Energy Supply Chain Design: A Dynamic Model for Energy Security, Economic Prosperity, and Environmental Sustainability

Raza Rafique KwonGi Mun Yao Zhao

Department of Supply Chain Management & Marketing Sciences, Rutgers University, Newark, NJ 07102
rrafique@rutgers.edu, kwongi.mun@rutgers.edu, yaozhao@andromeda.rutgers.edu

January 7, 2015

Abstract

Many developing countries in Asia and Africa suffer severe energy deficiencies despite their ample reserves of energy resources (e.g., coal, gas/oil and hydro), the so-called dilemma of "*resource rich, energy poor*" by The Economist. A leading driver of the dilemma is the vicious energy-economy cycle, where the poor economic status, inefficient utilization of limited budget, and energy deficiency reinforced each other and have led these countries into a cycle of economic downfall. How to turn this vicious cycle into a prosperity cycle? It is a classic question but not well answered in the energy policy/economics literature and barely studied in the operations management literature. We extend supply chain management concepts to address the unique features of the energy sector and present a new class of mathematical models for designing coal-fired energy supply chain. The model captures the interaction among different parts of an integrated energy supply chain from coal mining to power generation and to power consumption. The model incorporates the unique economics of power generation and transmission such as yield losses, and the dynamic nature of an energy supply system such as limited reserves, and the causal relationships between energy consumption and economy. The model answers the classic question by determining the optimal way to build up an energy supply chain strategically under limited budgets for energy security, economic prosperity and environmental sustainability. Applying the model to Pakistan's recent energy crises, we show that the solutions differ structurally from the government's plan, and can significantly outperform the latter by reducing the energy gaps faster, boosting the economy stronger with much less greenhouse gas emissions.

1 Introduction

1.1 Motivation

Many developing countries in Asia and Africa are bestowed with abundant hydrocarbon resources, e.g., coal (oil) reserves of India and Pakistan (Sudan) are among the largest in the world. However, these countries also have the world's largest bulk of population denial of electricity (International Energy Agency (IEA) 2011). The Economist (Kiernan 2014) called such a dilemma "*resource rich, energy poor*".

One leading cause for the dilemma is the vicious energy-economy cycle. Electricity supply requires the build-up of an energy supply system from mining to power plants and to electricity transmission.

This is a formidable task that requires a significant and consistent investment over decades. Developing countries with high energy deficiency are often in deep debts and bear poor credit ratings, and thus can only provide limited budgets for energy infrastructure development. Limited funds along with their inefficient utilization result in ineffective energy supply systems which further worsen the economic status and budgetary situations. The causal relationship between energy consumption and economy (GDP) is well established in the energy policy and economics literature, see Section 2.2 for details. Thus, the poor economy, inefficient use of limited budget and energy deficiency interact with each other and have led these countries into a vicious cycle of economic downfall (Figure 1).

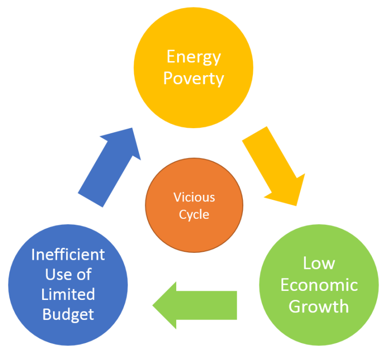


Figure 1: The Vicious Cycle (Rafique and Zhao 2011).

Facing energy crises, some countries essentially deny electricity access to bulks of their population which results in stagnated economic growth or recession; others have to import expensive oil/gas from fluctuated international market to meet their short-term needs. Either way, energy deficiency drove up unemployment and inflation rates which impair social and political stability. Some countries rely heavily on outside aids but at the risk of losing autonomy and independence.

The case of Pakistan is exemplary. With the world's 5th largest coal reserves, Pakistan is suffering a severe electricity shortage in recent years. For instance, electricity shortages in summer 2013 amounts to 6,500 (MWh) with an estimated total demand of 16,500 (MWh). As reported by National Electric Power Regulatory Authority (NEPRA) of Pakistan, load shedding is as common as 9-12 hours a day in major cities. The energy deficiency have put many industrial sectors of Pakistan on the verge of collapse. For example, the output of textile and fertilizer sectors has fallen by nearly 40%-50%, which, gives rise to a sharp increase in unemployment rate due to closure of hundreds of industrial units across the country. Textile Exporters Association estimates that about 150,000 jobs were lost in Faisalabad (the 3rd largest city in Pakistan) and the surrounding Punjab province over the last five years (Santana 2013). It is estimated that electricity shortages have resulted an approximate cost of around 2-4% of GDP annually in the past few years.

To resolve energy deficiency, Pakistan imported a significant amount of oil and gas from international market which resulted in an energy mix (Figure 2) heavily relying on thermal resources, such as oil (contributed 38.35%) and gas (25.5%), in sharp contrast to the 5% world's average of electricity generation through oil (IEA 2011 and NEPRA 2011). The heavy dependence on imported oil and gas for power generation is risky and pricey due to the highly volatile prices of oil in the global commodity market. As expected, electricity price has increased drastically in Pakistan together with the prices of numerous products and services that require electricity. The high inflation and unemployment rates combined are pushing millions of Pakistanis into poverty and riots. As reported by Kugelman (2013) in February 2013, Pakistan's minister for water and power warned that the energy crisis has become a national security issue.

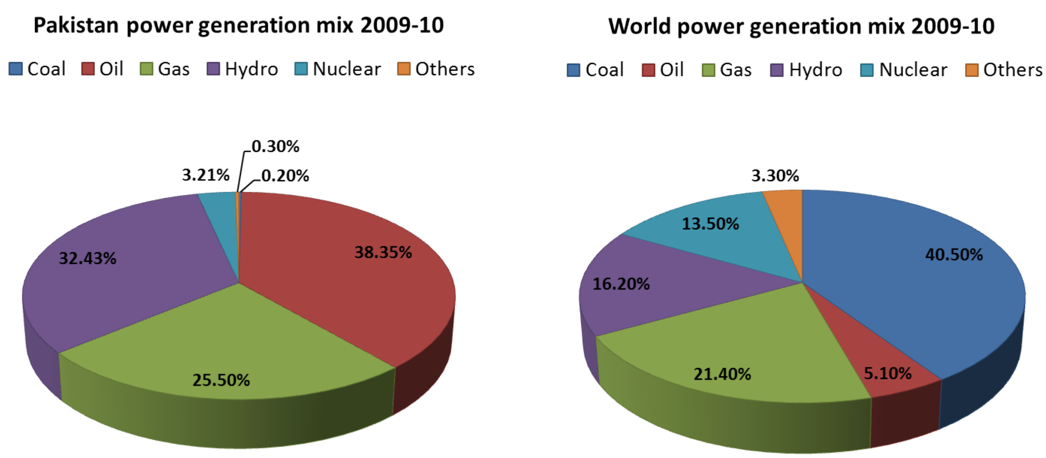


Figure 2: Energy Mix: Pakistan vs. World. Source: IEA 2011 and NEPRA 2011.

As described by Malik (2008), the current energy mix is not sustainable for Pakistan with a fast growing population of 180 million and a fast growing economy with a real GDP of $133 billion in 2011. Pakistan is in urgent need of an inexpensive and reliable source for its future energy security and economic prosperity. Fortunately, the country has one of the world's largest coal reserves with approximately 185 billion tons of coal, equivalent to about 300 billion barrels of oil, exceeding the combined oil reserves of Saudi Arabia and Iran. Pakistan's coal, however, is nearly untapped. By IEA (2011), the world's average of electricity generation from coal is 41% but in Pakistan, coal made only a negligible 0.2% contribution to the total energy supplied (Figure 2). Although increasing the reliance on coal for power generation instead of imported oil seems the only option for the long-term energy security of Pakistan (Malik 2010), building up a coal-fired energy supply chain from scratch presents a significant financial challenge especially given the country's heavy debts (Public debt 2012 is 50.4% of GDP) and poor credit ratings (Standard and Poor 2012: B-, domestic; B-, foreign). What Pakistan can afford at most is a limited budget of a few percent of its GDP each year.

To resolve the dilemma of "*resource rich, energy poor*" in Pakistan and countries alike, we must turn the vicious cycle into a prosperity cycle. To this end, this paper attempts to answer the following question: how to efficiently utilize limited funds to build up an energy supply chain gradually to ensure energy security, economic prosperity, and environmental sustainability? This paper focuses on coal - one of the most important sources for energy generation in the world. We shall discuss the generalization of the modeling framework to other energy sources in Section 6.

1.2 Coal-Fired Energy Supply Chain

A coal-fired energy supply chain includes the following major parts: coal reserves/mines, railway network, power plants, transmission network and demand zones (see Figure 3). The railway network transports coal from mines to power plants, and the transmission network transmits electricity generated by power plants to demand zones.

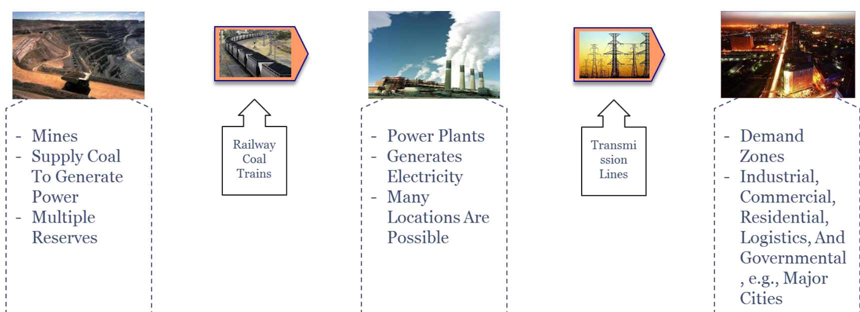


Figure 3: Coal-based energy supply chain.

Different parts of an energy supply chain have distinct economics. Specifically, setting up mining infrastructure requires a significant investment over multiple years depending on the depth and quality of the coal reserves as well as water resources. A standard 300 MWh power plant takes multiple years and billions to build and burns a few thousands tons of coal every day to run at its full capacity. To facilitate such a heavy load, a new railway network needs to be built and dedicated freight trains need to be purchased, at the cost of millions of dollars besides a distance-based operating cost. Power transmission line is much cheaper to built and operate than railway, but it is subject to yield losses. According to EIA (U.S. Energy Information Administration), the electricity transmission and distribution losses average about 6% per 100 miles in United States. World Bank data reports that transmission and distribution losses in developing countries, such as Pakistan, are around 20-25% due to aging infrastructure. Based on these numbers, we can reasonably estimate an average of 8% yield loss per 100 miles for new or upgraded transmission lines in developing countries. Finally, operating a mine or a power plant requires a variable cost per unit of output.

An energy supply chain has many unique features that distinguish it from the well-studied material supply chains (i.e., logistics networks).

1. *A material supply chain deals with production, distribution and transportation of physical goods. In contrast, a large part of an energy supply chain deals with energy generation and transmission.*

2. *Yield loss of power transmission makes an energy supply chain "leaky" while a material supply chain holds material conservation. Thus a longer distance in material supply chains leads to a higher transportation cost, but a longer distance in power transmission means that more power plants need to be built (and more coal to be burnt) to meet the same demand.*

3. *Coal reserves (the source of energy) are limited and may run out. However, factories and warehouses in a material supply chain can run for indefinite times.*

4. *Energy consumption has a strong impact on GDP, which may affect the budget for energy system development and in turn energy consumption. This dynamic feedback loop is likely much weaker in many material supply chains.*

To design an energy supply chain, we shall make decisions on reserve selection (which reserves to mine and when), the number of power plants (PPs) and their locations, rail and power transmission networks over time. An energy supply chain is an integrated system where decisions on one part may affect other parts. For example, if PPs are placed near mines but farther away from demand zones, railway cost for coal transportation will be lower but the yield loss will be higher and so more PPs must be built. Conversely, if PPs are placed near demand zones, then yield loss will be lower but the railway cost will be higher. Defining coal-transportation related costs as inbound costs and transmission-yield induced costs as outbound costs, an effective design of energy supply chains requires a delicate balance of the trade-off between the inbound and outbound costs. This can be a substantial challenge because (1) an energy supply chain is a complex network with geographically dispersed demand zones and reserves, (2) an energy supply system is dynamic in nature due to the limited reserves, fast growing demand, and the causal relationship between energy consumption, GDP and budget.

