Incentives and Alignment in Collaborative Projects

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Abstract

Collaboration and partnerships prevail in large and complex projects but may incur externalities that encourage deliberate delays and cost overruns. This paper characterizes the externalities under the popular loss-sharing partnership and designs alignment strategies in a project operations context. By integrating the economic theory of teams with project operations specifics on time-cost trade-off, we discover various forms of mis-aligned incentives in project execution, such as, the Prisoners' Dilemma, and the new Supplier's Dilemma and Coauthors' Dilemma. These dilemmas reveal how individual firms can be motivated to execute their tasks against the best interest of the project and how collaboration may impact project duration and cost adversely. To align the incentives in collaborative projects, we derive a payment distribution scheme to separate the consequences of one's action from the aggregated project outcome and present a new "fair-sharing" partnership to ensure that each firm is fully responsible for its actions.

Keywords: Collaboration, partnership, project management, incentive alignment.

1 Introduction

Over the last three to four decades, advances in technology and the networked economy have led the business models in many industries to evolve from "one-firm-does-all" to collaboration. Examples can be found in book publishing, commercial aerospace, and engineering-procurement-construction (EPC) industries. While projects across these industries vary significantly in content and scale, they share important common features: First, they require diverse knowledge and expertise that few companies or individuals possess them all; Second, they demand a significant investment of time and/or capital up front, which mandates market expansion a necessity for success.

The book publishing industry is popularized by books with many coauthors. Using textbooks on operations management as an example, a quick search by the key-word "operations management" on Amazon.com in September 2013 returns a list of textbooks which are the most relevant (definition: (1) production & operations section (2) hardcover (3) four stars & up). Among the 48 textbooks that top the list, 17 (35.42%) are single authored, 19 (39.58%) have two authors, and the rest have three or more authors. Thus, coauthored books account for a majority (about 65%) of the most relevant textbooks on operations management. Replicating the search by "supply chain management" and "marketing science" returns similar results.

In the commercial aerospace industry, suppliers are playing an increasingly important role in the development of new aircrafts. Recent examples are Boeing 787 Dreamliner, Airbus 380, China Comac C919 and Airbus 350. In particular, the Boeing 787 Dreamliner outsourced 65% of the development work to more than 100 suppliers from 12 countries (see Horng and Bozdogan (2007) and Exostar (2007)). Tier 1 suppliers design and fabricate 11 major subassemblies, Boeing integrates and assembles the airplane. To manage the collaboration, Boeing made the suppliers stakeholders of the program by engaging them in a strategic partnership where the suppliers and Boeing are responsible for the non-recurring development cost of their tasks and must wait until the completion of the project to get paid (see Xu and Zhao, 2011).

In the EPC industries, the $150 billion international space station (ISS) is a representative example where the design and construction of ISS are spread out to fifteen countries around the world. The elements of ISS are not assembled on the ground but launched from different countries and mated together on orbit. Each country invests its own money to design and make its elements and takes the responsibility of their maintenance. Five countries, including the U.S., Russia and Japan, are the principals (partners) of ISS due to their significant contributions (see NASA, 2013).

As we can see, collaboration is everywhere, especially popular in large and complex projects. Collaboration in project management can be defined as a joint effort by a group of people or organizations, working on their individual parts (deliverables) of a project". Collaboration includes two basic elements: workload distribution and incentive alignment. One common incentive alignment scheme in practice is the loss (and reward) sharing partnership. In such a partnership, the workload of a project, for instance, different tasks, is spread out to multiple parties (firms) where each party pays for its own tasks and shares the outcome (loss/revenue, credit or utility) when the project is completed. The loss sharing partnership is the foundation of the coauthorship in book publishing, the strategic partnership of the Boeing 787 Dreamliner program, and the collaborative agreement among multiple countries for the International Space Station.

Collaboration under the loss-sharing partnership offers significant benefits to the project and its participants: First, they allows the project to utilize the best in-class expertise and knowledge. For instance, authors with different expertise can combine their domain knowledge into one book. Second, a collaborative partnership allows multiple parties to share the up front investment and thus make feasible a costly project that is infeasible to any individual party, as in the ISS project. Finally, such a collaboration is essential to market expansion. As witnessed in the Boeing 787 Dreamliner program, the suppliers are the stakeholders (owners) of the program and thus are motivated to sell the plane in their own countries and keep the customers waiting despite the significant delay of the program.

Collaboration under the loss-sharing partnership is one way to outsource the workload of a project, subcontracting is another. They are different because in subcontracting, suppliers get paid when their tasks are completed; therefore, a supplier's interest is tied only to its tasks. However, in collaboration, suppliers' interests are tied to the project because they won't get paid until the project is done. This difference is important because collaboration motivates the parties more strongly than subcontracting to expand the market (so everyone gets more) and keep customers waiting until the final completion of the project (so everyone loses less).

Despite irresistible benefits, collaboration may impact project schedule and cost adversely. For instance, coauthorships may bring delays to publication due to the frustration of who should do the work. Even with clear split of workload, too many coauthors may dilute the significance of the publication to such an extent that the work may never be completed. In the aerospace industry, Boeing 787 Dreamliner suffered significant cost overruns due to repeated delays, a majority of which were caused by non-technical but managerial slips of Boeing and its suppliers (see Zhao, 2014). Perhaps the impact of collaboration on project performance is most exemplary through the comparison between the 28-country European Union's Galileo satellite navigation system and the one-country China's BeiDou/Compass system (see Wikipedia, 2015). The latter was announced 3 years later than the former but has 16 satellites launched by December 2012 and has since entered services in the Asia-Pacific region. In contrast, Galileo just had its first 2 satellites launched

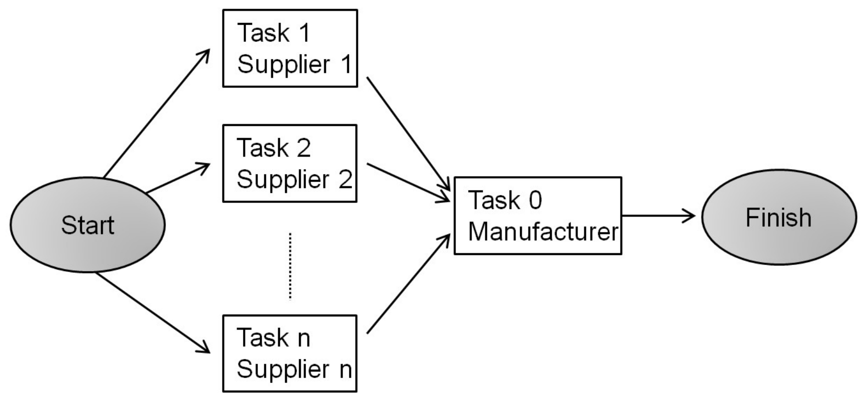

Figure 1: A typical collaborative project.

in 2014 and plans to start offering first services in 2015, despite EU's technical superiority over China. These examples indicate that collaboration may result in project delays and significant cost overruns. One important reason is that incentives are harder to align with more parties involved; and selfish motives can drive gaming behaviors in project execution against the best interest of the project.

To see incentive issues and gaming behaviors in collaboration more clearly, let's consider a typical project (see Figure 1) with both parallel and sequential tasks, each conducted by a different company. It is easily seen that the manufacturer can only start its task after all suppliers complete their tasks, and supplier 1 has to watch out for other suppliers' completion time to determine its own task duration. Under the loss-sharing partnership, because all firms share the consequences of project performance (delays and cost overruns), their schedule and cost rely on not only their own effort but also others' efforts. Depending on project network and cost structure, collaboration may thus introduce various forms of 'externalities" (an externality is the cost or benefit that affects a party who did not choose to incur that cost or benefit) to the participating firms.

Partnerships and externality are studied extensively in the economics and supply chain literature, however, they are not well understood in a project management context. In this paper, we combine the economic theory of teams with project operations specifics (e.g., project networks, cost structures and time-cost trade-off) to characterize the explicit forms of externality in project execution under the loss-sharing partnership, and identify alignment strategies in the project management context. Our objective is to provide insights into the following issues: (1) How does the loss-sharing partnership impact firms' incentives in project execution and thus the project duration and cost? (2) How to design a partnership to mitigate such externalities in collaborative projects? (3) How do project network and cost structure affect the results?

To this end, we thoroughly study the representative system in Figure 1 - a two-stage project network with parallel tasks (e.g., subsystems) in the first stage and an integration task (e.g., system integration) in the second stage. Each firm faces a time-cost trade-off and must balance its task duration against its share of cost. We consider various network and cost structures and characterize the subgame perfect equilibriums. Under loss sharing, we find an inherent mismatch between individual firms' best interest and that of the project. Depending on project cost and network structures, we discover various mis-aligned incentives in project execution: (1) The Prisoners' Dilemma: even though keeping the optimal task schedule benefits the project, it can be in each firm's best interest to delay; (2) The Supplier's Dilemma: if costs are time-dependent, a supplier may have to delay (even at a loss) in order to raise the penalty too high for the manufacturer to delay, to avoid a greater loss; (3) The Coauthors' Dilemma: a firm cannot expedite the project even if it expedites its tasks; this is true because if it expedites, others will delay. The second and third dilemmas are new implications of externality in the project management context that are meaningful in practice.

As a remedy, we design a new "fair-sharing" partnership to have all firms fully responsible for the consequences of their actions, as such, it minimizes externalities and aligns firms' interest with that of the project. At the core of fair sharing, there lies a payment distribution scheme that identifies the consequences of everyone's action from the aggregated project outcome. Because firms' actions are intertwined in the final project outcome, the subtle separation of the cause and effect for each firm depends on the project network structure.

The paper is organized as follows. In §2, we review the related literature; which is followed by §3 where we introduce our models and methodology. In §4, we study firms' incentive issues and strategic gaming behaviors under the loss sharing partnership. In §5, we present the fair-sharing partnership and prove its effectiveness. We conclude the paper in §6 with a brief summary of our results.

2 Literature

This paper is related to the literature of project management, economics theory of teams, development chain management and project/supply chain interfaces. We shall review related results in each literature and point out the contribution of our work.

Classic project management literature. The most well known results in this literature include the critical path method (CPM), project evaluation and review techniques (PERT), time-cost analysis (TCA), and resource constrained project scheduling (RCPS). This literature focuses on the scheduling and planning of project(s) for a single firm and thus the main issue is on optimization. We refer the reader to Nahmias (2008) and Jozefowska and Weglarz (2006) for a summary and review of this literature. Our paper draws project management specifics, e.g., cost structure, project network and time-cost trade-off, from this literature but studies gaming behaviors, incentive issues and alignment strategies in a multi-firm collaborative project.

Classic economics literature of teams. The economics literature of teams discusses incentives and contracts in general teamwork settings. This literature is vast, we refer the reader to several seminal papers, e.g., Holmstrom (1982), Demski and Sappington (1984), McAfee and McMillan (1986), and Holmstrom and Milgrom (1991), for principal-agent models and moral hazard games; and Bhattacharyya and Lafontaine (1995), Kim and Wang (1998), and Al-Najjar (1997) for the double moral hazard games. However, this literature ignores project management specifics, such as project network and cost structure. Thus it is not known how the general economics theory works in a project management context, and what it implies in terms of project metrics such as delays and cost overruns. Our work enriches this literature by explicitly considering project management specifics which lead to many novel results.

Bidding and subcontracting in project management. This body of literature studies project management issues involving multiple firms, such as project bidding and subcontracting. Elmaghraby (1990) studies project bidding under deterministic and probabilistic activity durations from the contractor's perspective, while Gutierrez and Paul (2000) compares fixed price contracts, cost-plus contracts and menu contracts in project bidding from the project owner's perspective. Paul and Gutierrez (2005) studies how to assign tasks to contractors for projects with parallel or serial tasks. Szmerekovsky (2005) studies the impact of payment schedule on contractors' performance. In this model, the owner selects the payment terms in the first place, the contractor then decides the schedule to maximize its net present value. Aydinliyim and Vairaktarakis (2010) considers a set of manufacturers who outsource certain operations to a single third party by booking its capacity, and the third party identifies a schedule that minimizes the total cost for all manufacturers. Our paper differs from this literature in two ways: first, we consider collaboration and partnerships which are structurally different from subcontracting as shown in §1. Second, all partners considered in this paper have to contribute to the workload and share the outcome, while in the subcontracting literature, the project owner does not contribute to the workload but only supervises the contractors' work.

Development chain management. This stream of literature studies issues in the development of new products within a single firm and more recently involving multiple firms. For instance, Bhaskaran and Krishnan (2009) studies a development chain with two firms, a focal firm and a partner firm. Their model considers the cost, time, and quality triangle under three partnerships: revenue sharing, investment sharing and innovation sharing. They show that simple revenue sharing does not work well and leads to underinvestment in quality improvements. Alternatively, the investment sharing and innovation sharing, are better than revenue sharing in collaboration. Our paper differs from this literature by incorporating project management specifics, such as, project network (the precedence structure of tasks) and time-cost trade-off (concepts developed in the classic project management literature), into the model and analysis.

