |x_1| - 1) > \pi_0(x_1, |x_1|)$. Hence, when $x_1 \to -\infty$, $x_0^*(x_1) < |x_1|$. In other words, when $x_1 \to -\infty$, $x_0^*(x_1) > 0$ and $x_1 + x_0^*(x_1) < 0$.

Now we start from $x_1 \to -\infty$ and increase $x_1$ by one unit each time to see how $x_0^*(x_1)$ and $x_1 + x_0^*(x_1)$ will change.

When $x_1 \to -\infty$, $x_0^*(x_1) > 0$, $x_1 + x_0^*(x_1) < 0$, so that $\pi_0(x_1, x_0^*(x_1)) = s_0^1 + \ldots + s_0^{x_0^*(x_1)} + r_0^1 + \ldots + r_0^{|x_1+x_0^*(x_1)|}$. $x_0^*(x_1)$ being the best response requires conditions $\pi_0(x_1, x_0^*(x_1)) > \pi_0(x_1, x_0^*(x_1) - 1)$ and $\pi_0(x_1, x_0^*(x_1)) > \pi_0(x_1, x_0^*(x_1) + 1)$ which are equivalent to $s_0^{x_0^*(x_1)} > r_0^{|x_1+x_0^*(x_1)-1|}$ and $r_0^{|x_1+x_0^*(x_1)|} > s_0^{x_0^*(x_1)+1}$. Let $x_1' = x_1 + 1$, to find the manufacturer's best response, we compare the following pay-offs as(assuming $x_0^*(x_1) - 2 \geq 0$ and $|x_1 + x_0^*(x_1) + 1| > 0$):

(1) $\pi_0(x_1', x_0^*(x_1) - 2) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)-2|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)-2}$.

(2) $\pi_0(x_1', x_0^*(x_1) - 1) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)-1|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)-1}$.

(3) $\pi_0(x_1', x_0^*(x_1)) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)}$.

(4) $\pi_0(x_1', x_0^*(x_1) + 1) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)+1|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)+1}$.

Because $r_0^{|x_1+1+x_0^*(x_1)-2|} < s_0^{x_0^*(x_1)} < s_0^{x_0^*(x_1)-1}$ and $r_0^{|x_1+1+x_0^*(x_1)|} > r_0^{|x_1+x_0^*(x_1)|} > s_0^{x_0^*(x_1)+1}$, we have $\pi_0(x_1', x_0^*(x_1) - 2) < \pi_0(x_1', x_0^*(x_1) - 1)$ and $\pi_0(x_1', x_0^*(x_1)) > \pi_0(x_1', x_0^*(x_1) + 1)$. We can easily verify that when $x_0^*(x_1) - 2 = -1$ and $|x_1 + x_0^*(x_1) + 1| = 0$, these inequalities still hold. By the unimodality property of Lemma 5, $x_0^*(x_1) - 1 \leq x_0^*(x_1 + 1) \leq x_0^*(x_1)$. In other words, when $x_1$ increases by one unit, the manufacturer's best response is to either reduce the corresponding $x_0^*(x_1)$ by one unit or keep it the same until $x_0^*(x_1)$ reaches 0.

At $x_0^*(x_1) = 0$, to find the manufacturer's best response for $x_1' = x_1 + 1$, we still compare four pay-offs, $\pi_0(x_1', x_0^*(x_1) - 2)$, $\pi_0(x_1', x_0^*(x_1) - 1)$, $\pi_0(x_1', x_0^*(x_1))$, and $\pi_0(x_1', x_0^*(x_1) + 1)$. By the same logic stated in the previous paragraph, we have $x_0^*(x_1) - 1 \leq x_0^*(x_1 + 1) \leq x_0^*(x_1)$ for either $x_1 < 0$ or $x_1 = 0$.

Applying similar approach stated in the previous paragraphs to analyze the rest possible scenarios: (1) $x_1 < 0$, $x_0^*(x_1) \geq 0$, $x_1 + x_0^*(x_1) \geq 0$; (2) $x_1 \geq 0$, $x_0^*(x_1) \geq 0$, $x_1 + x_0^*(x_1) \geq 0$; (3) $x_1 \geq 0$, and $x_0^*(x_1) < 0$, $x_1 + x_0^*(x_1) > 0$, we can always show that when $x_1$ increases by one unit, the manufacturer's best response is to reduce the corresponding $x_0^*(x_1)$ by one unit or keep it the same.

In summary, for $x_1 \in (-\infty, +\infty)$, when $x_1$ increases by one unit, $x_0^*(x_1)$ will either decrease by one unit or remain the same and therefore $x_1 + x_0^*(x_1)$ will not decrease. □

**Proof of Theorem 6**

In the proof of Theorem 5, we have shown that when $x_1 \to -\infty$, $x_1 + x_0^*(x_1) < 0$. On the other hand, when $x_1 = 0$, $x_0^*(0)$ should be greater than or equal to 0 so as to guarantee original schedule to be the global optimum; $x_1 + x_0^*(x_1)$ is therefore greater than or equal to 0. From Theorem 5, we know that $x_1 + x_0^*(x_1)$ will increase by one unit or hold still each time when $x_1$ increases by one unit. There must exist $x_L$ such that $x_L = \max\{x_1 | x_1 + x_0^*(x_1) = 0, x_1 \leq 0\}$.