Pakistan exemplifies the challenges in energy supply chain design. Pakistan has two provinces that contributed significantly to the country's economy: Punjab province (the industrial center contributing 60% GDP) in the north and Sindh province (the commercial center contributing 20% of GDP) in the south (see Figure 4). Punjab (Sindh) accounts for about 60-75% (20%, respectively) of the country's total power consumption. Geographically, major cities of these two provinces scatter across the entire country with a distance (via railway) between Lahore (the capital city of Punjab) and Karachi (the capital city of Sindh) of about 800 miles.

Figure 4: Pakistan coal reserves and major GDP provinces. Modified from "Nations Online Project".

Pakistan has three major coal reserves which account for approximately 98% of the country's total coal reserves (see Figure 4): Thar, Sonda/Lakhra and Salt Range. Thar, the largest reserve (essentially unlimited), is located in the southeast corner of the country far away from all major demand zones. To set up Thar for mining, a significant amount of infrastructure must be built which will cost at least \$6 billion and take 5 years. Salt Range, located in a close proximity to the largest demand zones in Punjab, is the smallest reserve but only costs \$0.5 billion to set up and takes less than 3 years. The medium reserve at Sonda/Lakhra is near the smaller demand zones in Sindh and would cost \$2 billion to set up and take 3 years.

The geographically dispersed demand zones (large or small) and reserves (expensive or inexpensive) of Pakistan showcase the challenges in deciding which reserve(s) to mine and where to locate power plants. Specifically, the fact that the larger reserves (Thar, Sonda/Lakhra) are closer to the smaller demand zones (Sindh) in the south and the smallest reserve (Salt Range) is closer to the largest demand zones (Punjab) in the north makes such decisions intriguing and the trade-off between the inbound and outbound costs hard to balance. Pakistan also has a rapidly growing population and thus faces an increasing demand of energy at an average of 5-7% annually (Alter and Syed 2011, Iqbal, Nawaz and

Anwar 2013).

To save the railway costs and reduce waste from the smaller reserve(s) that may run out, Pakistan government's plan is to solve the energy crisis once and for all by mining the distant largest reserve at Thar and building all power plants nearby. More than 800 miles of transmission lines are planned to transmit the power from Thar to Punjab province in the north and the rest of the country. Although this plan is quite intuitive, it raises two concerns: (1) the accumulated yield loss of power transmission amounts to nearly 50% over the 800-mile distance. Despite savings from railway, one has to double the number of power plants to get the same output at Punjab. (2) The reserves at Thar are least ready and require the longest duration and heaviest investment to mine.

While cost is an important factor, time is equally critical in designing energy supply chains. Giving the fact that Pakistan's economy is on the verge of collapse, a timely influx of new energy will not only save the country from bankruptcy but could also jump-start the economy, which in turn allows more budget to be allocated to the energy sector in the future. Hence, investing in a nearby small reserve that may run out (like Salt Range) may not be a waste because it is inexpensive and fast, and so can meet immediate needs. Of course, for such a strategy to work, one must design the energy supply chain in a way so as to facilitate possible switch to larger reserves in the future.

The complexity and dynamic nature of an energy supply chain in general and the situation of Pakistan in particular lead us to more specific questions on energy supply chain design: how to design an energy supply chain to optimally balance the trade-off between inbound and outbound costs? Where to location PPs (near reserves or demand zones)? How to combine large and small reserves to take advantage of the economy-energy interaction? And what is the difference that an optimal design can make relative to intuitive designs such as the plan of Pakistan government?

1.3 Summary of Results

In this paper, we apply supply chain management principles and mathematical programming to the energy sector and present a new class of mathematical models for designing coal-fired energy supply chains. For the first time, the model captures the trade-off between inbound and outbound costs in an integrated energy supply system from coal mining to power generation and to power consumption. Also for the first time, the model incorporates the dynamic interaction between energy consumption and economy, and considers limited reserves and the need to switch after they run out. The resulting mathematic model is a multi-period mixing integer program (MIP) aiming at minimizing energy gaps among all demand zones by building up an energy supply chain gradually under limited budgets. Decisions are on reserve selection, power plant location, coal transportation and power transmission linkages.

Performance metrics are cost per MWh consumed, energy gap, GDP growth, and coal burnt per MWh consumed (to measure environmental impact).

The mathematical model produces solutions drastically different from the government's plan. In all cases (under different budgets, planning horizons, demand growth rates), the optimal solutions utilize a tiered strategy which first explores the smallest nearby reserve in Punjab (in the north), then the medium reserve in Sindh (in the south), and finally the largest reserve in Thar (if the planning horizon is sufficiently long). Power plants are spread out in both north and south near demand zones and are supplied locally from the small and medium reserves respectively. After the small reserve in the north runs out, all power plants will be supplied by the medium and large reserves in the south. The optimal solutions outperform government's plan significantly by reducing the energy gaps much faster, boosting the economy much stronger with much less greenhouse gas emissions per MWh consumed.

The rest of this paper is organized as follows: In Section 2, we review the related work in material supply chain design literature and energy policy/economics literature, and describe in detail our contributions. In Sections 3-4, we present assumptions and justifications, the conceptual model, and the mathematical model in full detail. In Section 5, we demonstrate the effectiveness of the model and its solutions through the real-life example of Pakistan in comparison to the government's plan. Section 6 concludes this paper.

2 Literature Review

This work is related to two broad streams of literature: the design of material supply chains, such as logistics networks and integrated supply chains; and energy economics and policy. We shall review related work in both streams and point out the contribution of this work.

2.1 Design of Material Supply Chains

Facility location decisions play a critical role in the strategic design of material supply chains, such as logistics networks and integrated supply chains of physical goods. A key question answered by the literature is where to locate plants, warehouses and other facilities in a material supply chain either for a single period or over multiple periods.

For instance, Geoffrion and Graves (1974) studies the optimal location of intermediate distribution facilities between plants and customers. A multi-commodity capacitated single-period version of this problem is formulated as a mixed integer linear program and solved by Benders Decomposition. Pirkul and Jayaraman (1996) develops a mixed integer programming model for a multi-commodity and multi-echelon distribution system with the objective of minimizing the transportation and distribution costs as well as the fixed costs for opening and operating the facilities. The model is solved by Lagrangian relaxation and a heuristic. The literature went beyond distribution systems to more integrated supply chains with both production/logistics and inventory (safety-stock) costs. For instance, Daskin, Coullard and Shen (2002) considers a distribution-center location model which explicitly incorporates safety-stock costs and economies of scale in transportation. The model is formulated as a non-linear integer-programming problem and solved by a Lagrangian relaxation algorithm. Shen, Coullard and Daskin (2003) studies a joint location-inventory problem where some retailers can serve as distribution centers to achieve risk pooling effect. The problem is formulated as a set-covering integer-programming model and solved by column generation algorithms. We refer the reader to Daskin, Snyder and Berger (2005), and Shen (2007) for reviews of the literature. Shu, Teo and Shen (2005) studies the stochastic transportation-inventory network design problem involving one supplier and multiple retailers, and show that by exploiting certain structures, the problem can be solved efficiently. Snyder (2006) surveys the literature of stochastic and robust facility location models.

Wesolowsky (1973) starts the dynamic facility location literature by studying the single facility location problem that permits location changes for a multi-period planning horizon. An algorithm is developed to optimize the sequence of locations in order to meet changes in cost, volume and location of destinations. Wesolowsky and Truscott (1975) extends this model to locate multiple facilities among many possible sites to serve different demand zones. Van Roy and Erlenkotter (1982) solves a capacitated dynamic location problem with opening and closing decisions using a dual-based branch-and-bound procedure. Love, Morris, and Wesolowsky (1988) provides an early review of this literature.

Hinojosa, Puerto and Fernandez (2000) studies a mixed integer programming model to build facilities at multiple echelons of a distribution system over time. A dynamic, multiple objective, mixed-integer programming model is developed by Melachrinoudis and Min (2000) to solve the multi-period relocation problem. More related work can be found in Canel and Khumawala (1997, 2001) which solve a multi-period international facilities location problem, Klose and Drexl (2005) which addresses concerns like which customers should be serviced from which facility (or facilities), Troncoso and Garrido (2005) which considers specific production and logistics issues in the forest industry, and Dias, Captivo, and Climaco (2007) which solves a dynamic location problem with opening, closure and reopening of facilities by primal-dual heuristic approach.

The dynamic facility location literature also considers integrated supply chains with both production/logistics and inventory issues. For instance, Gen and Syarif (2005) studies an optimization model to integrate facility location decisions with inventory management for multiple products and multiple time periods. Meixell and Gargeya (2005) reviews decision support models for global supply chain design and connects the research literature to practical issues. Altiparmak, Gen, Lin and Paksoy (2006) proposes a solution procedure based on genetic algorithms to find the set of Pareto-optimal solutions for multi-objective supply chain network design problem. Fleischmann, Ferber and Henrich (2006) develops a strategic-planning model for BMW to optimize the allocation of products to global production sites over a finite planning horizon. We refer the reader to Shapiro (2007) and Simchi-Levi, Kaminsky and Simchi-Levi (2009) for a thorough review of supply chain modeling and strategies.

The material supply chain design literature provides important modeling and solution methodologies that can be useful in designing an energy supply chain. However, an energy supply chain is structurally different from a material supply chain (§1.2) and thus demands new models, performance metrics and insights. For instance, the unique feature of yield losses in power transmission gives rise to a new trade-off in energy supply chains (§1.2) that connects the decisions of reserve selection, power plant locations, and rail/power line linkages. The limited reserves mandate the consideration of mine switching over time and thus shape the dynamic nature of the model. The interaction between energy consumption and economy (GDP) not only endogenizes the budget (in contract to exogenous budget often assumed in the material supply chain literature), but also introduces a dynamic feedback loop that could play a significant role in system design and configuration. Finally, energy infrastructure development must account for environmental issues in addition to conventional cost factors.

The energy sector is gaining increasing attention from operations and supply chain management researchers. We refer the reader to Hu, Kapuscinski and Lovejoy (2011) for a study of auctions in the wholesale electricity markets, Secomandi and Seppt (2013) for a monograph that provides an integrated finance and operations perspective, and Fang, Misra, Xue, Yang (2012) for a survey on smart grid - how to improve efficiency and reliability of existing energy systems. To the best of our knowledge, energy supply chain design is not studied in the operations and supply chain management literature. In this paper, we extend the literature of supply chain management from physical goods to energy and energy resources by developing a new class of location models to capture the unique features of the energy supply chain.

2.2 Energy Economics and Policy

The energy policy and economics literature studies the specific features of an energy supply chain. Such studies are either empirical or analytical but often focus on individual parts of an energy supply chain rather than the supply chain as a whole.