Project management and supply chain interfaces. This literature considers project management specifics and studies the management of projects that involve multiple firms from a supply chain perspective. It is a fairly new research area in the operations management literature. For instance, Bayiz and Corbett (2005) introduces a principal-multi-agent game to project management by considering projects with either two sequential tasks or two parallel tasks. They analyze the effectiveness of the fixed-price contracts versus incentive contracts in a subcontracting arrangement. Kwon, Lippman, McCardle, and Tang (2010) analyzes delay payment versus no delay payment in a project management setting where parallel tasks are done by different suppliers. They consider a simultaneous game among suppliers while the manufacturer does not contribute to the project but only selects payment regimes. By assuming exponentially distributed task durations, they showed that the delayed payment regime can be more preferred by the manufacturer when its revenue is low, and the impact of the number of suppliers on the effectiveness of the regimes depends on information possessed by the suppliers on others' progresses.

The delayed payment regime is similar to the loss-sharing partnership as it requests each supplier to be paid only after all suppliers have completed their tasks. Our paper differs from Kwon, Lippman, McCardle, and Tang (2010) in three aspects: (1) The manufacturer in our model contributes to the workload and the loss-sharing partnership mandates everyone (including the manufacturer) to be paid only after all suppliers and the manufacturer have completed their tasks. Thus, our project network not only includes parallel tasks but also sequential tasks. As we shall show, the analysis of sequential tasks is more intriguing and leads to many new and meaningful discoveries, such as the Prisoners' Dilemma and the new Coauthors' Dilemma. (2) Our inclusion of time-dependent unit cost for tasks (to model increasing incremental cost of project delays and task expedition) captures some delicate dynamics between the suppliers and the manufacturer, as summerized by the new Supplier's Dilemma. (3) We suggest a new "fair-sharing" partnership, and prove its potential to mitigate the externalities and align incentives in project execution for collaborative projects.

3 The Model and Preliminaries

In this section, we introduce the fundamentals of our model. First, we present the project management specifics such as the project cost structure and project network (the precedence structure). Second, we precisely define the loss-sharing and fair-sharing partnerships. Finally, we present the game theoretical model and our methodological approach.

Project Cost Structure. We consider predictable task durations and classify project costs into two categories (Nahmias (2008)): direct cost and indirect cost. Direct cost includes all spending directly contributing to a task, such as the cost of management, labor, material and shipping. Normally, a longer task duration is coupled with a lower direct cost. Indirect cost includes all spending not directly contributing to tasks but depending on the project duration, such as the overhead (e.g., rent, utilities, benefits), interests and financial costs, delay penalty and order cancelation loss. Normally, a longer project duration is coupled with a higher indirect cost. We refer the reader to Nahmias (2008) for more details.

Consistent to a majority of practical situations, we assume that direct cost is convex and decreasing as task duration increases and indirect cost is convex and increasing as project duration increases (Figure 2, Nahmias (2008)). If task i is delayed by one period, firm i saves s_i in the direct cost. If the project is delayed by one period, it suffers a penalty p in the indirect cost. Conversely,

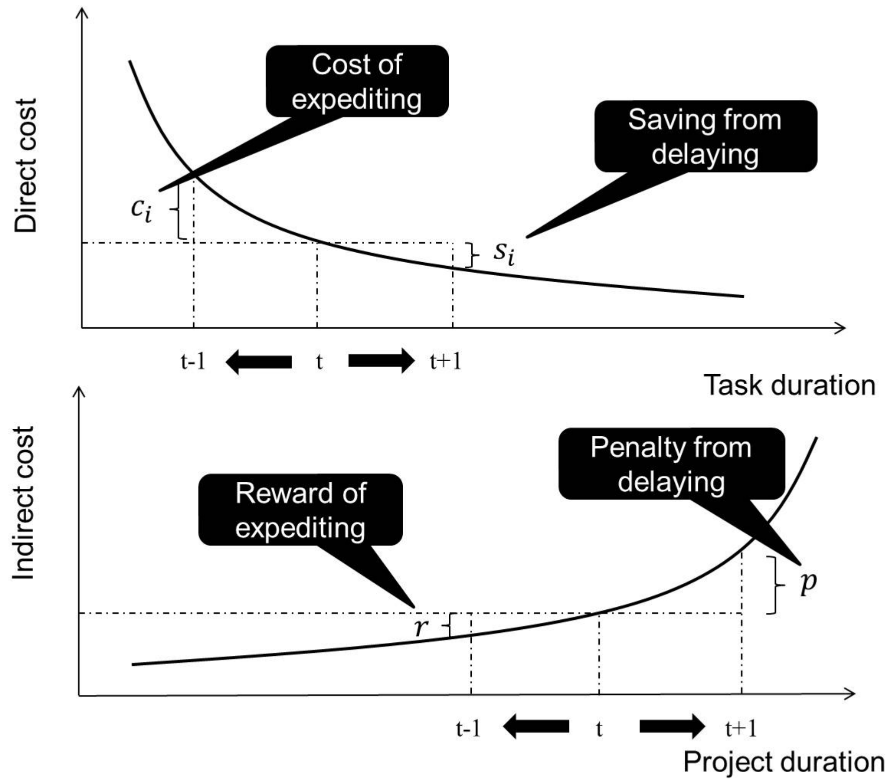

Figure 2: Project Cost Structure.

if task i is expedited by one period, firm i incurs a cost c_i for expediting. If the project is completed one period earlier, it receives a reward r.

Project Network. We consider projects with a network (precedence) structure shown in Figure 1. It has two stages: At stage 1, there are several tasks to be completed in parallel, similar to the design and fabrication of subsystems in the 787 Dreamliner program, the writing of individual chapters in a coauthored book, and the development of subsystems and components of the International Space Station (ISS). At stage 2, there is the task of integration and assembly of all parts completed in stage 1, similar to the system integration task in the 787 Dreamliner program, the integration and proofreading of a coauthored book, and the final assembly and testing task of the ISS. Clearly the task at stage 2 cannot start until all tasks at stage 1 are completed.

Figure 1 shows the general project network, where $n = 1$ denotes the case with only one task at stage 1, and thus the project network reduces to two sequential tasks. When $n \geq 2$, there are multiple tasks at stage 1, and the project network has an assembly structure. We will discuss these special cases in the paper.

The Loss-Sharing Partnership. In this partnership, each firm pays for the direct and indirect costs of its own task(s), and get paid when the project is done. Intuitively, if a firm delays its task, it saves on its direct cost but everyone (including the delayed firm) suffers an increase in indirect cost (a penalty) if the firm's delay results in a project delay. Thus other firms completed their tasks on time may be penalized by this firm's delay, and this delayed firm is not fully responsible for all the consequences of its action as the penalty is shared among all firms.

The Fair-Sharing Partnership. This partnership differs from the loss sharing partnership in the way that it ensures that every firm is fully responsible for the consequence of its action. Intuitively, if one firm causes damage to others, it has to compensate others; if it brings benefit to others, it receives compensation from others. For this purpose, this partnership requires a payment distribution scheme to separate the consequences of each firm's action from the aggregated project outcome. We refer the reader to §5 for the exact mechanism of this partnership.

Game Theoretical Framework. We assume that each task in the 2-stage project network is assigned to a different firm. For the ease of exposition, we use "supplier(s)" to name the firm(s) responsible for the task(s) at stage 1 and "manufacturer" to name the firm responsible for the task at stage 2. By the structure of the project network, a two-phase game theoretic model is appropriate for predicting the behaviors of the supplier(s) and the manufacturer in equilibrium. The sequence of events is described as follows (see also Figure 3): At the beginning of phase 1, suppliers choose task durations and carry out their tasks. After all suppliers complete their tasks, phase 1 is concluded. At the beginning of phase 2, the manufacturer chooses the task duration and conducts its task. When the manufacturer completes its task, phase 2 is drawn to an end and the project is completed. In this game, the suppliers take the lead by taking actions first (anticipating the manufacturer's response) and the manufacturer follows by responding to suppliers' actions accordingly. We assume information symmetry thus the direct and indirect cost functions of all parties are public knowledge. Under either partnership, each firm aims to maximize its own profit by optimally balancing its task duration and its share of the total cost. We shall derive subgame perfect Nash equilibrium (SPNE) for each case considered in this paper and compare the resulting project performance to the optimum. If the SPNE is not unique, we shall compare different SPNEs and report the Pareto or strong equilibrium.

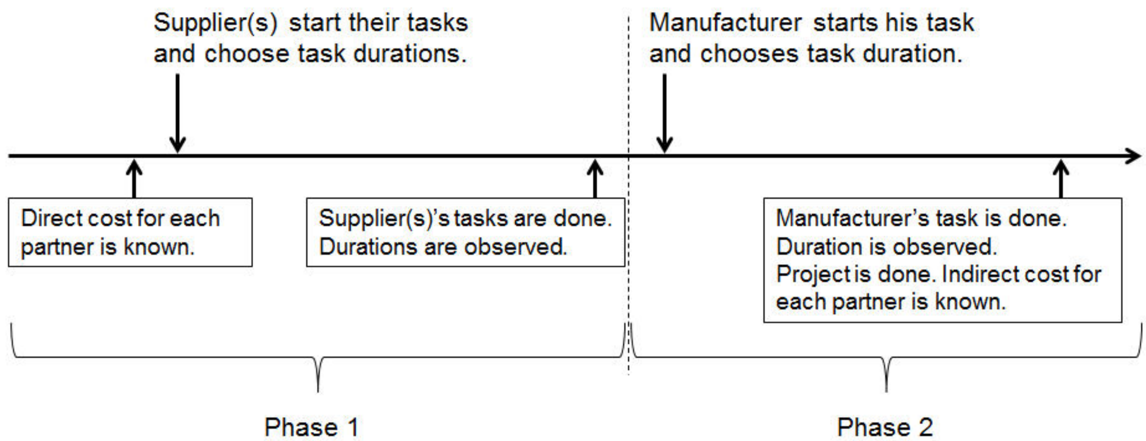

Figure 3: Sequence of Events.

Methodological Approach. To better illustrate the impact of the partnerships on project duration and cost, we assume that the project starts with the optimal schedule as planned under the centralized control. We call this schedule and the task durations the "original schedule" and "original task durations" respectively. The optimal original schedule serves as a benchmark - by analyzing how the partnerships may (or may not) motivate the firms to deviate from the optimal schedule in their attempt to maximize their own profit, we can make the impact of the partnerships on project performance readily visible. In addition, such an optimal project schedule is achievable under information symmetry and can be reasonably assumed as our focus is on project execution.

To highlight managerial insights, we first analyze simpler models, i.e., the one-period models in which each firm can delay or expedite its original task duration by at most one period. Then we relax this constraint to allow the firms to delay or expedite multiple periods. To study the impact of cost structure and project network on the firms' behaviors, we consider both constant (time-independent) and varying (time-dependent) costs per unit of time, and both one supplier and multi-supplier cases.

4 The Loss-Sharing Partnership

In this section, we study firms' incentive issues and strategic behaviors under the loss-sharing partnership. We start with the base model in §4.1 with one supplier and time-independent cost. In this model, each firm can either "keep" the original task duration (which is globally optimal, by assumption) or "delay" it by one period. In §4.2, we relax the time-independent cost assumption in the base model to allow time-dependent costs, for instance, penalty per period may increase as the project delays more. In §4.3, we consider the base model but allow each firm an additional option of "expediting" its task by one period. In §4.4, we extend the base model to include multiple suppliers, and in the last subsection, §4.5, we consider a general model and develop structural results and algorithms for the equilibrium.

4.1 The Base Model – The Prisoners' Dilemma in Project Execution

In this section, we consider the base model (defined by Assumption 1). Our objective is to understand the impact of loss sharing on project duration and cost.

Assumption 1 *At stage 1 of the project network, there is only one task. Each task cannot be expedited but can be delayed by at most one period. If the project is delayed, it is subject to a penalty which is time independent.*

In this model, the supplier and manufacturer only have two options (actions) available: "keep" (keeping the original task duration) or "delay" (delaying it by one period). We use K for "keep" and D for "delay" for simplicity. We assume that firm i is responsible for task i for $i = 0, 1$ where firm 1 (or 0) refers to the supplier (or manufacturer, respectively). The action set, [supplier's action, manufacturer's action], is {[K, K], [D, D], [K, D], [D, K]}. When task i is delayed, firm i receives a saving of s_i in terms of its direct cost. When the project is delayed, a penalty of p per period (the additional indirect cost) is shared by all firms, where firm i pays p_i and $p_0 + p_1 = p$.

Recall that, by assumption, the project starts with an original schedule that is optimal under the centralized control. In other words, the action set [K, K] has a pay-off higher than those under [D, K], [K, D] and [D, D] for the project as a whole. To this end, we need the following necessary condition,

Condition 1 *Global Optimum - Base Model: $s_1 < p$, $s_0 < p$.*

We can easily verify Condition 1 as follows: at [K, K], there is neither a saving nor a penalty for the project, and thus the pay-off of the project relative to the original schedule is zero. At [D, K], task 1 is delayed by one period but task 0 is kept at its original duration. Thus, we receive a saving of s_1 from task 1 but must pay a penalty of p because the project is delayed by one period. The pay-off of the project is $s_1 - p$ and thus $s_1 < p$ is a necessary condition for [K, K] to outperform [D, K] from the project's perspective. Repeating a similar logic to [K, D] and [D, D] leads to Condition 1.