For any $x_1 < x_L$ and $x_1 + x_0^*(x_1) \leq -1$, the supplier's pay-off is $\pi_1(x_1, x_0^*(x_1)) = r_1^1 + \ldots + r_1^{|x_1+1+x_0^*(x_1)|} + r_1^{|x_1+x_0^*(x_1)|} - c_1^1 - \ldots - c_1^{|x_1+1|} - c_1^{|x_1|}$. When the supplier expedites one period less, $x_1 + 1$, the manufacturer's best response is either $x_0^*(x_1)$ or $x_0^*(x_1) - 1$ by Theorem 5. (1) If the manufacturer's best response is $x_0^*(x_1)$, then the supplier's pay-off is $r_1^1 + \ldots + r_1^{|x_1+1+x_0^*(x_1)|} - c_1^1 - \ldots - c_1^{|x_1+1|}$. Note that $r_1^{|x_1+x_0^*(x_1)|} \leq r_1^1 < c_1^1 < c_1^{|x_1|}$ by Condition 5(1), the supplier actually improves its pay-off by increasing $x_1$ to $x_1+1$. (2) If the manufacturer's best response is $x_0^*(x_1)-1$, the supplier's pay-off is $r_1^1 + \ldots + r_1^{|x_1+1+x_0^*(x_1)-1|} - c_1^1 - \ldots - c_1^{|x_1+1|}$. The supplier also improves its pay-off. Therefore the supplier could continuously improve its pay-off by increasing $x_1$ until $x_1 + x_0^*(x_1) = -1$.

When $x_1 < x_L$ and $x_1 + x_0^*(x_1) = -1$, the supplier's pay-off is $r_0^1 - c_1^1 - \ldots - c_1^{|x_1+1|} - c_1^{|x_1|}$. If it expedites one period less, $x_1 + 1$, the manufacturer's best response is either $x_0^*(x_1)$ or $x_0^*(x_1) - 1$. The former one yields the supplier a pay-off of $-c_1^1 - \ldots - c_1^{|x_1+1|}$. Note that $r_1^1 < c_1^1 \leq c_1^{|x_1|}$. The supplier has a higher pay-off at $x_1 + 1$ than that at $x_1$. The latter one is the same as the case discussed in the previous paragraph which is shown that the supplier could improve its pay-off from $x_1$ to $x_1 + 1$. In other words, at $x_1 + x_0^*(x_1) = -1$, the supplier could also improve its pay-off until $x_1 + x_0^*(x_1) = 0$.

When $x_1 < x_L$ and $x_1 + x_0^*(x_1) = 0$, the supplier's pay-off is $-c_1^1 - \ldots - c_1^{|x_1|}$. If the supplier expedites one period less, as long as $x_1 + x_0^*(x_1)$ is still equal to 0, the supplier always gets its pay-off improved.

In summary, $x_L$ always exists and for any $x_1 < x_L$, we have $\pi_1(x_1, x_0^*(x_1)) < \pi_1(x_L, x_0^*(x_L))$. $\square$

### Proof of Theorem 7

It is obvious that the manufacturer has a dominant strategy of "keep". Using a backward induction, the pay-off matrix between supplier 1 and supplier 2 is:

| 1\2 | K | D |
|---|---|---|
| K | $0, 0$ | $0, s_2 - p$ |
| D | $s_1 - p, 0$ | $s_1 - \beta_1 p, s_2 - \beta_2 p$ |

By Condition 4, [D, K] or [K, D] cannot be the equilibrium because $s_1 < p$ and $s_2 < p$. [D, D] cannot be the equilibrium either because $s_1 - \beta_1 p$ and $s_2 - \beta_2 p$ cannot be larger than 0 at the same time, otherwise $s_1 + s_2 < p$ from Condition 4 is violated. We could verify that [K, K] is the only equilibrium. Note that we do not have to specify $\beta_1$ and $\beta_2$ completely. □

**Proof of Theorem 8**

Not every supplier would like to expedite because $-\sum_{j=1}^{|x_i|} c_i^j + \alpha_i \sum_{j=1}^{|x_s|} r^j$ is not positive for every $i$, otherwise we violate the assumption that the original schedule is the optimal schedule. So $x_s \geq 0$ and thus no supplier would like to expedite. On the other hand, no supplier would like to delay because those suppliers who delayed have to share the penalty. By Condition 5, at least one of them is losing money. Because this fact applies to any group of suppliers who delay, no supplier would like to delay and so "Keep" is the dominant strategy for every supplier.

Knowing that suppliers will always keep, the manufacturer's pay-off is: (1) $\sum_{i=1}^{|x_0|} r^i - \sum_{i=1}^{|x_0|} c_0^i$ if it expedites $|x_0|$; (2) 0 if it keeps; (3) $-\sum_{i=1}^{|x_0|} p^i + \sum_{i=1}^{|x_0|} s_0^i$ if it delays. By Condition 5(1), the pay-offs in (1) and (3) are all less than 0. Thus, the best strategy for the manufacturer is "keep". □