The causal relationship between energy consumption and economy (GDP) is one of most widely studied relationships in this literature. In the economic theory, energy is considered as an input factor in the production function along with capital and labor. Therefore energy consumption is regarded as one of the key drivers of economic growth. Solow (1956) is among the first to develop a theory based on Cobb-Douglas equations to study the influence of energy on the economy. Ever since, the relationship is empirically estimated and justified by many authors using various data sets. For instance, Oh and Lee (2004) performs a multivariate analysis on Korea over the period 1970-1999, which suggests a long-run bidirectional causal relationship between energy and GDP, and a short-run unidirectional causality running from energy to GDP. Narayan, Narayan and Popp (2010) conducts a multi-country analysis and confirms that energy consumption has a positive impact on real GDP in countries like Japan, Malaysia, Pakistan, Sri Lanka, Thailand, and Vietnam. Menegaki (2014) performs a meta-analysis of 51 studies published in the last two decades, and shows that on average, 1% increase in capital increases the elasticity of GDP with respect to energy consumption by 0.85%. Multiple studies focus on Pakistan and justify the causality from electricity consumption to economic growth or industrial output (Shahbaz and Lean 2012, Shahbaz, Zeshan and Afza 2012, Tang and Shahbaz 2013).

Analytical studies and mathematical modeling in the energy economics and policy literature focus on three important parts of an energy supply chain: (i) power plant operations and locations, (ii) power plant fuel transportation, and (iii) electricity transmission.

The literature studies location issues of power plants related to solar, nuclear, wind and thermal sources. Dutton, Hinman and Millhamet (1974) studies the optimal location of nuclear-power plants with respect to construction, operating, and transmission costs. The mathematical model was solved by the simplex method in conjunction with a branch and bound procedure. Barda, Dupuis and Lencioniet (1990) uses the industrial feasibility standard approach to evaluate the best possible location of power plants. The paper considers gas transportation by pipelines that differs from coal energy economics. Rietveld and Ouwersloot (1992) proposes stochastic dominance concepts to rank alternatives among possible locations for nuclear power plants. An integrated hierarchical approach is presented by Azadeh, Ghaderi and Maghsoudi (2008) to select the best-possible location for solar power plants with the lowest costs. This literature also studies power plant operations. For instance, Liu, Huang, Cai, Cheng, Niu, and An (2009) develops a mathematical programming based optimization model for coal and power management to improve the efficiency of a coal-based power plant. Godoy, Benz and Scenna (2012) provides a non-linear programming model to optimize the long-term operations of natural gas combined-cycle power plants.

The literature also studies the fuel transportation and power transmission issues. For instance, Mathur, Chand and Tezuka (2003) studies the optimal utilization and transportation of thermal coal and develops a framework of the general transportation problem based on a linear programming model.

Bowen, Canchi, Lalit, Preckel, Sparrow and Irwin (2010) presents a mathematical programming based multi-period planning model to optimize and expand power transmission system in India with growing demand for electricity. Paulus and Truby (2011) studies the impact of energy transport decisions on the global steam coal market by a spatial equilibrium model. Rosnes and Vennemo (2012) builds an optimization model to estimate the cost of providing electricity to Sub-Saharan Africa over a 10-year period. These papers consider existing power plants and thus power plant location is not an issue.

All aforementioned analytical papers study individual parts of an energy supply chain rather than the energy supply chain as a whole. Recently, the potential of such an integrated approach is acknowledged by Halldorsson and Svanberg (2012), which conceptually explains how supply chain management may have a great potential in applications to the production, accessibility and use of energy, from the point of origin to the point of consumption. The paper also points out that "supply chain research has only to a limited extent explored the nature of energy and energy resources."

Our work expands the energy economic and policy literature to study an integrated energy supply chain from coal mining to power consumption based on supply chain management principles and mathematical programming models. For the first time, we incorporate the new trade-off between inbound and outbound costs as well as the dynamic nature of energy supply systems, such as, limited reserves and energy-GDP interaction, in deciding reserve selection and power plant locations. Applying the model to the real-life situation of Pakistan, we present novel solutions that significantly outperform the conventional wisdom in resolving the energy crises and shed new insights on energy supply chain design.

3 Preliminaries

In this section, we make assumptions for a coal-fired energy supply chain and justify them by practices and standards in the energy sector. We then present a conceptual framework to outline the structure of the mathematical model.

3.1 Assumptions and Justifications

A coal-fired energy supply chain is a network of typically three echelons: coal reserves, power plants and demand zones. The coal reserves (upper-most echelon) are mined and coal is transported by trains via railway to the power plants (middle echelon), where the coal is burnt and the generated electricity is transmitted by power lines to the demand zones (lowest echelon). Figure 5 provides an overview of the network structure.

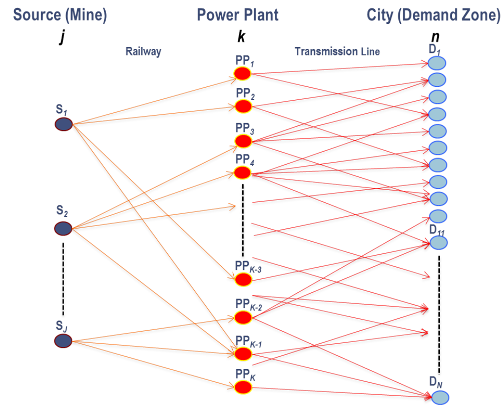


Figure 5: Conceptual Model: Coal-fired Energy Supply Chain.

Inspired by practice in Pakistan and standards in the energy sector, we make the following "network" assumptions.

Assumption 1 *Assumptions on the coal-fired energy supply chain:*

1. *We assume multiple coal reserves (each at a different location) for potential mining as indexed by j where $j = 1, 2, ..., J$. The amount of coal available to mine at location j is denoted as CR_j (in ton).*

2. *We assume multiple demand zones (each at a different location) as indexed by n where $n = 1, 2, ..., N$.*

3. *The potential locations for power plants include all reserves and demand zones which are indexed by k where $k = 1, 2, ..., K$. Although there is no limit on the number of power plants that can be built at the reserves, there are limits at demand zones (cities) due to environmental concerns.*

4. *A coal mine can supply multiple power plant locations, and a power plant can be supplied by multiple mines.*

5. *A power plant can provide power to multiple demand zones, and a demand zone can receive power from multiple power plant locations.*

6. *All power plants use the latest IGCC (integrated gasification combined cycle) technology with a capacity of 300 MWh. All power plants operate at full capacity with a daily consumption of coal at 2,000 tons.*

7. *The railway linkage between a mine and a power plant location is dedicated and requires dedicated coal freight trains.*

8. *Building a power plant at a mine requires new transmission; building it at a demand zone requires upgrading of existing transmission network.*

9. *The yield loss of transmission lines is* 8% *for every 100 miles.*

The first two assumptions in Assumption 1 are sufficiently general to cover real-life situations in Pakistan as well as other developing countries around the world. The third assumption is based on Rafique and Zhao (2011) which provides a thorough case study of the energy crises in Pakistan. This assumption does not lose generality as we can always include potential locations for power plants as demand zones. The limitations on the number of power plants are based on pollution and environment concerns (McMullan, Williams and McCahey 2001, Kavouridisa and Koukouzasb 2008, Chen and Xu 2010). The fourth and fifth assumptions are based on the common industry practice (Rosnes and Vennemo 2012, Bowen et al. 2010). To justify the sixth assumption, we note that a power plant with the IGCC technology and 300 MWh capacity is the current standard in practice (Susta 2008). The seventh assumption can be justified by the heavy load required by power plants. The eighth assumption comes from the facts that reserves are likely unexplored in many developing countries and thus have no transmission infrastructure available, whereas in the demand zones, such infrastructure may be established and thus only an upgrade is needed to deliver a higher volume. The ninth assumption is justified in Section 1.2.

We shall consider a planning horizon of multiple periods (years), and make the following "regularity" assumptions:

Assumption 2 *Regularity assumptions:*

1. *The ration of budget allocated to coal-fired energy infrastructure development is RA percent (1%, 2%, 3%, 4%, or 5%) of the real annual GDP (we use real GDP rather than nominal GDP to eliminate the impact of inflation).*

2. *The impact of energy consumption on real GDP is country-specific.*

3. *Once we start construction or updating an energy infrastructure, we must complete the job without preemption.*

4. *Power sources other than coal run as BAU (business as usual).*

To justify Assumption 2, we first note that developing countries suffering from energy deficiency can only provide limited funding for energy system development. Using Pakistan as an example, the budgetary allocation to its energy sector in the last few years ranges from 10% to 15% (Federal Budget

Publications 2014-15). However, most of the budget is spent on maintenance and operations of existing infrastructures. Only a small amount is dedicated to new ventures. Thus any budget allocation of a higher than 5% of GDP to the new energy projects may be unrealistic. The impact of energy consumption on the economy (GDP) is observed and justified by the energy economics literature (see Section 2.2 for details) and confirmed by our regression study of Pakistan (Section 5). We make the third assumption for convenience because of the unpredictable political circumstances in the next 25 or 50 years. We make the fourth assumption in order to focus on the energy sources of coal.

3.2 The Conceptual Framework

The mathematical model of an integrated energy supply chain is complex (see Section 4). We shall present a conceptual framework first to outline its structure and show intuitively how different parts are connected and interacting.

The **objective** of the mathematical model is to minimize the total discounted energy gap among all demand zones over a given planning horizon. Energy gaps are defined as the differences between projected demand and available supplies. The discounted factors represent a smaller importance for a more distant year into the future.

From the upper-most echelon to the lowest echelon of an energy supply chain, the **decision variables** are: where and when to set up a mine; the location, number and timing of power plant construction; which mine supplies which power plants; and finally which power plant supplies a demand zone and in which year.

The **constraints** can be categorized by echelons and their linkages in the energy supply chain.

1. **Mine constraints**: Reserves have limited supplies.

2. **Railway constraints**: Railway and train have capacity limits.

3. **Power plant constraints**: A limited number of power plants is allowed to built in each potential location.

4. **Network constraints**: A power plant's electricity output can't exceed the required input of energy sources (coal) and its capacity.

5. **Demand and transmission constraints**: The system can't supply more power (electricity) than what is needed for each demand zone.

6. **Budget constraints**: The budget is limited, and energy consumption has an impact on the GDP.

While many of these constraints come from capacity limits at various parts of the supply chain, the network constraint connects coal supply with power generation and transmission, and the budget constraint provides a feedback loop from energy consumption to GDP and to the budget.

The first two categories of constraints can be divided into finer subcategories, specifying availability, capacity and budget requirement of mining, railway and coal trains. For instance, the availability constraints of a reserve indicate that it can be setup only once; the capacity constraints honor the limit of the reserve; and the budget constraints calculate the required budget. The subcategories of capacity and budget constraints also apply to power plants.

4 Mathematical Model

In this section, we present a mathematical model for the optimal design of coal-fired energy supply chains. We define indices in Table 1, which is followed by key decision variables in Table 2. All decision variables are non-negative.