Now we are ready to study the firms' strategic behaviors under the loss-sharing partnership and their impact on project performance. Before introducing the general theory, we first present an example (see Figure 4) to illustrate the key insight. In this example, task 1 has an original duration of 9 weeks, which can be delayed to 10 weeks with a saving of $s_1 = \$900$. Task 0 has an original duration of 5 weeks which can be delayed to 6 weeks with a saving of $s_0 = \$1200$. The project is due in 14 weeks; each week of delay incurs a penalty of $p = \$1600$ for the project. Clearly, Condition 1 is satisfied in the example, and so it is in the project's best interests to keep the original schedule.

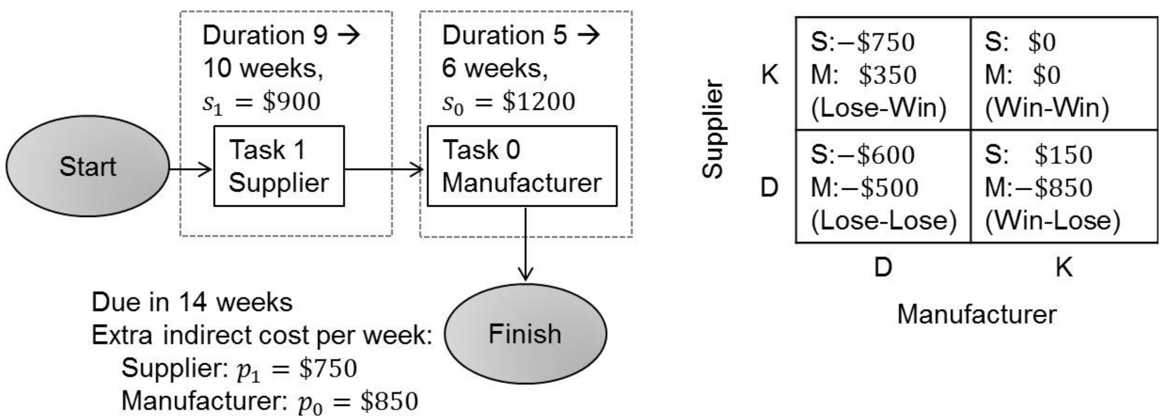

Figure 4: An example of the base Model and its pay-off matrix. (K:keep, D:delay)

Under the lost sharing partnership, we assume that upon each week of the project's delay, the supplier's share of the penalty is $p_1 = \$750$ and the manufacturer's share is $p_0 = \$850$. To see what the supplier and the manufacturer would do in their own best interests (i.e., the equilibrium), we consider the following four scenarios:

- **Win-Lose**: Firm 1 (the supplier) delays but firm 0 (the manufacturer) keeps its original task duration. In this scenario, firm 1 saves \$900 but must pay \$750 with a net gain of \$150. However, firm 0 must pay \$850 for firm 1's delay. The firms' pay-offs (relative to the original schedule) are $(\pi_1, \pi_0) = (150, -850)$ and the project's pay-off is $-\$700$ (relative to the original schedule).

- **Lose-Win**: Firm 1 keeps its original task duration but firm 0 delays. In this scenario, firm 0 saves \$1200 but must pay \$850 with a net gain of \$350. However, firm 1 must pay \$750 for the delay caused by firm 0. The firms' pay-offs are $(-750, 350)$ and the project's pay-off is $-\$400$.

- **Lose-Lose**: Both firms delay. In this scenario, the project is delayed by two weeks and the firms' pay-offs are $(-600, -500)$. This is the worst scenario for the project as a whole with a total loss of \$1100.

- **Win-Win**: Both firms keep their original task duration. This is the best scenario for the project where both the firms and the project lose nothing with a pay-off of zero.

Figure 4 summarizes the action sets and the corresponding pay-off matrix. We can see that no matter what the supplier does, the manufacturer's optimal strategy is always to "delay". In other words, "delay" is the dominant strategy for the manufacturer. Similarly, the supplier's best strategy is also to "delay" regardless of the manufacturer's response. Thus, although the "Win-Win" scenario has the best outcome for the project, it is unstable – each firm will find every excuse to delay. The "Lose-Lose" scenario, although having the worst outcome for the project, is the subgame perfect Nash equilibrium (SPNE), as in a typical (sequential) Prisoners' Dilemma.

We now present the general theory for the base model. Note that in this game, the supplier leads and the manufacturer follows (see §3). If the project is finished on time, there is no penalty. For every period of the project delay, the supplier pays a penalty of p_1 and the manufacturer pays the rest which is p_0. The firm whichever delays obtains a saving from the direct cost of its own task. For example, if the supplier delays but the manufacturer keeps the original duration of its task, the supplier saves s_1 from its direct cost which brings its pay-off to be $s_1 - p_1$, and the manufacturer bears a pure penalty of p_0. Figure 5 shows the extensive form of the game in the base model.

We derive the following results on the dominant strategies and equilibrium (all proofs of this paper are presented in the Appendix unless otherwise mentioned).

Lemma 1 (Dominant Strategy): *Under Condition 1, when $s_i < p_i$, "keep" is the dominant strategy for firm i, $i = 0, 1$; when $s_i > p_i$, "delay" is the dominant strategy for firm i, $i = 0, 1$.*

For simplicity, we use "S" ("M") to denote the supplier (the manufacturer, respectively).

Theorem 1 (Equilibrium): *For the base model, under Condition 1, the subgame perfect Nash equilibrium (SPNE) is given by,*

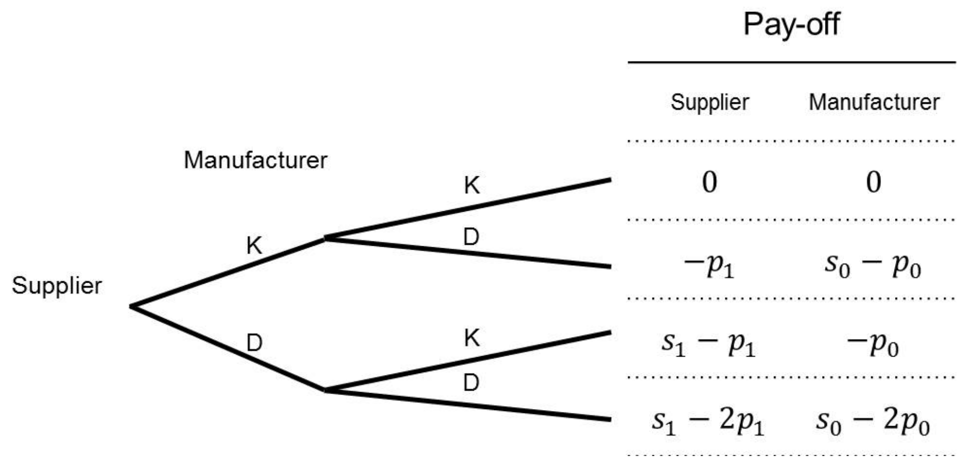

Figure 5: The extensive form of the game in the base model.

Case	*Condition on S*	*Condition on M*	*Optimal strategy for S*	*M's best response*
1	$s_1 < p_1$	$s_0 < p_0$	K	K
2	$s_1 > p_1$	$s_0 < p_0$	D	K
3	$s_1 < p_1$	$s_0 > p_0$	K	D
4	$s_1 > p_1$	$s_0 > p_0$	D	D

Based on these results, we present the following key insight for the base model under the loss-sharing partnership:

The Prisoners' Dilemma in Project Execution: *In the base model, for a schedule to be optimal under centralized control, we need $s_1 < p, s_0 < p$ (Condition 1). For the optimal schedule to be the SPNE under loss sharing, a much stronger condition is required, that is, $s_1 < p_1$ and $s_0 < p_0$ where $p_1 + p_0 = p$. Thus, if $s_1 > p_1$ and $s_0 > p_0$ but $s_1 < p$ and $s_0 < p$, then it is in each firm's best interests to delay although being on time benefits the entire project.*

4.2 The Base Model with Time-dependent Costs – The Supplier's Dilemma

In this section, we relax the "time-independent cost" assumption in the base model to study the impact of time-dependent penalty costs on the dominant strategies and the Prisoners' Dilemma in project execution. We define the model by Assumption 2.

Assumption 2 *Assumption 1 holds here except that project delay penalties are time dependent.*

Let p^1 (or p^2) be the penalty for the 1st (the 2nd, respectively) period of project delay; and let p_i^1 and p_i^2) be the corresponding penalties shared by firm i, where $p_1^1 + p_0^1 = p^1$ and $p_1^2 + p_0^2 = p^2$. The assumption of starting with the optimal schedule and the assumptions of convex and increasing cost functions (see §3) require,

Condition 2 *(1) Global Optimum - Time-Dependent:* $s_1 < p^1$, $s_0 < p^1$. *(2) Monotonicity - Time-Dependent:* $p^1 < p^2$, $p_1^1 < p_1^2$, $p_0^1 < p_0^2$.

To see the impact of time-dependent penalty costs, we slightly modify the example in §4.1 (shown in Figure 4). In this modified example, everything remains the same except that (1) the saving per week for task 1 is reduced to $s_1 = \$600$ from \$900; (2) the second period delay penalty of the project, p^2, is increased from \$1600 to \$2500, where the supplier bears $p_1^2 = \$1100$ and the manufacturer pays $p_0^2 = \$1400$. Figure 6 depicts the modified example. Clearly, Condition 2 is satisfied in this example, and it is in the project's best interests to keep the original schedule.

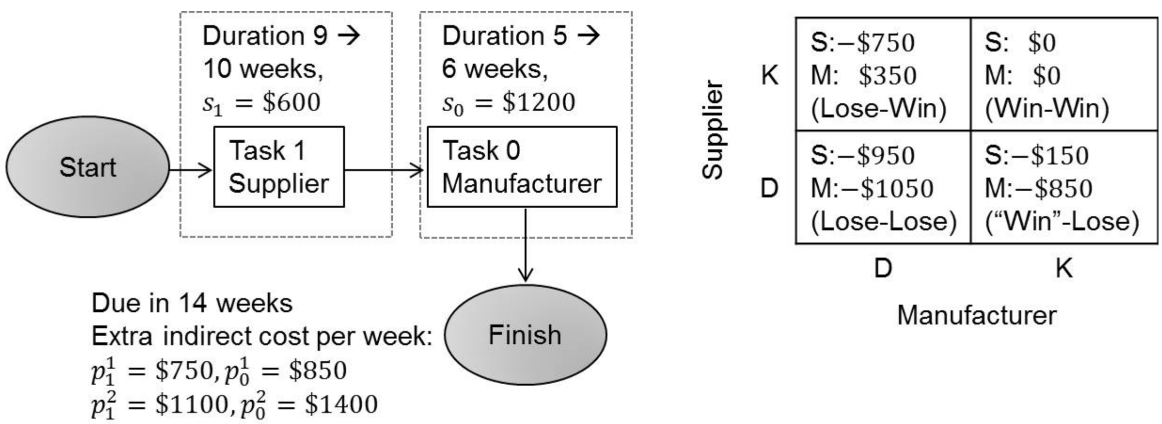

Figure 6: An example for the base model with time-dependent costs and its pay-off matrix. (K:keep, D:delay)

We consider the following four scenarios under the loss-sharing partnership,

- **"Win"-Lose**: firm 1 (the supplier) delays but firm 0 (the manufacturer) keeps its original task duration. In this scenario, firm 1 saves \$600 but must pay \$750 with a net loss of \$150, while firm 0 must pay \$850. The firms' pay-offs (relative to the original schedule) are $(\pi_1, \pi_0) = (-150, -850)$ and the project's pay-off is $-\$1000$ (relative to the original schedule).

- **Lose-Win**: firm 1 keeps its original task duration but firm 0 delays. This scenario is identical to the "Lose-Win" scenario of the example in §4.1 with the firms' pay-offs being $(-750, 350)$

and the project's pay-off being $-\$400$.

- **Lose-Lose**: both firms delays. In this scenario, the project is delayed by two weeks and the firms' pay-offs are $(-1250, -1050)$. This is the worst scenario for the project as a whole with a total loss of \$2300.

- **Win-Win**: both firms keep. The firms' pay-offs are $(0, 0)$.

Figure 6 summarizes the action set and the pay-off matrix. Clearly, if the supplier (firm 1) keeps its original task duration, the manufacturer's best response is to "delay" because its saving exceeds its penalty of the **1st** period of project delay. However, if the supplier delays, the manufacturer's best response is to "keep" its original task duration because now its penalty of the **2nd** period of project delay exceeds its saving. Thus in order to avoid a greater loss, the supplier has to delay (even at a loss) to raise the penalty so high that the manufacturer would have to keep. We call such a phenomenon the "Supplier's Dilemma". It is easy to verify that the SPNE in this example is [D, K].

We now analyze the base model with time-dependent costs in general. We note that the only difference between this model and the base model in §4.1 is that when both firms delay, the delay penalty is $p_i^1 + p_i^2$ for firm i. Figure 7 shows the extensive form of the game between the supplier and the manufacturer.