Index	**Name**	**Set**
j	mines (reserves)	$\{1, 2, ..., J\}$
k	power plant locations	$\{1, 2, ..., K\}$
n	demand zones	$\{1, 2, ..., N\}$
t	time	$\{1, 2, ..., T\}$

Table 1: Indices

Mines			
w_{jt}	1 if mine (reserve) j enters service in year t, 0 otherwise	Binary	N/A
q_{jkt}	Coal shipped from mine j to power plant location k in year t	Continuous	Unit: *ton*
Railway			
x_{jkt}	1 if the railway b/t j and k enters service in year t, 0 otherwise	Binary	N/A
nt_{jkt}	New trains purchased to transport coal b/t j and k in year t	Integer	N/A
Power Plants / Power Supplies			
y_{kt}	No. of power plants entering service at location k in year t	Integer	N/A
e_{kt}	Electricity generated at location k in year t	Continuous	Unit: MWh
p_{knt}	Electricity supplied from location k to demand zone n in year t	Continuous	Unit: MWh

Table 2: Decision Variables

4.1 Objective Function

Let ω be the yield of power transmission every 100 miles ($\omega = 92\%$), G_{nt} be the energy gap at demand zone n in year t (in MWh), and D_{kn} be the distance between power plant location k and demand zone n (in 100 miles). Let β_t be a series of time discounted factors decreasing in years, the objective function, i.e., the total discounted energy gap for all demand zones over a finite planning horizon T is,

$$\sum_{t=1}^{T}[\beta_t \cdot \sum_{n=1}^{N}\{G_{nt} - (\sum_{k=1}^{K}\omega^{D_{kn}} \cdot p_{knt})\}] \longrightarrow Min \tag{1}$$

The second term in the parenthesis represents the total energy consumed at demand zone n in year t.

4.2 Mine Constraints

The first set of constraints for mines is on their limited reserves, that is, the amount of coal extracted from mine j up to time t cannot be greater than the total reserve of mine j, CR_j (in *ton*).

$$\sum_{k=1}^{K}\sum_{\tau=1}^{t} q_{jk\tau} \leq CR_j \cdot \sum_{\tau=1}^{t} w_{j\tau} \qquad \text{for} \quad j = 1, \ldots, J \quad \text{and} \quad t = 1, \ldots, T, \tag{2}$$

where the left-hand-side is the amount of coal extracted from mine j up to year t. Clearly, coal can only be extracted from mine j if the mine is set up (that is, $w_{j\tau} = 1$ for some $\tau \leq t$).

The second set of constraints for mines specifies their availability. Because a mine can only be setup once, thus

$$\sum_{t=1}^{T} w_{jt} \leq 1 \qquad \text{for} \quad j = 1, \ldots, J. \tag{3}$$

Because each mine requires time to setup, w_{jt} must satisfy the following initial conditions: $w_{jt} = 0$, for $t = 1, 2, .., T_j^{CM}$ where T_j^{CM} is the reserve/mine specific setup time.

The last set of constraints for mines calculates their capital and operating costs. For mine j, the setup cost in year t (in \$1,000), b_{jt}^{CM1}, can be written as,

$$b_{jt}^{CM1} = I_j^{CM} \cdot \sum_{\tau=t+1}^{t+T_j^{CM}} w_{j\tau} \qquad \text{for} \quad t = 1, \ldots, T - T_j^{CM}, \tag{4}$$

where I_j^{CM} is the annual capital investment for setting up mining infrastructure at mine j assuming that the total investment is evenly distributed over the duration.

Because setting up the mines takes multiple years, it is logical to assume that we cannot start setting up the mining infrastructure at mine j after the $T - T_j^{CM}$th year as the mine will be ready beyond the planning horizon and thus cannot contribute to the objective function. Therefore the ending conditions for mine j are

$$b_{jt}^{CM1} = I_j^{CM} \cdot \sum_{\tau=t+1}^{T} w_{j\tau} \qquad \text{for} \quad t = T - T_j^{CM} + 1, ..., T - 1, \tag{5}$$

and

$$b_{jT}^{CM1} = 0. \tag{6}$$

Finally, the operating cost of all mines in year t, b_t^{CM2}, is given by

$$b_t^{CM2} = OC^{CM} \cdot \sum_{j=1}^{J} \sum_{k=1}^{K} q_{jkt} \qquad \text{for} \quad t = 1, \ldots, T, \tag{7}$$

where OC^{CM} is the unit operating cost at mines.

4.3 Railway Constraints

We shall only consider railway upgrading in this section. New railway construction uses the same equations but with different cost and time parameters. The first set of constraints specifies the availability of the railway. Let D_{jk} be the distance between mine j and power plant location k (in 100 *miles*) and M be a large number. Note that x_{jkt} is the indicator on the availability of the railway between j and k in year t, and nt_{jkt} is an integer variable representing new coal trains purchased on this railway (Table 2), then

$$\sum_{t=1}^{T} x_{jkt} \leq 1 \qquad \text{where} \quad D_{jk} > 0, \quad \text{for} \quad j = 1, \ldots, J \quad \text{and} \quad k = 1, \ldots, K. \tag{8}$$

Constraint 8 ensures that the railway between mine j and power plant location k is set up only once.

$$q_{jkt} \leq CR_j \cdot \sum_{\tau=1}^{t} x_{jk\tau} \qquad \text{where} \quad D_{jk} > 0, \quad \text{for} \quad j = 1, \ldots, J, \quad k = 1, \ldots, K \quad \text{and} \quad t = 1, \ldots, T. \tag{9}$$

Constraint 9 indicates that coal can only be transported if the corresponding railway is set up.

$$nt_{jkt} \leq M \cdot \sum_{\tau=1}^{t} x_{jk\tau} \qquad \text{where} \quad D_{jk} > 0, \quad \text{for} \quad j = 1, \ldots, J, \quad k = 1, \ldots, K \quad \text{and} \quad t = 1, \ldots, T. \tag{10}$$

Constraint 10 connects trains to the availability of railway. Because a railway takes multiple years to setup, we have the following initial conditions: $x_{jkt} = 0$ for $t = 1, 2, ..., T^{RW}$ where T^{RW} is the railway setup time.

The second set of constraints is on railway capacity which depends on the frequency and capacity of trains. Let RT_{jk} be the round trip time between j and k (in *hour*), then Constraint 11 specifies an upper limit on the number of trains between j and k.

$$\sum_{\tau=1}^{t} nt_{jk\tau} \leq BF^{TR} \cdot RT_{jk} \qquad \text{for} \quad j = 1, \ldots, J, \quad k = 1, \ldots, K \quad \text{and} \quad t = 1, \ldots, T, \tag{11}$$

where $\sum_{\tau=1}^{t} nt_{jk\tau}$ is the total number of trains purchased up to year t to operate on the railway between j and k, BF^{TR} stands for "buffer of trains", which is the maximum number of trains allowed to pass through location k in one hour. Assuming a 10-minute minimum time interval between consecutive trains, $BF^{TR} = 6$. We can choose $BF^{TR} \cdot RT_{jk}$ for the M in constraint 10 for the railway between mine j and power plant location k.

For the railway between j and k, we further define $C_{jk}^{TR} = C^{TR} \cdot F_{jk}$ to be the annual capacity of one train (in *ton*) where C^{TR} is the load of one train, and F_{jk} (depending on RT_{jk} and can be country specific) is the maximum frequency (number of round-trips) of a train in one year. Then

$$q_{jkt} \leq C_{jk}^{TR} \cdot \sum_{\tau=1}^{t} nt_{jk\tau} \quad \text{where} \quad D_{jk} > 0, \quad \text{for} \quad j = 1, \ldots, J, \quad k = 1, \ldots, K \quad \text{and} \quad t = 1, \ldots, T. \tag{12}$$

Constraint 12 specifies the maximum railway capacity based on train capacity and frequency.

The last set of constraints for railways calculates their capital and operating costs. Let b_t^{RW1} (b_t^{RW2}) be the setup cost (operating cost, respectively) of railways in year t (in \$1,000), and b_t^{TR} be the cost of purchasing trains in year t (in \$1,000).

$$b_t^{RW1} = \sum_{j=1}^{J} \sum_{k=1}^{K} \left(\frac{SC_{jk}^{RW}}{T^{RW}} \cdot \sum_{\tau=t+1}^{t+T^{RW}} x_{jk\tau} \right) \quad \text{for} \quad t = 1, \ldots, T - T^{RW}, \tag{13}$$

$$b_t^{RW2} = OC^{RW} \cdot \sum_{j=1}^{J} \sum_{k=1}^{K} (D_{jk} \cdot q_{jkt}) \quad \text{for} \quad t = 1, \ldots, T, \tag{14}$$

$$b_t^{TR} = I^{TR} \cdot \sum_{j=1}^{J} \sum_{k=1}^{K} nt_{jkt} \quad \text{for} \quad t = 1, \ldots, T, \tag{15}$$

where SC_{jk}^{RW} is the setup cost of railway between j and k, OC^{RW} is the unit operating cost for railway system, and I^{TR} is the purchasing cost of one train.

Because railway upgrading takes multiple years, so similar to coal mines, we have the following ending conditions.

$$b_t^{RW1} = \sum_{j=1}^{J} \sum_{k=1}^{K} \left(\frac{SC_{jk}^{RW}}{T^{RW}} \cdot \sum_{\tau=t+1}^{T} x_{jk\tau} \right) \quad \text{for} \quad t = T - T^{RW} + 1, ..., T - 1, \tag{16}$$

$$b_T^{RW1} = 0. \tag{17}$$

4.4 Network Constraints

This set of constraints ensures that the power plants are adequately supplied from the mines to run at their full capacity, and the electricity generated at each location, e_{kt}, is transmitted to demand zones.

$$e_{kt} \leq \frac{3}{20 \cdot 365} \cdot \sum_{j=1}^{J} q_{jkt} \quad \text{for} \quad k = 1, \ldots, K \quad \text{and} \quad t = 1, \ldots, T, \tag{18}$$

where $\frac{3}{20 \cdot 365}$ is the conversion rate between coal and power as 2000 tons of coal is needed every day for a power plant to maintain its full capacity at 300 MWh year around.

$$e_{kt} \leq 300 \cdot \sum_{\tau=1}^{t} y_{k\tau} \quad \text{for} \quad k = 1, \ldots, K \quad \text{and} \quad t = 1, \ldots, T. \tag{19}$$

Constraint 19 limits the electricity generated by the power plant capacity.

$$\sum_{n=1}^{N} p_{knt} \leq e_{kt} \qquad \text{for} \quad k = 1, \ldots, K \quad \text{and} \quad t = 1, \ldots, T. \tag{20}$$

Constraint 20 ensures that the amount of electricity transmitted is less than electricity generated at each location.

4.5 Power Plant and Transmission Constraints

The first set of power plant constraints is on their location dependent limitations, UB_k, which is the maximum number of power plants that can be built in location k.

$$\sum_{\tau=1}^{t} y_{k\tau} \leq UB_k \qquad \text{for} \quad k \in \mathcal{K} \quad \text{and} \quad t = 1, \ldots, T. \tag{21}$$

The initial condition for y_{kt} is $y_{kt} = 0$ for $t = 1, 2, ..., T^{PP}$ where T^{PP} is the setup time for a power plant.

The second set of constraints for power plants calculates their capital and operating costs. Let I^{PP} be the annual setup cost of a power plant, OC^{PP} be the annual operating cost per MWh generated (for a standard 300 MWh power plant), and SC_k^{TL} be the location-dependent setup/upgrading cost for transmission line and grid station. Then the cost of building power plants in year t, b_t^{PP1}, and the cost for building associated grid stations and power plant operations, b_t^{PP2}, are

$$b_t^{PP1} = I^{PP} \cdot \sum_{k=1}^{K} \sum_{\tau=t+1}^{t+T^{PP}} y_{k\tau} \qquad \text{for} \quad t = 1, \ldots, T - T^{PP}, \tag{22}$$

$$b_t^{PP2} = \sum_{k=1}^{K} \{(SC_k^{TL} \cdot y_{kt}) + (OC^{PP} \cdot e_{kt})\} \qquad \text{for} \quad t = 1, \ldots, T, \tag{23}$$

where the operating cost depends on the electricity generated, e_{kt}. Because constructing transmission lines and grid stations typically take a shorter time than power plants, we assume that such auxiliary infrastructures are scheduled so as to match the completion time of the corresponding power plant.

Because a power plant takes multiple years to build, similar to coal mines, we have the following ending conditions.

$$b_t^{PP1} = I^{PP} \cdot \sum_{k=1}^{K} \sum_{\tau=t+1}^{T} y_{k\tau} \qquad \text{for} \quad t = T - T^{PP} + 1, \ldots, T - 1, \tag{24}$$

$$b_T^{PP1} = 0. \tag{25}$$

4.6 Demand Constraints

Demand constraints ensure that the total amount of electricity supplied at each demand zone is less than its energy gap.

$$\sum_{k=1}^{K}(\omega^{D_{kn}} \cdot p_{knt}) \leq G_{nt} \qquad \text{for} \quad n = 1, \ldots, N \quad \text{and} \quad t = 1, \ldots, T. \tag{26}$$

4.7 Budget Constraints

The first set of budget constraints limits the total spending on energy infrastructure development in each year by the budget.

$$\sum_{j=1}^{J} b_{jt}^{CM1} + b_t^{CM2} + b_t^{RW1} + b_t^{RW2} + b_t^{TR} + b_t^{PP1} + b_t^{PP2} \leq g_{t-1} \cdot RA_t \qquad \text{for} \quad t = 1, \ldots, T, \tag{27}$$

where RA_t is the ration in year t, that is, the % of GDP allocated to the energy sector for these projects; g_t is year t's real GDP.

The second set of budget constraints connects GDP in year t, g_t, to year $(t-1)$'s energy consumption.

$$g_1 = g_0 + Coef \cdot \sum_{k=1}^{K}\sum_{n=1}^{N}(\omega^{D_{kn}} \cdot p_{kn1}), \tag{28}$$

where g_0 is the initial GDP and $Coef$ is the elasticity of GDP with respect to energy consumption, that is, the slope of the country or economy specific regression model with dependent variable being real GDP and independent variable being energy (electricity) consumption.

$$g_t = g_{t-1} + Coef \cdot \sum_{k=1}^{K}\sum_{n=1}^{N}\{\omega^{D_{kn}} \cdot (p_{knt} - p_{knt-1})\} \qquad \text{for} \quad t = 2, \ldots, T. \tag{29}$$

Specifically, GDP growth in year t depends on how much more electricity is consumed in this year than the previous year.

5 Solution and Impact

In this section, we apply the mathematical model (developed in Section 4) to Pakistan and demonstrate its potential impact. Section 5.1 presents the real-life situations of Pakistan, and Section 5.2 provides solutions and insights.

5.1 The Case of Pakistan

A map of Pakistan coal reserves and major demand zones is shown in Figure 6.

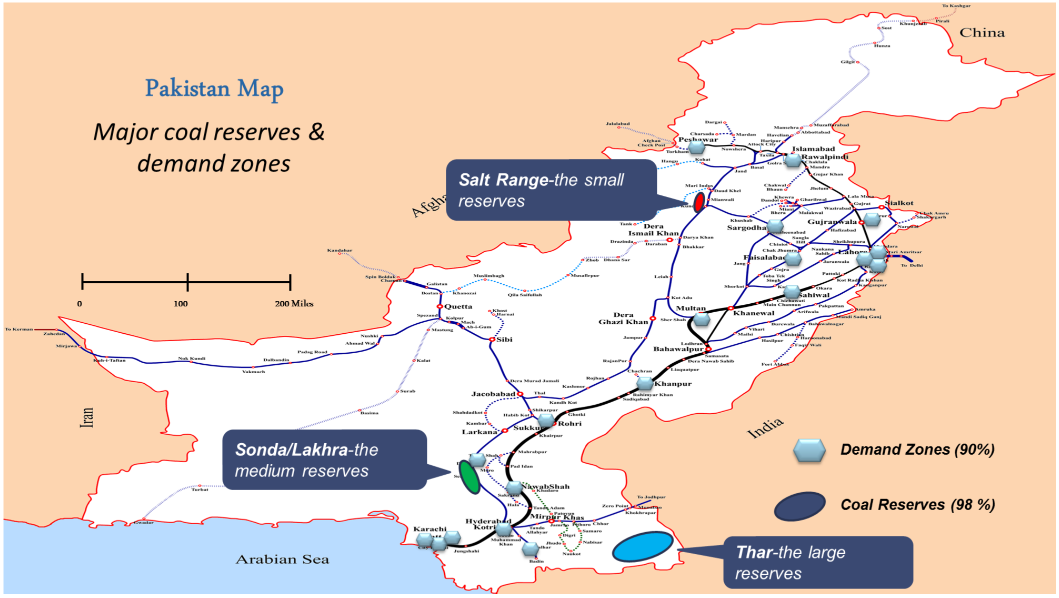


Figure 6: Pakistan map with major coal reserves and demand zones. Modified from "Pakistan Railways Network Map" by Adnanrail Licensed under Creative Commons Attribution-Share Alike 3.0 via Wikimedia Commons.

A thorough case study of Pakistan's energy crises by Rafique and Zhao (2011) provides the following observations.

Observation 1 *Observations on the coal-fired energy supply chain of Pakistan:*

1. *Three largest coal reserves (Figure 6): Thar, Sonda/Lakhra and Salt Range, account for 98% of Pakistan's total coal reserves,* $J = 3$. *The reserves that are of sufficient quality for power generation are listed in Table 3. As we can see, Thar and Sonda/Lakhra reserves have ample reserves to meet demand in a planning horizon of 25 or 50 years but Salt Range does not.*

2. *There are 19 demand zones that account for 90% of the country's total energy consumption (Figure 6),* $N = 19$. *14 of them are major energy-consumption cities and 5 of them are smaller cities but ideal locations for power plants. Demand for energy is estimated to grow at a rate of 5-7% annually (Section 1.2).*

3. *The potential locations for power plants include all reserves and demand zones, thus there are 22 locations* ($K = 22$)*.*

4. *There is no limit on the number of power plants that can be built at the three coal reserves, that is,* $UB_j = +\infty$ *for* $j \in \mathcal{K}_R$ *where* $\mathcal{K}_R$ *is the set of power plant locations at coal reserves. For demand zones, at most ten power plants* ($UB_j = 10$) *can be built in each of the five smaller cities* $j \in \mathcal{K}_S$*; at most five power plants* ($UB_j = 5$) *can be built in the other bigger cities* $j \in \mathcal{K}_B$*.*

We consider a planning horizon of either 25 years or 50 years ($T = 25$ or 50).

Name	**Province**	**Reserves (Million Tons)** $CR_j/10^6$	**Years of 10,000 MWh generated**
Salt Range	*Punjab*	213	9
Sonda and Lakhra	*Sindh*	8,440	350
Thar	*Sindh*	175,000	7,251

Table 3: Pakistan Coal Reserves. Source: Rafique and Zhao (2011).

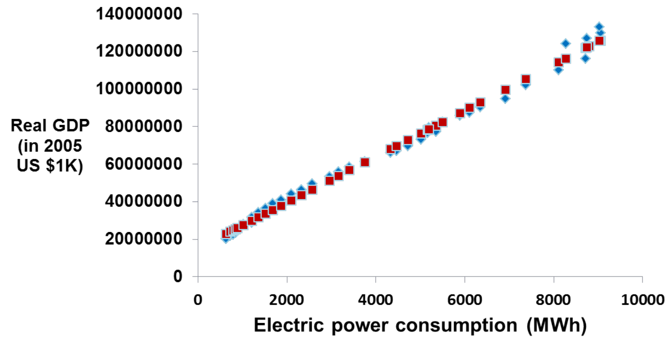
Table 4 specifies the model parameters for Pakistan.

Mines:		
T_1^{CM}	Setup time for *Thar* reserve	5 years
T_2^{CM}	Setup time for *Sonda/Lakhra* reserve	3 years
T_3^{CM}	Setup time for *Salt Range* reserve	3 years
I_1^{CM}	Annual investment for mine setup at *Thar*	$6,000,000/5$ in *\$1,000*
I_2^{CM}	Annual investment for mine setup at *Sonda/Lakhra*	$2,000,000/3$ in *\$1,000*
I_3^{CM}	Annual investment for mine setup at *Salt Range*	$500,000/3$ in *\$1,000*
OC^{CM}	Unit operating cost at mines	5.33×10^{-3} per *ton* in *\$1,000*
Railway / Train:		
T^{RW}	Setup time for railways	3 (5) years for upgrading (for new)
RT_{jk}	Round trip time between j and k	in *hour*
C^{TR}	Load of one train	15,000, in *ton*
F_{jk}	Maximum annual frequency of a train on the railway between j and k	N/A
SC_{jk}^{RW}	Setup cost of the railway between j and k	in *\$1,000*
OC^{RW}	Unit operating cost for railway coal transport	0.04×10^{-3} per *ton* per *mile*, in *\$1,000*
I^{TR}	Purchasing cost of one train	50,000, in *\$1,000*
Power Plant / Transmission Line:		
T^{PP}	Setup time for a standard 300 MWh power plant	3 years
I^{PP}	Annual setup cost of a standard 300 MWh power plant	1,000,000/3 in *\$1,000*
OC^{PP}	Annual operating cost of a standard 300 MWh power plant	2% of the total setup cost
SC_k^{TL}	Setup cost of transmission and grid station at location k for a new power plant	in *\$1,000*
Demand / Distances		
G_{nt}	Energy gap at demand zone n in year t	in MWh
D_{jk}	Distance between j and k	in 100 *miles*
D_{kn}	Distance between k and n	in 100 *miles*

Table 4: Parameters for Pakistan. Sources: Rafique and Zhao (2011).

Note that the mine setup costs, I_j^{CM}, include water supply related costs. The matrices of D_{jk} and D_{kn} are determined by the geology and transportation network of Pakistan. $RT_{jk} = 2D_{jk}/0.5 + LT + UT$ where 0.5 refers to the average train speed of 50 miles/hour, and LT (UT) refers to loading (unloading, respectively) time. $F_{jk} = 24/RT_{jk} \cdot 365$ where the numbers 24 and 365 refer to 24 hours a day and 365 days a year respectively. G_{nt} depends on the demand of the starting year and the projected growth rate. The cost matrix of railway, SC_{jk}^{RW}, is calculated by the distance matrix D_{jk} and a setup cost of \$0.73 million/mile for upgrading (or \$13 million/mile for new construction) (Ministry of Pakistan Railway). The setup cost of transmission and grid station for a new power plant, SC_k^{TL}, is calculated by the distance from location k to the nearest grid station and a transmission line cost of \$1.8 million/mile for new (or \$0.6 million/mile for upgrading) as well as a local grid station cost of \$33,000/ MWh (American Electric Power, Transmission Facts).

Our empirical study of Pakistan from 1971 to 2011 shows a strong correlation between energy consumption and real GDP (see Figure 7), which confirms the impact of energy consumption on Pakistan's economy (see also Shahbaz and Lean 2012, Shahbaz, Zeshan and Afza 2012, Tang and Shahbaz 2013).



SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9958799
R Square	0.99177677
Adjusted R Square	0.99156592
Standard Error	3229972.25
Observations	41

	Coefficients	Standard Error	t Stat	P-value
Intercept	14951524.8	891458.2	16.77199	1.99E − 19
Electric power consumption (MWh)	12254.8004	178.685	68.58327	2.81E − 42

Figure 7: A regression model between real GDP (in \$1000) and energy consumption (in MWh) for Pakistan. Source: World Bank 2012.

The elasticity of GDP on energy consumption (slope of the regression model) is Coef = 12,254.80044. We select 2011 as the starting year ($t = 0$), and so $g_0 = 133,000,000$ (in *\$1000*).