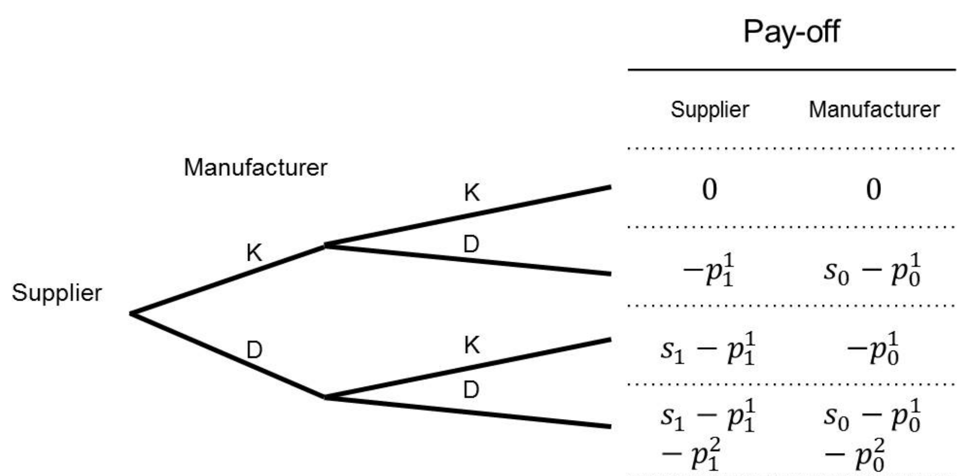

Figure 7: The extensive form of the game in the base model with time-dependent costs.

We can derive the following results on the dominant strategies and equilibrium.

Lemma 2 (Dominant Strategy): *In the base model with time-dependent costs, under Condition 2, when $s_0 < p_0^1$, "Keep" is the dominant strategy for the manufacturer; when $s_0 > p_0^2$, "Delay" is the dominant strategy for the manufacturer.*

Theorem 2 (Equilibrium): *For the base model with time-dependent costs, under Condition 2, the subgame perfect Nash equilibrium is given by:*

Case	*Condition on S*	*Condition on M*	*Optimal strategy for S*	*M's best response*
1	$s_1 < p_1^1$	$s_0 < p_0^1$	K	K
2	$s_1 > p_1^1$	$s_0 < p_0^1$	D	K
3		$p_0^1 < s_0 < p_0^2$	D	K
4	$s_1 < p_1^2$	$s_0 > p_0^2$	K	D
5	$s_1 > p_1^2$	$s_0 > p_0^2$	D	D

Theorem 2 is similar to Theorem 1 except for one new case (3rd case in Theorem 2): when $p_0^1 < s_0 < p_0^2$ (the manufacturer's saving is greater than its penalty of the 1st period project delay but less than its penalty of the 2nd period project delay, as illustrated in the example), the manufacturer's best strategy depends on the supplier's action. If the supplier keeps its original task duration, the manufacturer will delay; otherwise, the manufacturer will keep its original task duration. Thus, in this case, the supplier must take the manufacturer's response into account in making its own decision.

Based on these results, we present the following key insight for the base model with time-dependent costs under the loss-sharing partnership:

The Supplier's Dilemma: *if $p_0^1 < s_0 < p_0^2$, the supplier has to delay (even at a loss) to raise the penalty too high for the manufacturer to delay, to avoid a greater loss.*

4.3 The Base Model with Expediting and Reward – The Coauthors' Dilemma

In this section, we relax the base model by allowing each firm an additional option: expediting by one period (see Assumption 3). With the new action of "expediting", the project could be completed earlier than the original schedule. The question is, will this happen in equilibrium under loss sharing?

Assumption 3 *Assumption 1 holds here except that each task can be expedited by at most one period, and there is a reward per period if the project is expedited.*

We use "E" to denote "expediting". Let c_0 (or c_1) be the cost of expediting (i.e., the additional direct cost) for task 0 (or 1, respectively). Let r be the reward for the project per period expedited, and r_0 and r_1 be rewards received by the firms where $r_1 + r_0 = r$. When a firm expedites, the pay-off functions are different from previous sections where firms cannot expedite. Specifically, if the supplier expedites, the action set [E, K] yields $-c_1 + r_1$ for the supplier and r_0 for the manufacturer, [E, D] yields $-c_1$ for the supplier and s_0 for the manufacturer, and [E, E] yields $-c_1 + 2r_1$ for the supplier and $-c_0 + 2r_0$ for the manufacturer. If the manufacturer expedites, the pay-off functions could be derived in a similar way.

As in all previous sections, we assume that the project starts with an original schedule that is optimal under the centralized control. To this end, Condition 3 (Global Optimum) provides a necessary condition. For instance, [E, K] should yield less profit for the entire project than [K, K], which requires $-c_1 + r_1 + r_0 < 0$, and [E, D] should yield less profit for the project than [K, K], which requires $s_0 < c_1$. Condition 3 (Monotonicity) comes from the assumption of convex and increasing indirect cost and convex and decreasing direct cost (see §3). Condition 3 (Loss Sharing) indicates that the monotonicity condition on the project's reward and penalty also applies to each firm's share of the reward and penalty.

Condition 3 *(1) Global Optimum - Expediting:* $s_1 < p$, $s_0 < p$; $r < c_1$, $r < c_0$; $s_1 < c_0$, $s_0 < c_1$. *(2) Monotonicity - Expediting:* $r < p$; $s_1 < c_1$, $s_0 < c_0$. *(3) Loss Sharing - Expediting:* $r_1 < p_1$, $r_0 < p_0$.

The extensive form of the game is shown in Figure 8. For instance, if the supplier expedites while the manufacturer keeps its original task duration, the supplier gets an award of r_1 but must pay an expediting cost of c_1; the manufacturer gets an award of r_0 without any cost.

We can derive the following results on the dominant strategies and equilibrium.

Lemma 3 (Dominant Strategy)*: In the base model with expediting and reward, under Condition 3, when* $s_i > p_i$*, "delay" is the dominant strategy for firm* i*,* $i = 1, 0$*; when* $s_i < r_i < p_i < c_i$*, "keep" is the dominant strategy for firm* i*,* $i = 1, 0$.

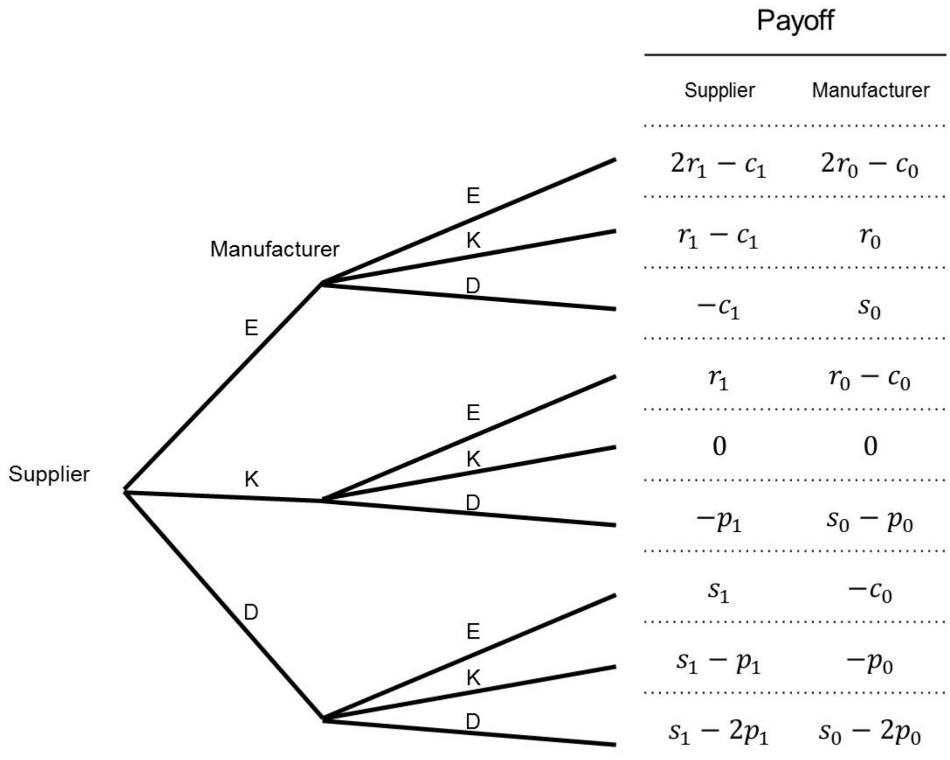

Figure 8: The extensive form of the game in the base model with expediting and reward.

Lemma 3 differs from Lemma 1 on the conditions for "keep" because we must consider not only "delay" but also "expediting" in this model.

Theorem 3 (Equilibrium): *For the base model with expediting and reward, under Condition 3, the subgame perfect Nash equilibrium is given by,*

Case	*Condition on S*	*Condition on M*	*Optimal strategy for S*	*M's best response*
1		$c_0 < p_0$	*D*	*E*
2	$s_1 < p_1$	$s_0 < p_0 < c_0$	*K*	*K*
3	$s_1 > p_1$	$s_0 < p_0 < c_0$	*D*	*K*
4	$c_1 < p_1$	$s_0 > p_0$	*E*	*D*
5	$s_1 < p_1 < c_1$	$s_0 > p_0$	*K*	*D*
6	$s_1 > p_1$	$s_0 > p_0$	*D*	*D*

Theorem 3 is similar to Theorem 1 except for the 1st (equilibrium: [D, E]) and 4th (equilibrium: [E, D]) cases that involve expediting. We shall explain the intuition behind these two new cases.

- **1st case, $c_0 < p_0$, [D, E] is the equilibrium**: In this case, the manufacturer faces a delay penalty that is greater than its expediting cost, and so it would do anything to prevent the project from being delayed. Taking advantage of the manufacturer's weakness, the supplier could delay regardless of its own cost structure, and earn a net saving without any penalty. Thus, even if the manufacturer expedites its task, the project will not be expedited because the supplier will delay.

  An example in the book publishing industry: Let's consider a coauthor and a lead author working sequentially on a textbook. The coauthor writes parts of the book and must pass on the manuscripts to the lead author to integrate and complete. The lead author is responsible for the delivery and is very concerned about the deadline. Thus the lead author will do anything possible to finish the book on time. Knowing this, the coauthor will delay as much as what the lead author can catch up without a penalty.

- **4th case, $c_1 < p_1$ and $p_0 < s_0$, [E, D] is the equilibrium**: In this case, "delay" is the dominant strategy for the manufacturer (by Lemma 3). In addition, the supplier faces a delay penalty that is greater than its expediting cost, and so the supplier will have to expedite to prevent the project from being delayed.

  An example of the academic thesis completion: Let's consider a PhD student and his/her advisor. The student shall write the PhD thesis and handle it over to the advisor to read and approve. The student needs to graduate and will do anything possible to complete his/her thesis on time. The advisor, on the other hand, is well established and much less concerned. Knowing that the advisor is the bottleneck, the student has to work extra hard in the hope of getting the thesis done on time.

Theorem 3 implies that in the base model with expediting and reward, the project will never be expedited in the equilibrium under the loss-sharing partnership as compared to the optimal schedule. We summarize the results in this section by the following dilemma:

The Coauthors' Dilemma: *A firm cannot expedite the project even if it expedites its tasks, because if it does so, other firms will delay.*

Although it is obvious that the optimal schedule should outperform the loss-sharing partnership in cost, it is not clear how loss sharing may affect the project duration. Theorem 3 and the

Coauthors' Dilemma reveal a non-trivial and interesting insight, that is, loss sharing tends to increase the project duration relative to the optimal schedule rather than shortening it.

4.4 The Base Model with Multiple Suppliers – The Worst Supplier Dominance

In this section, we extend the base model to include two suppliers at stage 1 to study the impact of parallel tasks. The analysis of a N-supplier system is similar. The model is defined in Assumption 4 where the suppliers play a simultaneous game among themselves in phase 1 anticipating the manufacturer's response to their aggregated actions in phase 2.

Assumption 4 *Assumption 1 holds here except that stage 1 has two tasks each conducted by a unique supplier, and the manufacturer can only start its task after both suppliers complete their work.*

We denote supplier 1 (2)'s saving in the direct cost from delay to be s_1 (s_2) per period. The project penalty shared by the supplier 1 (or 2) is p_1 (or p_2 respectively) where $p_1 + p_2 + p_0 = p$. A necessary condition for the original schedule to be optimal under the centralized control is provided as follows,

Condition 4 *Global Optimum - Two Suppliers:* $s_1 + s_2 < p$, $s_0 < p$.

Without the loss of generality, we assume that the original durations of tasks 1 and 2 are identical (otherwise, the system reduces to the base model as we can ignore the supplier with a shorter duration). The same assumption applies to systems with more than two suppliers which will be discussed later in the paper.

We have the following results on the dominant strategies and equilibrium.

Lemma 4 (Dominant Strategy)*: In the base model with two suppliers, under Condition 4, when* $s_0 < p_0$*, "keep" is the dominant strategy for the manufacturer; when* $s_0 > p_0$*, "delay" is the dominant strategy for the manufacturer. When* $s_i > p_i$*, "delay" is the dominant strategy for supplier* i*.*

Lemma 4 differs from Lemma 1 because of the assembly-like structure at stage 1 – there is no unilateral condition for a supplier to keep the original duration of its task as the stage 1's on time performance depends on both suppliers' actions.

Theorem 4 (Equilibrium): *For the base model with two suppliers, under Condition 4, the subgame perfect Nash equilibrium is given by,*

Case	*Condition on S*	*Condition on M*	*Optimal strategy for S1, S2*	*M's best response*
1	$s_1 < p_1$ *and* $s_2 < p_2$	$s_0 < p_0$	*K, K*	*K*
2	$s_1 > p_1$ *or* $s_2 > p_2$	$s_0 < p_0$	*D, D*	*K*
3	$s_1 < p_1$ *and* $s_2 < p_2$	$s_0 > p_0$	*K, K*	*D*
4	$s_1 > p_1$ *or* $s_2 > p_2$	$s_0 > p_0$	*D, D*	*D*

Note that with multiple suppliers, the SPNE is no longer unique due to the simultaneous game played among the suppliers in phase 1. We only report Pareto optimum equilibrium here.

Theorem 4 illustrates the impact of the project network on the equilibrium and project performance, that is, the project is more likely to be delayed with multiple suppliers. For the original schedule to be the SPNE, we require $s_1 < p_1$ and $s_2 < p_2$ (i.e., penalty exceeds saving for both suppliers) and $s_0 < p_0$. If the saving exceeds penalty for any supplier, all suppliers will have to delay in equilibrium. This observation gives rise to the following key insight:

The Worst Supplier Dominance: if one supplier delays, the other supplier(s) have to follow.

4.5 The General Model

In previous sections, we reveal many managerial insights from the base model and its extensions. In this section, we put all the extensions together into a general model where we also allow each firm to delay or expedite its task by multiple periods (see Assumption 5). The questions are, first, do the results obtained from the special cases in previous sections (§4.1-4.4), especially the Coauthors' Dilemma, hold in the general model? Second, how to compute the project schedule in equilibrium?

Assumption 5 *The system has multiple suppliers and one manufacturer; each task can be either expedited or delayed by multiple periods; the cost structure, including penalty, reward, saving and expediting costs, are time dependent.*

We first consider the system with a single supplier. For the ease of exposition, we use the notation of (x_1, x_0) to represent the actions of the supplier and the manufacturer, where x_1 (or x_0) is an integer and its absolute value represents the number of periods expedited or delayed by the supplier (the manufacturer, respectively) relative to the original schedule. A negative integer means expediting, a positive integer means delaying, and zero means keeping the original task duration.

In this game, the supplier is the first mover and takes an action x_1. Let's define the manufacturer's best response (to x_1) as $x_0^*(x_1)$. The project duration will therefore be changed by $x_1 + x_0^*(x_1)$. We use superscripts on s_i, c_i, r and p to index the associated periods. For example, if task i is delayed by two periods, then the total saving should be $s_i^1 + s_i^2$ where s_i^1 (s_i^2) is the saving from the 1^{st} (2^{nd}) period of delay. if task i is expedited by two periods, then c_i^1 (c_i^2) is the cost for the 1^{st} (2^{nd}) period of expediting. Lastly, we define $\pi_1(x_1, x_0)$ ($\pi_0(x_1, x_0)$) to be the pay-off function for the supplier (the manufacturer, respectively).

For this system, Condition 5 (Global Optimum) is necessary for the original schedule to be optimal under the centralized control; Condition 5 (Monotonicity) comes from the convex increasing indirect cost and convex decreasing direct cost; finally, Condition 5 (Loss Sharing) indicates that the monotonicity condition on project reward and penalty also applies to each party's reward and penalty.

Condition 5 *(1) Global Optimum - General:* $\sum_{i=0}^{n} \pi_i(x_1, \cdots, x_n, x_0) \leq 0$ *for any* x_i*,* $i = 0, 1, \ldots, n$*; (2) Monotonicity - General:* $r^k > r^{k+1}$*,* $p^k < p^{k+1}$*,* $s_i^k > s_i^{k+1}$*,* $c_i^k < c_i^{k+1}$ *for any positive integer* k *and any* $i = 0, 1, \ldots, n$*, and* $r^1 < p^1$*,* $s_i^1 < c_i^1$ *for any* $i = 0, 1, \ldots, n$*; (3) Loss Sharing - General:* $r_i^k > r_i^{k+1}$*,* $p_i^k < p_i^{k+1}$ *and* $r_i^1 < p_i^1$ *for* $i = 0, 1, \ldots, n$*.*

We first characterize the pay-off function for the manufacturer for a given action of the supplier.

Lemma 5 *Given* $x_1 = a$*,* $\pi_0(a, x_0)$ *is a uni-modal function of* x_0*.*

Lemma 5 indicates that the manufacturer has a unique best response to each of the supplier's actions. The following lemma shows some monotonicity properties of the manufacturer's best response function.

Theorem 5 (Monotonicity Property)*: As* x_1 *increases,* $x_0^*(x_1)$ *is non-increasing but* $x_1 + x_0^*(x_1)$ *is non-decreasing.*

Theorem 5 implies that if the supplier delays more, the manufacturer will delay less, but the project will be delayed for a longer time.

In the case that the task duration is sufficiently long and so x_1 is effectively unbounded from below, the following theorem specifies a limit by which the supplier would expedite its task.

Theorem 6 (Expedition Limit): *There exists a* $x_L = \max\{x_1 | x_1 + x_0^*(x_1) = 0\} > -\infty$ *such that if* $x_1 \leq x_L$*, the supplier will be better off if it increases* x_1 *to* x_L*.*

Combining Theorems 5-6, we arrive at the following key insight,

Corollary 1 (The General Coauthor's Dilemma): *No matter by how much each firm expedites its task, the project will never be expedited (relative to the original schedule) in equilibrium under the loss-sharing partnership.*

For the system with multiple suppliers, we define $x_s = \max\{x_1, \ldots, x_n\}$. We can show that Theorems 5-6 hold if we replace x_1 by x_s.

To numerically compute the equilibrium (the SPNE), we design an algorithm which enumerates x_1 between x_L and a pre-specified maximum allowable project delay, to find the optimal x_1^* for the supplier. Here is the key idea: we start by setting $x_1 = 0$. First, we search the region of $x_1 < 0$ until x_1 reaches x_L (if $x_L < 0$); second, we search the region of $x_1 > 0$ until we reach the maximum allowable project delay. We keep updating the best π_1 found to date and the corresponding x_1 and x_0, denoted by $(\pi_1^{\max}, x_1^*, x_0^*)$, until the enumeration is completed.

Let U be the maximum allowable project delay, the implementation details of this algorithm are described as follows:

Algorithm

- Step 1 - initialization: set $x_1 \leftarrow 0$. If $s_0^1 < p_0^1$, $x_0^*(0) \leftarrow 0$ otherwise $x_0^*(0)$ equals to i that satisfies $s_0^i > p_0^i$ and $s_0^{i+1} < p_0^{i+1}$. Initialize $\{\pi_1^{\max}, x_1^*, x_0^*\}$ with $\{\pi_1(0, x_0^*(0)), 0, x_0^*(0)\}$. Let $k \leftarrow x_0^*(0)$.

- Step 2 - search the region of $x_1 < 0$: $x_1 \leftarrow x_1 - 1$. Find $x_0^*(x_1)$ by comparing $\pi_0(x_1, k)$ and $\pi_0(x_1, k+1)$: if the former is greater, k remains; otherwise $k \leftarrow k + 1$. Compute $\pi_1(x_1, k)$, and update $\{\pi_1^{\max}, x_1^*, x_0^*\}$ with $\{\pi_1(x_1, k), x_1, k\}$ if $\pi_1^{\max} < \pi_1(x_1, k)$. If $x_1 + k > 0$, repeat Step 2, otherwise reset $x_1 \leftarrow 0$, $k \leftarrow x_0^*(x_1)$ and go to Step 3.

- Step 3 - search the region of $x_1 > 0$: if $x_1 + k \leq U$, find $x_0^*(x_1')$ by comparing $\pi_0(x_1', k)$ and $\pi_0(x_1', k-1)$: if the former is greater, k remains the same; otherwise $k \leftarrow k - 1$. Compute $\pi_1(x_1, k)$, and update $\{\pi_1^{\max}, x_1^*, x_0^*\}$ with $\{\pi_1(x_1, k), x_1, k\}$ if $\pi_1^{\max} < \pi_1(x_1, k)$. If $x_1 + k > U$, stop and output the current $\{\pi_1^{\max}, x_1^*, x_0^*\}$.

5 The Fair-Sharing Partnership

In this section, we introduce the new fair-sharing partnership, under the principle of having each party fully responsible for the consequences of its actions (**The Fair-Sharing Principle**). Thus, if one firm causes damages to other firms, it has to compensate the others. Conversely, if one firm brings benefits to other firms, it shall receive compensations from the others. To this end, we must design a payment distribution scheme that first separates the consequences (damages or benefits) of one's action from the aggregated project outcome (e.g., indirect cost), then allocates them "fairly" to each firm according to its actions. The scheme clearly depends on project network and cost structures. We shall first revisit the base model (see §4.1) in §5.1 to derive the key ideas, and then present a complete solution for the general model (see §4.5) in §5.2.

5.1 The Base Model Under Fair Sharing

In this section, we present the fair-sharing partnership for the base model (defined by Assumption 1 in §4.1). By the fair-sharing principle: if firm i delays, it not only pays its own share of the project delay penalty p_i, but also must reimburse firm j ($j \neq i$) her (firm j's) share of the penalty p_j due to firm i's delay. Thus, the idea is that each firm is fully responsible for the penalty incurred by its delay. Specifically,

The Payment Distribution Scheme (The Base Model): *if both firms keep their original task duration, no payment is transferred. If only the supplier delays its task, the supplier not only suffers a penalty of* p_1*, but also pays the manufacturer* p_0 *to compensate the manufacturer's loss due to the supplier's delay. Similarly, if only the manufacturer delays its task, the manufacturer suffers a penalty of* p_0 *and must pay the supplier* p_1*, the supplier's loss due to the manufacturer's delay. If both firms delay, each will compensate the other for the loss caused by its delay, that is, the supplier pays* p_0 *to the manufacturer and the manufacturer pays back the supplier* p_1*. In any event, if a firm delays, it will pay the full penalty* p*.*

The pay-off matrix is shown in Figure 9. As we can see that the pay-off of each firm depends only on its own action and thus the interference among the firms is minimized. It is obvious that the action set [K, K] is the SPNE under Condition 1 in §4.1. Thus fair sharing is capable of aligning individual firms' interests with that of the project in the base model.

Extension to Two Suppliers

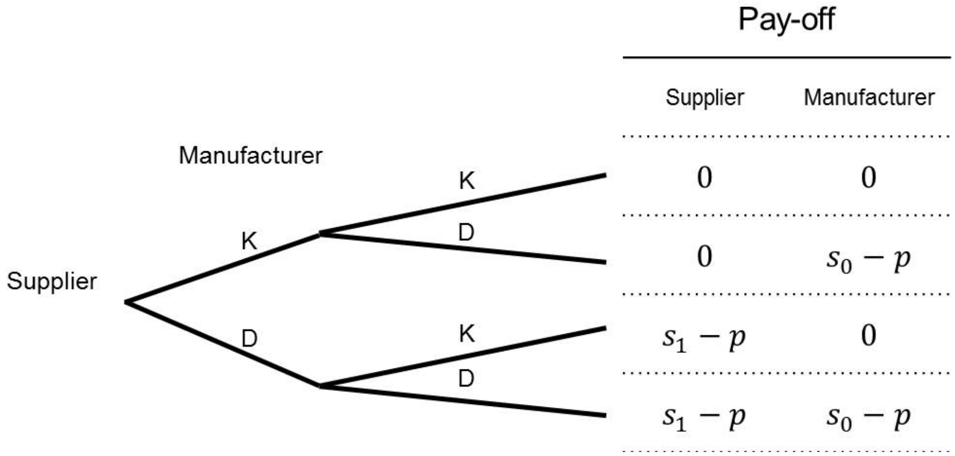

Figure 9: Extensive form of the game in base model under fair sharing.

The case of multiple suppliers complicates the payment distribution scheme because the impact of one supplier's action on the project outcome depends on the others' actions. To illustrate the key idea, we consider the base model with two suppliers (defined by Assumption 4 in §4.4). By the fair-sharing principle,

The Payment Distribution Scheme (The Base Model With Two Suppliers): *if the manufacturer delays, it pays p which is the delay penalty of the project. Likewise, if one of the suppliers delays while the other keeps its original task duration, the delayed supplier pays p. If both suppliers delay, they split the penalty according to a rationing rule ($\beta_1 > 0$, $\beta_2 > 0$) where $\beta_1 + \beta_2 = 1$ and supplier 1 (2) pays $\beta_1 p$ ($\beta_2 p$).*

The idea behind this scheme is that a supplier only shares the penalty of the project delay that it contributes to. In other words, if the stage 1 delay is caused by others, the supplier is not responsible. An analysis of the extensive form of the game leads to the following theorem.

Theorem 7 *Consider the base model with two suppliers. Under the fair-sharing partnership and Condition 4, the SPNE is to keep the original schedule (which is optimal under the centralized control) for any (β_1, β_2) as long as $\beta_1 > 0$, $\beta_2 > 0$ and $\beta_1 + \beta_2 = 1$.*

Note that Theorem 7 holds regardless of the value of $\beta_i, i = 1, 0$. Thus, the fair-sharing partnership provides the suppliers a flexibility in negotiating their shares of delay penalty.