5.2 Solutions and Insights

In this section, we present the solutions generated by the mathematical model for Pakistan and compare them to the government's plan in metrics such as cost per MWh consumed (for cost efficiency), net GDP (GDP less investment in coal-fired energy sector, for economic growth), energy gap (for energy security), and coal efficiency (coal burnt annually per MWh consumed, for carbon footprint). We shall also derive insights on how an energy supply chain should be built up strategically.

We consider various scenarios of budget from 1%, 2%, ..., to 5% of real GDP, planning horizon (25 or 50 years), and demand growth rate (5% or 7%). The mathematical model leads to a large-scale multi-period mixed integer program with 9,466 constraints, 3,317 integer variables, 2,573 binary variables and 11,025 continuous variables (for 25-year scenarios). The mathematical program is solved by Gomory cutting planes method and implemented by a code written in Python version 2.75 and Gurobi Solver version 5.6. Due to the complexity of the model, an optimal solution is not always achievable. We accept suboptimal but best solutions found if the values of their objective functions are sufficiently close to those of the optimal solutions or a certain limit of running time is reached. All computations are done on a desktop computer with an Intel Xeon 2620 2.0 GHz and 20 GB RAM. The computing time ranges from 4 minutes to 1,344 minutes (see Appendix for details).

The mathematical model provides intriguing solutions, which are drastically different from the government's plan. Recall that the government's plan explores only the largest reserve at Thar and builds all power plants at that location. For comparison, let's consider a representative scenario with 5% demand growth, a budget of 3% of GDP and a planning horizon of 25 years (see Figure 8). The optimal solution first mines the smallest reserve (Salt Range in the north) near the largest demand zones (the industrial hub in Punjab) that requires much less time and capital to setup than other reserves. The medium reserve at Sonda/Lakhra (in the south) near the commercial hub in Sindh is next explored, but the largest reserve at Thar (in the southeast corner) is not setup for mining throughout the planning horizon in this scenario. Power plants are first built at the largest demand zones in Punjab and supplied locally by the Salt Range mine so as to minimize the yield loss at an affordable coal transportation cost. After the medium reserve at Sonda/Lakhra is setup, power plants are then built at demand zones in Sindh and supplied locally by the Sonda/Lakhra mine. When the Salt Range mine runs out (it depletes in about 20 years in this scenario), the power plants in Punjab shall switch supply from Salt Range to Sonda/Lakhra in Sindh. Power plants may be built at coal reserves after nearby demand zones run out of space.

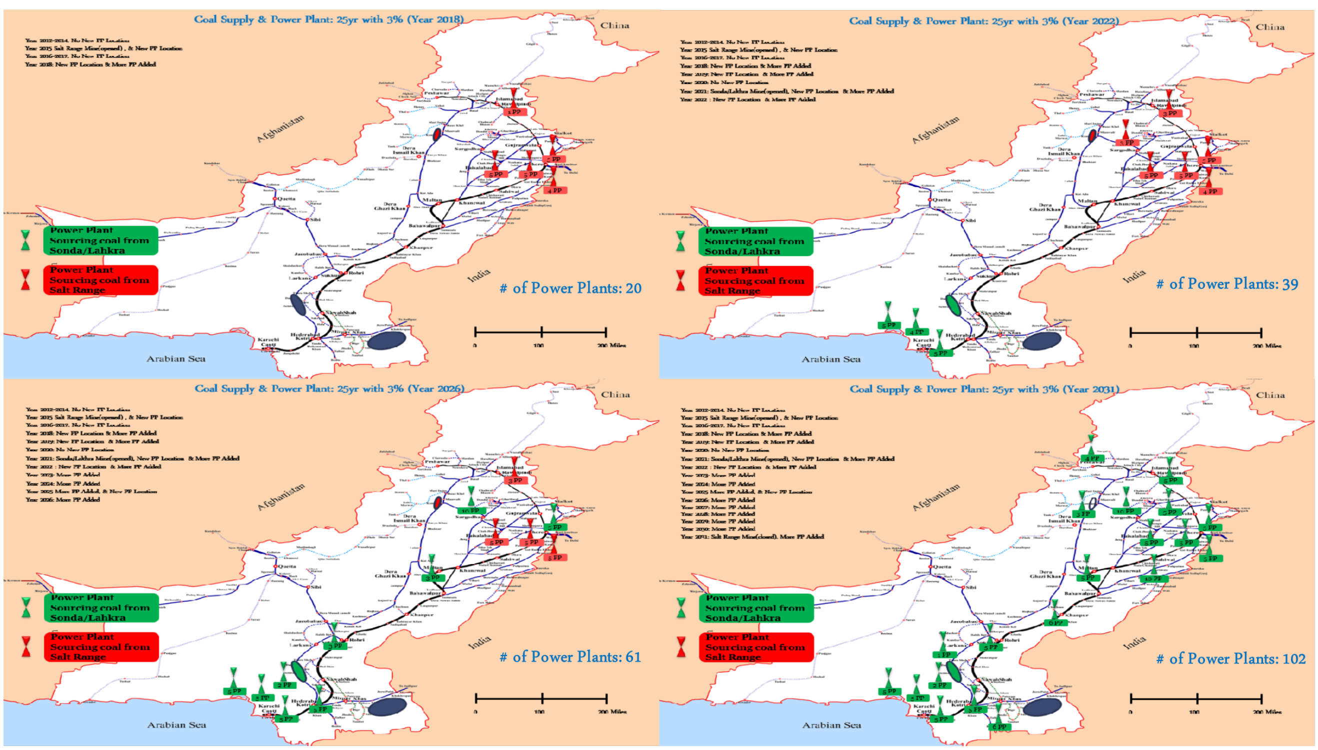


Figure 8: The optimal solution on reserve selection and power plant locations for the scenario with 5% demand growth, a budget of 3% GDP and a 25-year planning horizon. Circles - coal reserves; empty circles - reserves that run out; collate shapes - power plants (the box below indicates the number of power plants in service).

Figure 9 illustrates how the optimal solution supplies demand zones and reduces energy gaps in this scenario. Electricity is first supplied to the demand zones in Punjab, and then to demand zones in Sindh soon after. In about 23 years, the optimal solution reduces energy gaps at all demand zones to zero, and it remains that way till the end of the planning horizon. We must point out that energy gaps may not be reduced to zero in other scenarios with smaller budgets and/or higher demand growth rates.

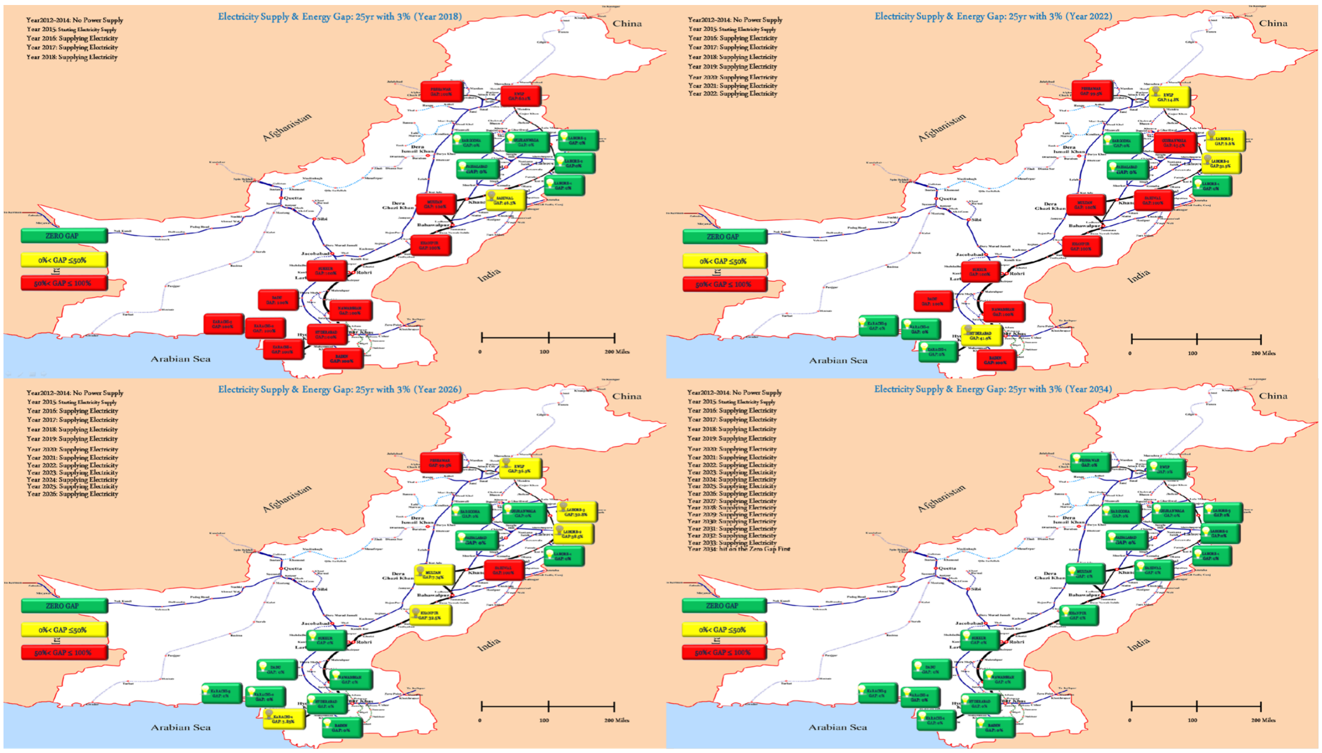


Figure 9: The optimal solution on transmission and energy gaps at demand zones for the scenario with 5% demand growth, a budget of 3% GDP and a 25-year planning horizon. Green - 0% gap, yellow - 1% to 50% gap, red - 51% gap and above.

The unique features of the energy supply chain play critical roles in shaping up the optimal solution. Specifically, power plants are first built at demand zones until the limit on the number of power plants is reached. This solution is driven by the *yield loss* as power plants are more expensive to build than railways. Despite the limited reserve at Salt Range, the optimal solution explores it starting from the beginning because of the *dynamic interaction* between energy consumption and economy. Although Salt Range has the smallest reserve, it is inexpensive and fast, and also close to the largest demand zones. In contrast, Thar has the largest coal reserve but it is not only remote from all demand zones but also requires the highest investment and longest time to set up the mining infrastructure. Intuitively speaking, "distant ocean cannot put off a nearby fire." Although the Salt Range reserve does not last for ever (and so we must be mindful about reserve switch down the road), it can jump-start the energy supply, which in turn fuels the economy and leads to a higher investment back to the energy sector in the future (to explore, for instance, the Thar reserve). Doing so can help turning the vicious energy-economic cycle into a prosperity cycle. Thus the capital spent to set up Salt Range is not a waste but a worthy investment.

To quantify the impact of the optimal solution, we compare it to the government's plan on four metrics (Figure 10): cost efficiency, i.e., cumulative cost over cumulative MWh consumed (a), net GDP (b), country-wide energy gap (c) and coal efficiency (d).