5.2 The General Model Under Fair Sharing

In this section, we present the fair-sharing partnership for the general model (defined by Assumption 5 in §4.5) and prove its effectiveness in aligning firms' incentives in project execution. Following the ideas derived from the base models, the payment distribution scheme for the general model shall follow a two-step procedure: first it splits project penalty/reward (changes in indirect cost) between stage 1 and stage 2 (step 1), then it distributes the penalty/reward of stage 2 among the suppliers (step 2).

The general model introduces several complexities to an effective payment distribution scheme. In addition to both parallel and sequential project networks, we must handle firms that can either delay or expedite multiple periods, and the penalty/reward that are time dependent. These complexities have the following implications on the fair-sharing partnership.

- Separating the consequences between the actions of stage 1 (suppliers) and that of stage 2 (manufacturer) is intriguing because one stage's expedition can be offset by the other's delay in various ways. One must also decide which period's delay or reward to be allocated to each firm.

- Distribution of penalty/reward among the suppliers is tricky because the suppliers' actions can be highly diversified. For instance, if some suppliers expedite, some keep, while others delay, it is unclear how a "fair" allocation of the penalty/reward works.

To gain insights into the fair-sharing partnership for the general model, let's consider an example as follows:

- **Case 1**: If stage 1 is expedited by 5 weeks but stage 2 is delayed by 2 weeks, the project is therefore expedited by 3 weeks. By the fair-sharing principle, stage 1 firms should be rewarded by $r^5, r^4, \ldots, r^1$, among which r^3, r^2, r^1 come from the project's earlier completion, but the rewards r^5 and r^4 are not materialized due to the delay at stage 2, and so must be paid by the firm (the manufacturer) at stage 2.

- **Case 2**: If stage 1 is delayed by 5 weeks but stage 2 is expedited by 2 weeks, the project is therefore delayed by 3 weeks. stage 1 firms must pay the penalties $p^1, p^2, \ldots, p^5$. However, p^4 and p^5 are not materialized by the stage 2 firm's expedition, and so must be paid to the stage 2 firm.

This example sheds lights on the payment distribution scheme in step 1 for the stage 1 firms (suppliers). Specifically, we shall have stage 1 firms fully responsible for the project penalty/reward assuming that the stage 2 firm keeps its original task duration. This is consistent to the payment distribution scheme in the Base Model. The stage 2 firm shall be responsible for any additional changes in the project outcome (indirect cost) due to its own action. Specifically, defining the change of project indirect cost (either reward or penalty) as B, fair sharing redistributes B among stages 1 and 2 firms where stage 1 firms (the suppliers) receive A_1, the stage 2 firm (the manufacturer) receives A_2, and $A_1 + A_2 = B$. For the ease of exposition, we also define $x_s = \max\{x_1, x_2, \ldots, x_N\}$ where x_s represents the change of stage 1 completion date as compared to the original schedule. Using this notation, we can outline the payment distribution scheme for the general model as follows.

The Payment Distribution Scheme (The General Model):

- **Step 1**: *we decide the payment transferred between the two stages by allocating B among stages 1 and 2. If stage 1 completion date is expedited by k periods ($x_s = -k$), stage 1 firms shall be compensated by the rewards (i.e., savings in the indirect cost) for the project for the first k periods, that is, $A_1 = r^1 + r^2 + \ldots + r^k$. If stage 1 completion date is delayed by k periods ($x_s = k$), stage 1 firms shall pay the penalty (i.e., the additional indirect cost) for the project for the first k periods, that is, $A_1 = p^1 + p^2 + \ldots + p^k$. After the suppliers' allocation A_1 is determined, the manufacturer's allocation $A_2 = B - A_1$ accordingly.*

- **Step 2**: *we decide the payment transferred within stage 1 firms by allocating A_1 among the suppliers. If stage 1 completion date is expedited by k periods ($x_s = -k$), then each supplier must have expedited its task by at least k periods. A_1 is the reward and should be shared among all the suppliers. If stage 1 completion date is delayed by k periods ($x_s = k$), then each supplier delays its task by at most k periods. A_1 is now the penalty and should be shared on a period-by-period basis among all delayed suppliers. For those suppliers who didn't delay in this case, they neither receive any reward nor share any penalty. More details are provided below.*

To specify the pay-off function of the manufacturer (the stage 2 firm), we must include any changes in its direct cost on the top of its share of the indirect cost $B - A_1$ (Table 1). In principle, the manufacturer is only responsible for the reward or penalty from the periods expedited or delayed by itself.

Stage 1	Stage 2 - The manufacturer	Pay-off of the manufacturer
E: $x_s < 0$	E: $x_0 < 0$	$\sum_{i=\lvert x_s\rvert+1}^{\lvert x_s\rvert+\lvert x_0\rvert} r^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$
	K: $x_0 = 0$	0
	D: $x_0 > 0$	$-\sum_{i=\lvert x_s+x_0\rvert+1}^{\lvert x_s\rvert} r^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$, if $x_s + x_0 \leq 0$
		$-\sum_{i=1}^{\lvert x_s\rvert} r^i - \sum_{i=1}^{\lvert x_s+x_0\rvert} p^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$, if $x_s + x_0 > 0$
K: $x_s = 0$	E: $x_0 < 0$	$\sum_{i=1}^{\lvert x_0\rvert} r^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$
	K: $x_0 = 0$	0
	D: $x_0 > 0$	$-\sum_{i=1}^{\lvert x_0\rvert} p^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$
D: $x_s > 0$	E: $x_0 < 0$	$\sum_{i=1}^{\lvert x_s\rvert} p^i + \sum_{i=1}^{\lvert x_s+x_0\rvert} r^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$, if $x_s + x_0 < 0$
		$\sum_{i=\lvert x_s+x_0\rvert+1}^{\lvert x_s\rvert} p^i - \sum_{i=1}^{\lvert x_0\rvert} c_0^i$, if $x_s + x_0 \geq 0$
	K: $x_0 = 0$	0
	D: $x_0 > 0$	$-\sum_{i=\lvert x_s\rvert+1}^{\lvert x_s\rvert+\lvert x_0\rvert} p^i + \sum_{i=1}^{\lvert x_0\rvert} s_0^i$

Table 1: The pay-off function of the manufacturer under fair sharing in the general model.

To specify the payment distribution scheme in Step 2, we expand the idea of the Base Model with two suppliers by distributing the project reward or penalty (A_1) among the suppliers using the following rules:

- A supplier only shares the penalty of the periods delayed by itself.
- If a supplier keeps its original task duration, it receives neither penalty nor reward.
- If a supplier expedites its task, it is rewarded only for the periods effectively expedited by stage 1 firms. In other words, it does not receive reward if its expedition is not effective – does not lead to an expedition of stage 1 completion.

Although the idea behind the payment allocation scheme of the project penalty/reward is simple, one must include the changes in the direct cost to calculate the pay-off function for each supplier. We discuss three cases:

- **Case 1:** $x_s < 0$. All suppliers share the expediting rewards of $|x_s|$ periods. The pay-off for supplier i is $\pi_i = -\sum_{j=1}^{|x_i|} c_i^j + \alpha_i \sum_{j=1}^{|x_s|} r^j$, where $\alpha_i > 0$ for $i = 1, \ldots, N$ and $\sum_{i=1}^{N} \alpha_i = 1$. Here α_i is supplier i's ration of the reward.
- **Case 2:** $x_s = 0$. If supplier i keeps its original task duration, its pay-off is $\pi_i = 0$; if supplier i expedites, its pay-off is $\pi_i = -\sum_{j=1}^{|x_i|} c_i^j$.

- **Case 3:** $x_s > 0$. If supplier i expedites, its pay-off is $\pi_i = -\sum_{j=1}^{|x_i|} c_i^j$; if it keeps, its pay-off is $\pi_i = 0$; if it delays, its pay-off is $\pi_i = \sum_{j=1}^{|x_i|} s_i^j - \sum_{j=1}^{|x_s|} \beta_i^j p^j$ where β_i^j is supplier i's ration for the penalty of the j^{th} period delayed. If $j > x_i$ (that is, this supplier does not contribute to the j^{th} period of delay), $\beta_i^j = 0$; otherwise $\beta_i^j > 0$ (ration of penalty) and β_i^j satisfies $\sum_{i=1}^{N} \beta_i^j = 1$ for all $j = 1, 2, \ldots, |x_s|$.

The cases are summarized in Table 2.

Stage 1	Supplier i	Pay-off of supplier i
E: $x_s < 0$	E: $x_i < 0$	$-\sum_{j=1}^{|x_i|} c_i^j + \alpha_i \sum_{j=1}^{|x_s|} r^j$
K: $x_s = 0$	E: $x_i < 0$	$-\sum_{j=1}^{|x_i|} c_i^j$
	K: $x_i = 0$	0
D: $x_1 > 0$	E: $x_i < 0$	$-\sum_{j=1}^{|x_i|} c_i^j$
	K: $x_i = 0$	0
	D: $x_i > 0$	$\sum_{j=1}^{|x_i|} s_i^j - \sum_{j=1}^{|x_s|} \beta_i^j p^j$

Table 2: The pay-off function of suppliers under fair sharing in the general model. Note: (1) $\alpha_i > 0$ and $\sum_{i=1}^{N} \alpha_i = 1$. (2) $\beta_i^j = 0$ if $j > x_i$, otherwise, $\beta_i^j > 0$. (3) $\sum_{i=1}^{N} \beta_i^j = 1$ for all $j = 1, 2, \ldots, |x_s|$.

For the fair-sharing scheme, we can prove the following theorem.

Theorem 8 *In the general model under the fair sharing scheme, "keep" for all firms is the unique SPNE.*

Theorem 8 implies that fair sharing is capable of aligning individual firms' incentives in project execution for the general model.

6 Conclusions

In this paper, we consider collaborative projects for which the workload and outcome are shared by multiple firms. Despite the "positive" connotation, we show that collaboration, via the popular loss-sharing partnership, may negatively affect project performance by distorting the firms' incentives and encouraging deliberate delays and cost overruns. As a remedy, we propose the new fair-sharing partnership and prove that under this partnership, when firms balance their own time and cost trade-off for their tasks, they execute the project in its optimal schedule. The key to incentive alignment lies in a payment distribution scheme that separates the consequences of a firm's action from the aggregated project outcome. By having all firms fully responsible for their own actions, fair sharing minimizes the interference among the firms, as such, regulates the gaming behaviors in collaborative projects.

Intuitively, fair sharing and loss sharing distribute the total penalty upon project delays in different ways. Although loss sharing may give the impression of a smaller penalty for individual firms than fair sharing because the damages are shared, this paper shows that, in fact, the reverse is true because if a firm does not pay for the damages of its own delay, it ends up paying more damages for others' delays.

The research bridging supply chain, economics and project management promises to be fruitful to both practitioners and academicians because of the high impact on practice, and the potential of making exciting theoretical discoveries by integrating these rich bodies of literature. This paper assumes information symmetry and predictable task durations which represent full visibility and predictability in practice. Going forward, it would be interesting to extend this research to the cases with partial visibility and/or predictability in future studies.

References

Al-Najjar, Nabil I. 1997. Incentive Contracts in Two-Sided Moral Hazards with Multiple Agents. *J. Econ. Theory* **74**(1) 174–195.

Aydinliyim, Tolga, GL Vairaktarakis. 2010. Coordination of Outsourced Operations to Minimize Weighted Flow Time and Capacity Booking Costs. *Manuf. Serv. Oper. Manag.* **12**(2) 236–255.

Bayiz, Murat, Charles J. Corbett. 2005. Coordination and Incentive Contracts in Project Management under Asymmetric Information. *SSRN eLibrary* .

Bhaskaran, Sreekumar R., V. Krishnan. 2009. Effort, Revenue, and Cost Sharing Mechanisms for Collaborative New Product Development. *Manage. Sci.* **55**(7) 1152–1169.

Bhattacharyya, Sugato, Francine Lafontaine. 1995. Double-sided Moral Hazard and the Nature of Share Contracts. *Rand J. Econ.* **26**(4) 761–781.

Demski, Joel S., David Sappington. 1984. Optimal Incentive Contracts With Multiple Agents. *J. Econ. Theory* **33**(1) 152–171.

Elmaghraby, Salah E. 1990. Project Bidding under Deterministic and Probabilistic Activity Durations. *Eur. J. Oper. Res.* **49**(1) 14–34.

Exostar, LLC. 2007. *Boeing 787: global supply chain management takes flight*. Case Study.

Gutierrez, Genaro, Anand Paul. 2000. Analysis of the Effects of Uncertainty, Risk-pooling, and Subcontracting Mechanisms on Project Performance. *Oper. Res.* **48**(6) 927–938.

Holmstrom, Bengt. 1982. Moral Hazard in Teams. *Bell J. Econ.* **13**(2) 324–340.

Holmstrom, Bengt, Paul Milgrom. 1991. Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *JL Econ. Org.* **7** 24.

Horng, T.C., K. Bozdogan. 2007. *Comparative analysis of supply chain management practices by Boeing and Airbus: long-term strategic implications*. Presentation at the MIT Lean Aerospace Initiative, April 18, 2007.

Jozefowska, Joanna, Jan Weglarz. 2006. *Perspectives in Modern Project Scheduling*. 1st ed. Springer, New York.

Kim, Son Ku, Susheng Wang. 1998. Linear Contracts and the Double Moral-Hazard. *J. Econ. Theory* **82** 342–378.

Kwon, H. Dharma, Steven A. Lippman, Kevin F. McCardle, Christopher S. Tang. 2010. Project Management Contracts with Delayed Payments. *Manuf. Serv. Oper. Manag.* **12**(4) 692–707.

McAfee, R. Preston, John McMillan. 1986. Bidding for contracts: a principal-agent analysis. *Rand J. Econ.* **17**(3) 326–338.

Nahmias, Steven. 2008. *Production and Operations Analysis (McGraw-Hill/Irwin Series Operations and Decision Sciences)*. McGraw-Hill/Irwin.

NASA. 2013. International Cooperation. http://www.nasa.gov/mission_pages/station/cooperation /index.html#.UmlVg_kegao.

Paul, Anand, Genaro Gutierrez. 2005. Simple Probability Models for Project Contracting. *Eur. J. Oper. Res.* **165**(2) 329–338.

Szmerekovsky, Joseph G. 2005. The Impact of Contractor Behavior on the Client's Payment-Scheduling Problem. *Manage. Sci.* **51**(4) 629–640.

Wikipedia. 2015. *Galileo (satellite navigation), BeiDou Navigation Satellite System.* http://en.wikipedia.org/wiki/Galileo$_{\%}28satellite_navigation\%29, http://en.wikipedia.org/wiki/BeiDou_Navigation_Satellite_System.$

Xu, Xin, Yao Zhao. 2011. *Build-to-Performance – Boeing 787 Dreamliner*. Rutgers Business School Case Study, Newark, NJ.

Zhao, Yao. 2014. *Why 787 Slips Were Inevitable?*. Rutgers Business School Working Paper, Newark, NJ.

Appendix

Proof of Lemma 1

For the supplier with $s_1 < p_1$, if the manufacturer chooses "keep", then $0 > s_1 - p_1$ and so the supplier will choose "keep"; if the manufacturer chooses "delay", then $-p_1 > s_1 - 2p_1$ so that the supplier will choose "keep" as well. Thus, the supplier has a dominant strategy of "keep" when $s_1 < p_1$. Similarly, we can prove that when $s_1 > p_1$, "delay" is the dominant strategy for the supplier.

For the manufacturer with $s_0 < p_0$, if the supplier chooses "keep", then $0 > s_0 - p_0$ and so the manufacturer will choose "keep"; if the supplier chooses "delay", then $-p_0 > s_0 - 2p_0$ so that the manufacturer will choose "keep" as well. Thus, the manufacturer has a dominant strategy of "keep" when $s_0 < p_0$. Similarly, we can prove that when $s_0 > p_0$, "delay" is the dominant strategy for the manufacturer. $\square$

Proof of Theorem 1

This theorem is a straightforward result of Lemma 1. $\square$

Proof of Lemma 2

For the manufacturer with $s_0 < p_0^1$, if the supplier chooses "keep", then $0 > s_0 - p_0^1$ and so the manufacturer will choose "keep"; if the supplier chooses "delay", then $-p_0^1 > s_0 - p_0^1 - p_0^2$ and so the manufacturer will choose “keep” as well. Thus, the manufacturer has a dominant strategy of “keep” when $s_0 < p_0^1$. Similarly, we can prove that when $s_0 > p_0^2$, “delay” is the dominant strategy for the manufacturer. $\square$

Proof of Theorem 2

Lemma 2 implies,

- when $s_1 > p_1^1$ and $s_0 < p_0^1$, the supplier has a dominant strategy of “delay” and the manufacturer has a dominant strategy of “keep”.
- when $s_1 > p_1^2$ and $s_0 > p_0^2$, the supplier has a dominant strategy of “delay” and the manufacturer has a dominant strategy of “delay”.

When $s_1 < p_1^1$ and $s_0 < p_0^1$, if the supplier chooses “keep”, then the manufacturer will choose “keep” as $0 > s_0 - p_0^1$; if the supplier chooses “delay”, then the manufacturer will choose “keep” as $-p_0^1 > s_0 - p_0^1 - p_0^2$. The former strategy gives the supplier a higher pay-off (0) than the latter strategy $(s_1 - p_1^1)$ and thus the supplier will choose “keep” and then the manufacturer will choose “keep”.

When $p_0^1 < s_0 < p_0^2$, if the supplier chooses “keep”, then the manufacturer will choose “delay” as $s_0 > p_0^1$; if the supplier chooses “delay”, then the manufacturer will choose “keep” as $s_0 < p_0^2$. The latter strategy gives the supplier a higher pay-off $(-p_1^1)$ than the former strategy $(s_1 - p_1^1)$ and thus the supplier will choose “delay” and then the manufacturer will choose “keep”.

When $s_1 < p_1^2$ and $s_0 > p_0^2$, the manufacturer has the dominant strategy of “delay”. Since $-p_1^1 > s_1 - p_1^1 - p_1^2$, the supplier will choose “keep”. $\square$

Proof of Lemma 3

When $s_0 > p_0$, we know that $s_0 > r_0$ and $r_0 < c_0$ from Condition 3. If the supplier chooses “expediting” or “keep”, the manufacturer always gets the highest pay-off if it delays. If the supplier chooses “delay”, because $p_0 < s_0 < c_0$, “delay” yields the highest pay-off for the manufacturer. Thus, the manufacturer has a dominant strategy of “delay” in this scenario. Similarly, we can prove that the supplier has a dominant strategy of “delay” when $s_1 > p_1$. By a similar analysis, we could prove that when $s_i < r_i < p_i < c_i$, “keep” is the dominant strategy for firm i, $i = 1, 0$. $\square$

Proof of Theorem 3

All potential actions are listed below:

S	M	S's Pay-off	Conditions	M's Best Response	M's Pay-off
	E	$2r_0 - c_0$			
E	K	r_0	if $r_0 > s_0$	K	$r_1 - c_1$
	D	s_0	if $r_0 < s_0$	D	$-c_1$
	E	$r_0 - c_0$			
K	K	0	if $p_0 > s_0$	K	0
	D	$s_0 - p_0$	if $p_0 < s_0$	D	$-p_1$
	E	$-c_0$	if $p_0 > s_0$; if $p_0 > c_0$	E	s_1
D	K	$-p_0$	if $p_0 > s_0$; if $p_0 < c_0$	K	$s_1 - p_1$
	D	$s_0 - 2p_0$	if $p_0 < s_0$	D	$s_1 - 2p_1$

- When $p_0 < s_0$, "delay" is the dominant strategy for the manufacturer by Lemma 3. The supplier's pay-off is $-c_1$ with "expediting", $-p_1$ with "keep", and $s_1 - 2p_1$ with "delay". We consider three cases:
  - (a) When $p_1 > c_1$, the supplier's optimal strategy is "expediting" because $c_1 > s_1$ by Condition 3(2) and so $-c_1$ is the largest payoff.
  - (b) When $s_1 < p_1 < c_1$, the supplier's optimal strategy is "keep".
  - (c) When $p_1 > c_1$, the supplier's optimal strategy is "delay".
- When $s_0 < p_0 < c_0$ and $r_0 > s_0$, "keep" is the dominant strategy for the manufacturer by Lemma 3. The supplier's pay-off is $r_1 - c_1$ with "expediting", 0 with "keep", and $s_1 - p_1$ with "delay". We consider two cases:
  - (a) When $p_1 > s_1$, the supplier's optimal strategy is "keep" because $r_1 < c_1$ by Condition 3(1).
  - (b) When $p_1 < s_1$, the supplier's optimal strategy is "delay" because $r_1 < c_1$.
- When $s_0 < p_0 < c_0$ and $r_0 < s_0$, there is no dominant strategy for the manufacturer. If the supplier chooses "expediting", the manufacturer will choose "delay". If the supplier chooses "keep" or "delay", the manufacturer will choose "keep". Thus, the supplier's pay-off is $-c_1$ with "expediting", 0 with "keep", and $s_1 - p_1$ with "delay".

- (a) When $p_1 > s_1$, the supplier's optimal strategy is "keep".
- (b) When $p_1 < s_1$, the supplier's optimal strategy is "delay".

- When $p_0 > c_0$ and $r_0 > s_0$, by $c_0 > s_0$ (Condition 3(2)) we obtain $p_0 > s_0$. If the supplier chooses "expediting", the manufacturer will choose "keep". If the supplier chooses "keep", the manufacturer will choose "keep". If the supplier chooses "delay", the manufacturer will choose "expediting". (Note: the manufacturer will do whatever it could to prevent project delay.) Given the manufacturer's optimal response, the supplier's pay-off is $r_1 - c_1$ with "expediting", 0 with "keep", and s_1 with "delay". Since $r_1 < c_1$ by Condition 3(1), the supplier's optimal strategy is "delay".

- When $p_0 > c_0$ and $r_0 < s_0$, by $c_0 > s_0$ (Condition 3(2)) we obtain $p_0 > s_0$. If the supplier chooses "expediting", the manufacturer will choose "delay". If the supplier chooses "keep", the manufacturer will choose "keep". If the supplier chooses "delay", the manufacturer will choose "expediting". (Note: the manufacturer will do whatever he could to prevent delay.) Given the manufacturer's optimal response, the supplier's pay-off is $-c_1$ with "expediting", 0 with "keep", and s_1 with "delay". Clearly, the supplier's optimal strategy is "delay".

Summarizing all cases, we have proved the theorem. □

Proof of Lemma 4

The extensive form of the game is shown in Figure 10.

By Lemma 1, the first two results are immediate, that is, when $s_0 < p_0$, "keep" is the dominant strategy for the manufacturer; when $s_0 > p_0$, "delay" is the dominant strategy for the manufacturer.

When $s_1 > p_1$, an enumerating over all options of supplier 2 and the manufacturer finds that supplier 1 archives the highest pay-off when it delays. □

Proof of Theorem 4

By Lemma 4, as long as one of the suppliers has a dominant strategy of "delay", the other has to delay as well. Otherwise, it suffers a pure penalty. Combining the dominant strategies leads to the theorem. □

Remarks: With two suppliers, the SPNE is no longer unique due to the simultaneous game played among the suppliers in phase 1. For instance, when $s_0 < p_0$, the manufacturer keeps its original

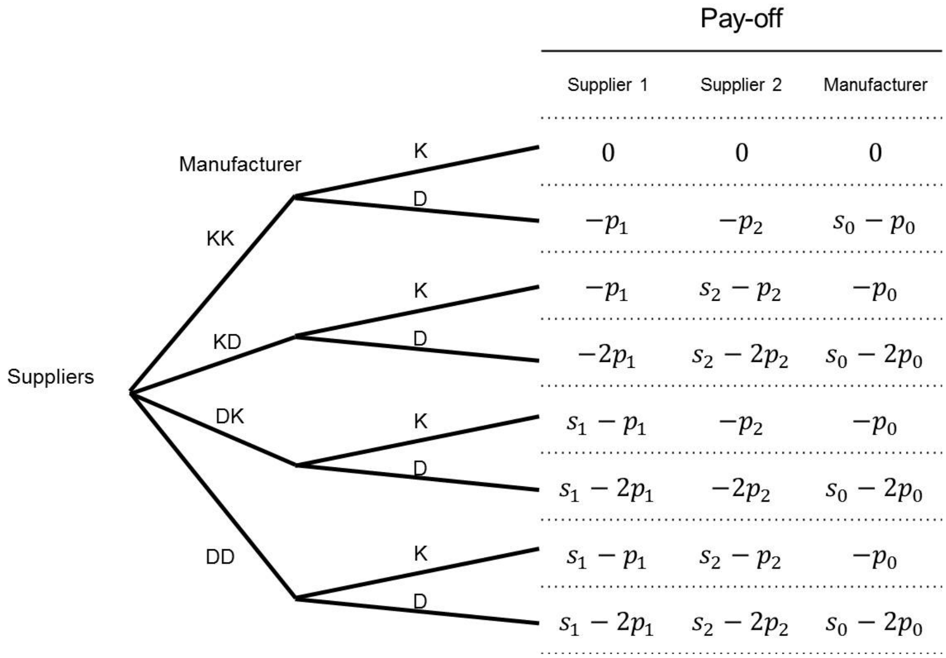

Figure 10: The extensive form of the game in the base model with multiple suppliers.

task duration, and the pay-off matrix for suppliers 1 and 2 is given by:

$1\backslash 2$	K	D
K	$0, 0$	$-p_1, s_2 - p_2$
D	$s_1 - p_1, -p_2$	$s_1 - p_1, s_2 - p_2$

Clearly, if $s_1 < p_1$ and $s_2 < p_2$, both [K, K] and [D, D] are SPNE. We only report [K, K] here because it is Pareto optimal but [D, D] is not.

Proof of Lemma 5

When $a < 0$, we consider three cases:

(1) If $x_0 \leq 0$, $\pi_0(a, x_0) = r_0^1 + \ldots + r_0^{|a|+|x_0|} - c_0^1 - \ldots - c_0^{|x_0|}$.

(2) If $0 < x_0 \leq |a|$, $\pi_0(a, x_0) = r_0^1 + \ldots + r_0^{|a|-x_0} + s_0^1 + \ldots + s_0^{x_0}$.