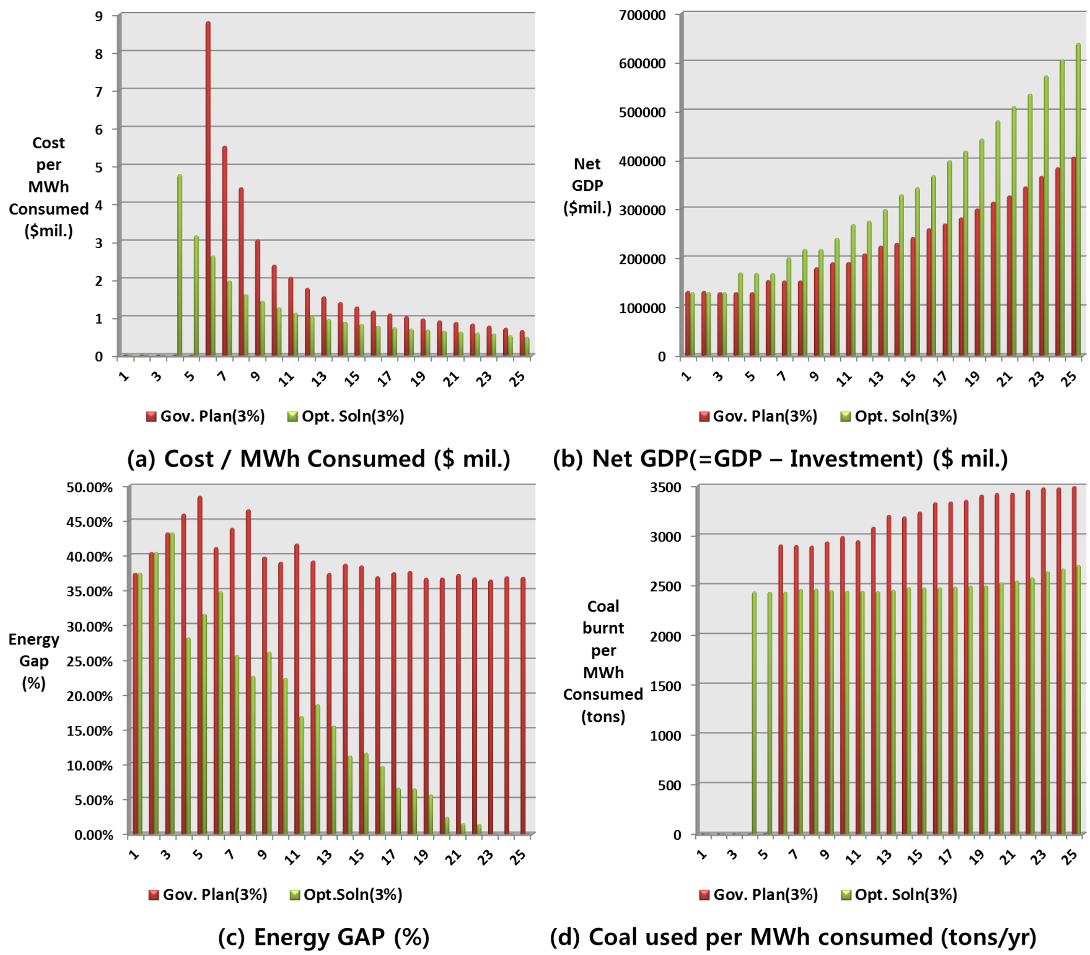


Figure 10: Optimal solution vs. Government's plan. The x-axis is on time (in year). 5% demand growth, 3% ration and 25-year planning horizon.

As we can see, the optimal solution significantly outperforms the government's plan by spending much less for each MWh consumed (Figure 10a), boosting the economy much stronger (Figure 10b), reducing the energy gaps much faster (Figure 10c) with less coal burnt per MWh consumed (Figure 10d). The optimal solution delivers much more electricity to the demand zones with a higher coal efficiency than the government's plan, and thus it is more sustainable in the sense of economy and environment. Specifically, the optimal solution can reduce the energy gap down to zero in about 23 years while the government's plan maintains an approximately 36.92% energy deficiency towards the end of planning horizon. Consequently, the optimal solution will generate a net GDP in the 25th year of $639 billion, as compared to $406 billion of the government's plan. Finally, for every MWh consumed, the government's plan requires 2,898-3,496 tons of coal annually but the optimal plan only requires about 2,439-2,702 tons.

In other budgetary scenarios of 5% demand growth and 25-year planning horizon, the solutions stay qualitatively the same. The most notable difference between the scenarios with less than 3% rations and the scenario with 3% ration is that in the former, we do not have enough money to reduce the energy gap to zero. For instance, in the scenario of 1% ration, the energy gap will rise in both the optimal solution and the government's plan to 65% and 70% respectively in 25th year. Interestingly, the optimal solution still significantly outperforms the government's plan on the net GDP ($228 billion vs. $195 billion) and coal efficiency (2,437-2,461 tons vs. 2,744-2,876 tons annually per MWh consumed). In the scenario of a 2% ration, the energy gap will rise in the government's plan to about 53% in the 25th year but decrease in the optimal solution to around 36%. Consequently, the optimal solution outperforms the government's plan on the net GDP ($412 billion vs. $305 billion) in the 25th year. In scenarios with more than 3% rations, the optimal solution will reduce energy gaps to zero much sooner than the government's plan with higher net GDP and coal efficiency. Thus, our model can be used to justify the budget required to bring down the energy gap to zero in targeted years. Increasing the planning horizon from 25 years to 50 years does not change the trend of the energy gap, net GDP and coal efficiency in all scenarios but widens the differences in the net GDPs (especially in scenarios with less than 3% rations). In addition, the optimal solution may explore Thar after the 25th year mark.

The improvement on GDP made by the optimal solution relative to the government's plan depends on the budget. Figure 11 shows the average GDP per year (in $ million) under the government's plan and the optimal solution for various budgetary conditions (% of GDP). The figure shows that although the optimal solution always outperforms the government's plan, it makes the greatest difference on the average GDP when the budget is neither too tight nor too generous. Intuitively, if the budget is very tight, it allows little flexibility for the optimal solution to improve; if the budget is very generous, cost efficiency as achieved by the optimal solution becomes relatively unimportant because funding is abundant.

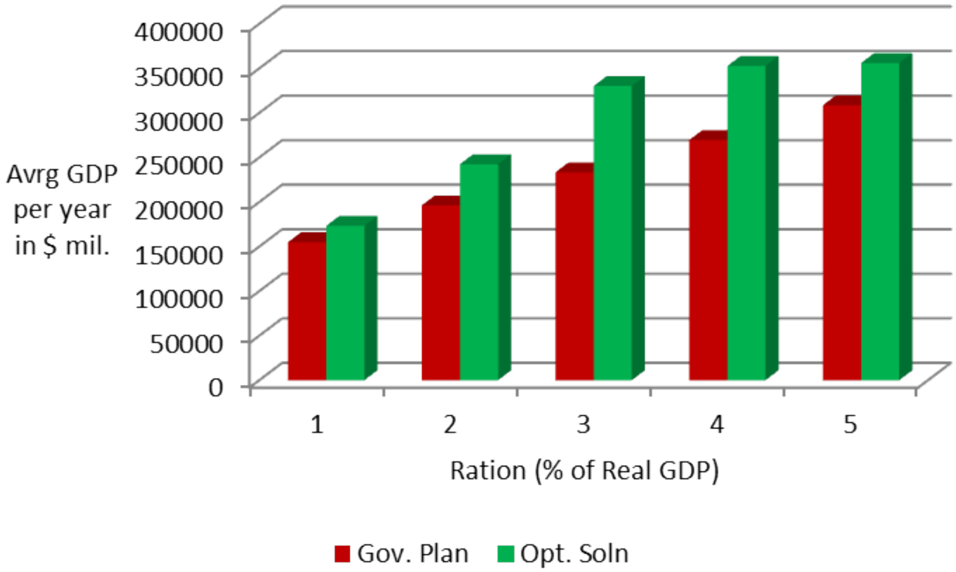


Figure 11: Comparing average GDP between government's plan and the optimal solution for various budgets. 5% demand growth and 25-year plan horizon.

For scenarios with 7% demand growth rate, we find, as expected, that a higher budget is required to reduce the energy gap to zero, although the optimal solutions and other insights remain qualitatively the same.

Implementation of the solutions in Pakistan is challenging and time consuming due to long construction cycle times, meager budgets, and politics played by provinces. In fact, the government's plan was a political decision because the Sindh province in the south wanted to boost its economy. The Punjab province in the north was suffering and wanted to explore the small nearby reserve at Salt Range but was concerned about the waste of exploring a limited reserve that may soon run out. Our model and solutions came in handy to support such a strategic move of Punjab and is also in the interest of Sindh as both the small (in Punjab) and medium (in Sindh) reserves are recommended. According to press media reports (The News International 2014), Punjab province has started exploring the nearby reserve at Salt Range and building coal-fired power plants close to demand zones, as indicated by our solutions.

6 Conclusion

Energy deficiency and economic crises are correlated and commonplace in developing countries around the world, especially those in South Asia, Middle East and sub-Saharan Africa. Fortunately, many of these countries are bestowed with abundant energy resources. A key to solving the energy deficiency (and therefore economic crises) is the design of energy supply chains to utilize these sources effectively under limited budget. An energy supply chain has unique features that distinguish it from the well-studied material supply chains, such as, yield loss of power transmission, limited reserves, and a strong interaction between energy consumption and GDP. In this paper, we construct a novel mathematic model to capture these features of energy supply chains. Applying to the real life situation of Pakistan, we demonstrate the potential of the model in breaking the vicious energy-economy cycle and in improving energy security, economic prosperity and environmental sustainability.

We can extend the mathematical model developed for coal to address similar issues with other energy resources such as oil, gas and hydro, etc. Different energy sources have difference economics in certain part(s) of the energy supply chain. For instance, gas can be transported either by pipelines or in the form of liquefied natural gas (LNG), which have distinct cost structures from coal; oil-based energy supply chains have similar features as gas-based ones except for an additional echelon of oil refineries. Hydro is one of the cleanest and most efficient energy sources but the location of hydro dams depends not only on energy needs but also irrigation in the agriculture sector and concerns of flood. We expect that the mathematical model, after proper customization to each energy source, may make significant differences in developing countries, and help them to resolve the dilemma of "*resource rich, energy poor*".

Although this study demonstrates the effectiveness of our model in addressing the energy gaps through coal-fired energy supply chain, in real world, a combination of coal and other energy options such as nuclear and renewable resources (hydro, solar and wind) is desirable for achieving a more balanced energy mix. In fact, an over-reliance on electricity generated through coal can pose a serious challenge on security, maintenance and environmental issues (Pakistan is currently free of this concern as the contribution of coal to the energy mix is only 0.2%). Ultimately, it is an energy-mix issue: how to optimally balance the energy mix from a portfolio of energy resources? How to optimally utilize different resources of energy under a limited budget? These questions clearly take the research to the next-level of complexity and we plan to answer them in future studies. We also plan to characterize the mathematical properties of energy supply chains to enable more efficient solution algorithms. Finally, this research may serve as a starting point at the interfaces between supply chain management and energy economics to aid policy makers in the energy sector.

Appendix

Table 5 summarizes the CPU times and optimality gaps for the scenarios discussed in Section 5.2.

Demand Growth	Planning Horizon	Ration (% of GDP)	Optimality Gap (%)	Running Time	Type and number of Variable
5%	25 years	1%	4.9937%	216.11 sec	Variable Type: 11,025 continuous, 3,317 integer (2,573 binary)
		2%	7.5839%	3,315.00 sec	
		3%	8.6058%	29,678.00 sec	
		4%	0.8686%	4,288.95 sec	
		5%	9.2671%	667.37 sec	
	50 years	1%	8.5173%	5,718.00 sec	Variable Type: 23,711 continuous, 7,093 integer (5,497 binary)
		2%	11.8825%	26,218.00 sec	
		3%	20.9940%	80,633.03 sec	
		4%	7.8265%	19,247.00 sec	
		5%	4.4076%	10,285.73 sec	

Table 5: CPU Time and Optimality Gaps

References

[1] Alter, N. and Syed, S. H. (2011). An empirical analysis of electricity demand in Pakistan. *International Journal of Energy Economics and Policy*, 1(4):116–139.

[2] Altiparmak, F., Gen, M., Lin, L., and Paksoy, T. (2006). A genetic algorithm approach for multi-objective optimization of supply chain networks. *Computers & Industrial Engineering*, 51(1):196–215.

[3] American Electric Power, Transmission Facts. `<http://www.aep.com/about/transmission/docs/transmission-facts.pdf>`. Accessed on Aug 12, 2012.

[4] Azadeh, A., Ghaderi, S., and Maghsoudi, A. (2008). Location optimization of solar plants by an integrated hierarchical DEA PCA approach. *Energy Policy*, 36(10):3993–4004.

[5] Barda, O. H., Dupuis, J., and Lencioni, P. (1990). Multicriteria location of thermal power plants. *European Journal of Operational Research*, 45(2):332–346.

[6] Bowen, B. H., Canchi, D., Lalit, V. A., Preckel, P. V., Sparrow, F., and Irwin, M. W. (2010). Planning India's long-term energy shipment infrastructures for electricity and coal. *Energy Policy*, 38(1):432–444.

[7] Canel, C. and Khumawala, B. M. (1997). Multi-period international facilities location: An algorithm and application. *International Journal of Production Research*, 35(7):1891–1910.

[8] Canel, C. and Khumawala, B. M. (2001). International facilities location: a heuristic procedure for the dynamic uncapacitated problem. *International Journal of Production Research*, 39(17):3975–4000.

[9] Chen, W. and Xu, R. (2010). Clean coal technology development in china. *Energy Policy*, 38(5):2123–2130.

[10] Daskin, M. S., Coullard, C. R., and Shen, Z.-J. M. (2002). An inventory-location model: Formulation, solution algorithm and computational results. *Annals of Operations Research*, 110(1-4):83–106.

[11] Daskin, M. S., Snyder, L. V., and Berger, R. T. (2005). Facility location in supply chain design. In *Logistics systems: Design and optimization*, pages 39–65. Springer.

[12] Dias, J., Eugénia Captivo, M., and Clímaco, J. (2007). Efficient primal-dual heuristic for a dynamic location problem. *Computers & operations research*, 34(6):1800–1823.

[13] Dutton, R., Hinman, G., and Millham, C. (1974). The optimal location of nuclear-power facilities in the pacific northwest. *Operations Research*, 22(3):478–487.

[14] Fang, X., Misra, S., Xue, G., Yang, D. (2012). Smart grid - the new and improved power grid: a survey. *IEEE Communications Surveys and Tutorials*, 14(4): 944–980.

[15] Federal Budget Publications 2014-15 (Ministry of Finance, Government of Pakistan, 2014). Available at <http://www.finance.gov.pk/fb_2014_15.html>. Retrieved on Oct 14, 2014.

[16] Fleischmann, B., Ferber, S., and Henrich, P. (2006). Strategic planning of BMW's global production network. *Interfaces*, 36(3):194–208.

[17] Geoffrion, A. M. and Graves, G. W. (1974). Multicommodity distribution system design by benders decomposition. *Management science*, 20(5):822–844.

[18] Gen, M. and Syarif, A. (2005). Hybrid genetic algorithm for multi-time period production/distribution planning. *Computers & Industrial Engineering*, 48(4):799–809.

[19] Godoy, E., Benz, S., and Scenna, N. (2012). Optimal economic strategy for the multiperiod design and long-term operation of natural gas combined cycle power plants. *Applied Thermal Engineering*.

[20] Halldorsson, A. and Svanberg, M. (2012). Energy resources: Trajectories for supply chain management. *Supply Chain Management: An International Journal*, 18(1):5–5.

[21] Hinojosa, Y., Puerto, J., and Fernández, F. R. (2000). A multiperiod two-echelon multicommodity capacitated plant location problem. *European Journal of Operational Research*, 123(2):271–291.

[22] Hu, S., Kapuscinski, R., Lovejoy, W. (2011). Price dispersion and electricity auctions: the strategic foundation and implication to market design. Working Paper, Ross School of Business, University of Michigan.

[23] IEA (International Energy Agency, 2011). World energy outlook. 2011. OECD Publications, Paris.

[24] Iqbal, N., Nawaz, S., and Anwar, S. (2013). Electricity demand in Pakistan: A nonlinear estimation. Technical report, Mimeo Pakistan Institute of Development Economics.

[25] Kavouridis, K. and Koukouzas, N. (2008). Coal and sustainable energy supply challenges and barriers. *Energy Policy*, 36(2):693–703.

[26] Kiernan, P. (2014). Africa: Resource rich, energy poor. Available at <http://geafricaascending.economist.com/2014/07/31/africa-resource-rich-energy-poor/>. Retrieved on December 31, 2014.

[27] Klose, A. and Drexl, A. (2005). Facility location models for distribution system design. *European Journal of Operational Research*, 162(1):4–29.

[28] Kugelman, M. (2013). Pakistan's energy crisis: From conundrum to catastrophe? Available at <http://www.nbr.org/downloads/pdfs/eta/Kugelman_commentary_03132013.pdf>. Retrieved on Jan 27, 2014.

[29] Liu, Y., Huang, G., Cai, Y., Cheng, G., Niu, Y., and An, K. (2009). Development of an inexact optimization model for coupled coal and power management in north china. *Energy Policy*, 37(11):4345–4363.

[30] Love, R. F., Morris, J. J., and Wesolowsky, G. O. (1988). *Facilities location*, volume 90. North-Holland Amsterdam.

[31] Malik, A. (2008). How Pakistan is coping with the challenge of high oil prices. *The Pakistan Development Review*, 46(4):551–575.

[32] Malik, A. (2010). Oil prices and economic activity in Pakistan. *South Asia Economic Journal*, 11(2):223–244.

[33] Mathur, R., Chand, S., and Tezuka, T. (2003). Optimal use of coal for power generation in India. *Energy policy*, 31(4):319–331.

[34] McMullan, J., Williams, B., and McCahey, S. (2001). Strategic considerations for clean coal R&D. *Energy Policy*, 29(6):441–452.

[35] Meixell, M. J. and Gargeya, V. B. (2005). Global supply chain design: A literature review and critique. *Transportation Research Part E: Logistics and Transportation Review*, 41(6):531–550.

[36] Melachrinoudis, E. and Min, H. (2000). The dynamic relocation and phase-out of a hybrid, two-echelon plant/warehousing facility: A multiple objective approach. *European Journal of Operational Research*, 123(1):1–15.

[37] Menegaki, A. N. (2014). On energy consumption and GDP studies; a meta-analysis of the last two decades. *Renewable and Sustainable Energy Reviews*, 29:31–36.

[38] Ministry of Pakistan Railway. `<http://www.railways.gov.pk>`. Accessed on Feb 08, 2011.

[39] Narayan, P. K., Narayan, S., and Popp, S. (2010). A note on the long-run elasticities from the energy consumption–GDP relationship. *Applied Energy*, 87(3):1054–1057.

[40] NEPRA (National Electric Power Regulatory Authority, 2011). State of industry report 2011. Available at `<http://nepra.org.pk/industryreports.htm>`. Retrieved on Feb 01, 2014.

[41] Oh, W. and Lee, K. (2004). Causal relationship between energy consumption and GDP revisited: the case of Korea 1970–1999. *Energy economics*, 26(1):51–59.

[42] Pakistan Railway Network Map by Adnanrail (2012). Licensed under Creative Commons Attribution-Share Alike 3.0 via Wikimedia Commons. Available at `<http://commons.wikimedia.org/wiki/File:Pakistan_Railways_Map.png>`. Accessed on Dec 05, 2013.

[43] Paulus, M. and Trüby, J. (2011). Coal lumps vs. electrons: How do Chinese bulk energy transport decisions affect the global steam coal market? *Energy Economics*, 33(6):1127–1137.

[44] Pirkul, H. and Jayaraman, V. (1996). Production, transportation, and distribution planning in a multi-commodity tri-echelon system. *Transportation Science*, 30(4):291–302.

[45] Rafique, R. and Zhao, Y. (2011). Pakistan Energy Crisis - Breaking the Vicious Cycle. Case Study, Rutgers Business School - Newark and New Brunswick.

[46] Rietveld, P. and Ouwersloot, H. (1992). Ordinal data in multicriteria decision making, a stochastic dominance approach to siting nuclear power plants. *European Journal of Operational Research*, 56(2):249–262.

[47] Rosnes, O. and Vennemo, H. (2012). The cost of providing electricity to Africa. *Energy Economics*, 34(5):1318–1328.

[48] Santana, R. (2013). Pakistan's power shortage problem is country's biggest threat. Available at `<http://www.huffingtonpost.com/2013/06/15/pakistan-power-shortage_n_3447599.html>`. Retrieved on Feb 01, 2014.

[49] Secomandi, N., and Seppi, D. J. (2013). Real options and merchant operations of energy and other commodities. forthcoming in *Foundations and Trends in Technology, Information and Operations Management*.

[50] Shahbaz, M., Zeshan, M., and Afza, T. (2012). Is energy consumption effective to spur economic growth in Pakistan? New evidence from bounds test to level relationships and granger causality tests. *Economic Modelling*, 29 (6):2310–2319.

[51] Shapiro, J. F. (2007). *Modeling the supply chain*, volume 2. Cengage Learning.

[52] Shen, Z.-J. M., Coullard, C., and Daskin, M. S. (2003). A joint location-inventory model. *Transportation Science*, 37(1):40–55.

[53] Shen, Z. (2007). Integrated supply chain design models: A survey and future research directions. *Journal of Industrial and Management Optimization*, 3(1):1.

[54] Shu, J., Teo, C. P., Shen, Z. J. (2005). Stochastic Transportation-inventory Network Design Problem. *Operations Research*, 53: 48–60.

[55] Simchi-Levi, D. (2009). *Designing and managing the supply chain concepts, strategies, and case studies.* Tata McGraw-Hill Education.

[56] Snyder, L. V. (2006). Facility location under uncertainty: A review. *IIE Transactions*, 38(7):547–564.

[57] Solow, R. M. (1956). A contribution to the theory of economic growth. *The quarterly journal of economics*, 70(1):65–94.

[58] Susta, M. R. (2008). Development in Supercritical Steam Tech. *Powergen Asia 2008, Kuala Lumpur, Oct. 2008.*

[59] Tang, C. F. and Shahbaz, M. (2013). Sectoral analysis of the causal relationship between electricity consumption and real output in pakistan. *Energy Policy*, 60:885–891.

[60] The Nations Online Project (1998-2011). Let's care for this planet. Copyright 1998-2011. `<http://www.nationsonline.org/oneworld/map/pakistan_map.htm>`. Accessed on Dec 31, 2014.

[61] The News International (2014). CM welcomes Chinese investment in coal mining. Tuesday, March 18. Extracted from `<http://www.thenews.com.pk/>` on March 20, 2014.

[62] Troncoso, J. J. and Garrido, R. A. (2005). Forestry production and logistics planning: an analysis using mixed-integer programming. *Forest Policy and Economics*, 7(4):625–633.

[63] Van Roy, T. J. and Erlenkotter, D. (1982). A dual-based procedure for dynamic facility location. *Management Science*, 28(10):1091–1105.

[64] Wesolowsky, G. O. (1973). Dynamic facility location. *Management Science*, 19(11):1241–1248.

[65] Wesolowsky, G. O. and Truscott, W. G. (1975). The multiperiod location-allocation problem with relocation of facilities. *Management Science*, 22(1):57–65.

[66] World Bank. (2012). The World Bank Data, Pakistan, (2012). `<http://data.worldbank.org/country/pakistan>`. Accessed on Sep 27, 2013.