(3) If $x_0 > |a|$, $\pi_0(a, x_0) = s_0^1 + \ldots + s_0^{x_0} - p_0^1 - \ldots - p_0^{a+x_0}$.

In case (1), when $x_0 \in (-\infty, 0)$, $\pi_0(a, x_0)$ is an increasing function in x_0 because $r_0^{|x_0|} < r_0^1 < c_0^1 < c_0^{|x_0|}$ by Condition 5. At $x_0 = 0$, $\pi_0(a, 0) > \pi_0(a, -1)$ because $r_0^1 < c_0^1$. Thus, $\pi_0(a, x_0)$ is a monotonically increasing function of x_0 on $x_0 \in (-\infty, 0]$. Note that $\pi_0(a, 0) = r_0^1 + \ldots + r_0^{|a|}$. It is easy to show that when $x_0 \to +\infty$, $\pi_0(a, x_0) \to -\infty$. There always exists $x_0 = \hat{x}_0 \in [0, +\infty)$ that maximizes $\pi_0(a, x_0)$.

We now show that $\pi_0(a, x_0)$ is monotonically increasing in $(-\infty, \hat{x}_0]$ and monotonically decreasing in $[\hat{x}_0, +\infty)$. We discuss three scenarios:

1°, if $\hat{x}_0 = 0$, we have $\pi_0(a, 0) > \pi_0(a, 1)$, indicating that $r_0^{|a|} > s_0^1$. Since both $\{s_0^t\}$ and $\{r_0^t\}$ are decreasing series in t, we have $r_0^{|a|-1} > r_0^{|a|} > s_0^1 > s_0^2 \Rightarrow \pi_0(a, 1) > \pi_0(a, 2)$. By induction, we could prove that $\pi_0(a, x_0)$ is decreasing in $[0, +\infty)$.

2°, if $0 < \hat{x}_0 < |a|$, we have $\pi_0(a, \hat{x}_0) > \pi_0(a, \hat{x}_0 - 1)$ and $\pi_0(a, \hat{x}_0) > \pi_0(a, \hat{x}_0 + 1)$, indicating that $s_0^{\hat{x}_0} > r_0^{|a|-(\hat{x}_0-1)}$ and $r_0^{|a|-\hat{x}_0} > s_0^{\hat{x}_0+1}$. Furthermore, as both $\{s_0^t\}$ and $\{r_0^t\}$ are decreasing series in t, we have $s_0^{\hat{x}_0-1} > s_0^{\hat{x}_0} > r_0^{|a|-(\hat{x}_0-1)} > r_0^{|a|-(\hat{x}_0-2)}$ and $r_0^{|a|-\hat{x}_0} > s_0^{\hat{x}_0+1} > s_0^{\hat{x}_0+2}$, which lead to $\pi_0(a, \hat{x}_0 - 1) > \pi_0(a, \hat{x}_0 - 2)$ and $\pi_0(a, \hat{x}_0 + 1) > \pi_0(a, \hat{x}_0 + 2)$. We first consider the left side of $\hat{x}_0$ and show $\pi_0(a, x_0)$ is monotonically increasing in $[0, \hat{x}_0]$ by induction. The induction assumption is $\pi_0(a, x_0') > \pi_0(a, x_0' - 1)$ where $x_0' \in (0, \hat{x}_0)$. We have $\pi_0(a, x_0') > \pi_0(a, x_0' - 1) \Rightarrow s_0^{x_0'} > r_0^{|a|-x_0'+1} \Rightarrow s_0^{x_0'-1} > s_0^{x_0'} > r_0^{|a|-x_0'+1} > r_0^{|a|-x_0'+2} \Rightarrow \pi_0(a, x_0' - 1) > \pi_0(a, x_0' - 2)$. In addition, when $x_0' = 1$, we could show that $\pi_0(a, 1) > \pi_0(a, 0)$. Thus, $\pi_0(a, x_0)$ is monotonically increasing in $[0, \hat{x}_0]$. Similarly, we could prove that $\pi_0(a, x_0)$ is monotonically decreasing in $[\hat{x}_0, +\infty)$. Recall that $\pi_0(a, x_0)$ is a monotonically increasing function of x_0 on $x_0 \in (-\infty, 0]$, therefore $\pi_0(a, x_0)$ is a concave unimodal function with the peak $x_0 = \hat{x}_0$ when $0 < \hat{x}_0 < |a|$.

3°, if $\hat{x}_0 \geq |a|$, by a similar analysis, it is easy to prove that $\pi_0(a, x_0)$ is a concave unimodal function with the peak $x_0 = \hat{x}_0$.

In summary, $\pi_0(a, x_0)$ is a unimodal function of x_0 when $a < 0$.

The proof for the case of $a \geq 0$ is similar and thus omitted. In conclusion, given $x_1 = a$, $\pi_0(a, x_0)$ is a uni-modal function of x_0. □

Proof of Theorem 5

We first show that when $x_1 \to -\infty$, $x_0^*(x_1) > 0$ and $x_1 + x_0^*(x_1) < 0$.

- When $x_1 < 0$, the supplier expedites; the manufacturer will never expedite because a negative x_0 yields $r_0^{|x_1+x_0|} < r_0^1 < c_0^1$. Consider the manufacturer's response in three scenarios: (1) $x_0 < |x_1|$, (2) $x_0 = |x_1|$, $\pi_0(x_1, x_0) = s_0^1 + \ldots + s_0^{|x_1|}$. (3) $x_0 > |x_1|$, $\pi_0(x_1, x_0) = s_0^1 + \ldots + s_0^{|x_0|} - p_0^1 - \ldots - p_0^{x_1+x_0}$.
- Scenario (1) yields the highest pay-off for the manufacturer when $x_1 \to -\infty$. Explanation: In

scenario (3), when $x_1 \to -\infty$, $x_0 \to +\infty$ and thus $s_0^{x_0} \to 0$ and $p_0^{x_1+x_0} \to +\infty$. It is clear that scenario (2) yields a higher pay-off than scenario (3). Next, let $x_0 = |x_1|-1$. $\pi_0(x_1, |x_1|-1) = s_0^1 + \ldots + s_0^{|x_1|-1} + r_0^1$. When $x_1 \to -\infty$, $s_0^{|x_1|} < r_0^1$ and thus $\pi_0(x_1, |x_1|-1) > \pi_0(x_1, |x_1|)$. Hence, when $x_1 \to -\infty$, $x_0^*(x_1) < |x_1|$. In other words, when $x_1 \to -\infty$, $x_0^*(x_1) > 0$ and $x_1 + x_0^*(x_1) < 0$.

Now we start from $x_1 \to -\infty$ and increase x_1 by one unit each time to see how $x_0^*(x_1)$ and $x_1 + x_0^*(x_1)$ will change.

When $x_1 \to -\infty$, $x_0^*(x_1) > 0$, $x_1 + x_0^*(x_1) < 0$, so that $\pi_0(x_1, x_0^*(x_1)) = s_0^1 + \ldots + s_0^{x_0^*(x_1)} + r_0^1 + \ldots + r_0^{|x_1+x_0^*(x_1)|}$. $x_0^*(x_1)$ being the best response requires conditions $\pi_0(x_1, x_0^*(x_1)) > \pi_0(x_1, x_0^*(x_1)-1)$ and $\pi_0(x_1, x_0^*(x_1)) > \pi_0(x_1, x_0^*(x_1)+1)$ which are equivalent to $s_0^{x_0^*(x_1)} > r_0^{|x_1+x_0^*(x_1)-1|}$ and $r_0^{|x_1+x_0^*(x_1)|} > s_0^{x_0^*(x_1)+1}$. Let $x_1' = x_1 + 1$, to find the manufacturer's best response, we compare the following pay-offs as(assuming $x_0^*(x_1) - 2 \geq 0$ and $|x_1 + x_0^*(x_1) + 1| > 0$):

(1) $\pi_0(x_1', x_0^*(x_1) - 2) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)-2|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)-2}$.

(2) $\pi_0(x_1', x_0^*(x_1) - 1) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)-1|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)-1}$.

(3) $\pi_0(x_1', x_0^*(x_1)) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)}$.

(4) $\pi_0(x_1', x_0^*(x_1) + 1) = r_0^1 + \ldots + r_0^{|x_1+1+x_0^*(x_1)+1|} + s_0^1 + \ldots + s_0^{x_0^*(x_1)+1}$.

Because $r_0^{|x_1+1+x_0^*(x_1)-2|} < s_0^{x_0^*(x_1)} < s_0^{x_0^*(x_1)-1}$ and $r_0^{|x_1+1+x_0^*(x_1)|} > r_0^{|x_1+x_0^*(x_1)|} > s_0^{x_0^*(x_1)+1}$, we have $\pi_0(x_1', x_0^*(x_1)-2) < \pi_0(x_1', x_0^*(x_1)-1)$ and $\pi_0(x_1', x_0^*(x_1)) > \pi_0(x_1', x_0^*(x_1)+1)$. We can easily verify that when $x_0^*(x_1) - 2 = -1$ and $|x_1 + x_0^*(x_1) + 1| = 0$, these inequalities still hold. By the unimodality property of Lemma 5, $x_0^*(x_1) - 1 \leq x_0^*(x_1 + 1) \leq x_0^*(x_1)$. In other words, when x_1 increases by one unit, the manufacturer's best response is to either reduce the corresponding $x_0^*(x_1)$ by one unit or keep it the same until $x_0^*(x_1)$ reaches 0.

At $x_0^*(x_1) = 0$, to find the manufacturer's best response for $x_1' = x_1 + 1$, we still compare four pay-offs, $\pi_0(x_1', x_0^*(x_1) - 2)$, $\pi_0(x_1', x_0^*(x_1) - 1)$, $\pi_0(x_1', x_0^*(x_1))$, and $\pi_0(x_1', x_0^*(x_1) + 1)$. By the same logic stated in the previous paragraph, we have $x_0^*(x_1) - 1 \leq x_0^*(x_1 + 1) \leq x_0^*(x_1)$ for either $x_1 < 0$ or $x_1 = 0$.

Applying similar approach stated in the previous paragraphs to analyze the rest possible scenarios: (1) $x_1 < 0$, $x_0^*(x_1) \geq 0$, $x_1 + x_0^*(x_1) \geq 0$; (2) $x_1 \geq 0$, $x_0^*(x_1) \geq 0$, $x_1 + x_0^*(x_1) \geq 0$; (3) $x_1 \geq 0$, and $x_0^*(x_1) < 0$, $x_1 + x_0^*(x_1) > 0$, we can always show that when x_1 increases by one unit, the manufacturer's best response is to reduce the corresponding $x_0^*(x_1)$ by one unit or keep it the same

same.

In summary, for $x_1 \in (-\infty, +\infty)$, when x_1 increases by one unit, $x_0^*(x_1)$ will either decrease by one unit or remain the same and therefore $x_1 + x_0^*(x_1)$ will not decrease. $\square$

Proof of Theorem 6

In the proof of Theorem 5, we have shown that when $x_1 \to -\infty$, $x_1 + x_0^*(x_1) < 0$. On the other hand, when $x_1 = 0$, $x_0^*(0)$ should be greater than or equal to 0 so as to guarantee original schedule to be the global optimum; $x_1 + x_0^*(x_1)$ is therefore greater than or equal to 0. From Theorem 5, we know that $x_1 + x_0^*(x_1)$ will increase by one unit or hold still each time when x_1 increases by one unit. There must exist x_L such that $x_L = \max\{x_1 | x_1 + x_0^*(x_1) = 0, x_1 \leq 0\}$.

For any $x_1 < x_L$ and $x_1 + x_0^*(x_1) \leq -1$, the supplier's pay-off is $\pi_1(x_1, x_0^*(x_1)) = r_1^1 + \ldots + r_1^{|x_1+1+x_0^*(x_1)|} + r_1^{|x_1+x_0^*(x_1)|} - c_1^1 - \ldots - c_1^{|x_1+1|} - c_1^{|x_1|}$. When the supplier expedites one period less, $x_1 + 1$, the manufacturer's best response is either $x_0^*(x_1)$ or $x_0^*(x_1) - 1$ by Theorem 5. (1) If the manufacturer's best response is $x_0^*(x_1)$, then the supplier's pay-off is $r_1^1 + \ldots + r_1^{|x_1+1+x_0^*(x_1)|} - c_1^1 - \ldots - c_1^{|x_1+1|}$. Note that $r_1^{|x_1+x_0^*(x_1)|} \leq r_1^1 < c_1^1 < c_1^{|x_1|}$ by Condition 5(1), the supplier actually improves its pay-off by increasing x_1 to x_1+1. (2) If the manufacturer's best response is $x_0^*(x_1) - 1$, the supplier's pay-off is $r_1^1 + \ldots + r_1^{|x_1+1+x_0^*(x_1)-1|} - c_1^1 - \ldots - c_1^{|x_1+1|}$. The supplier also improves its pay-off. Therefore the supplier could continuously improve its pay-off by increasing x_1 until $x_1 + x_0^*(x_1) = -1$.

When $x_1 < x_L$ and $x_1 + x_0^*(x_1) = -1$, the supplier's pay-off is $r_0^1 - c_1^1 - \ldots - c_1^{|x_1+1|} - c_1^{|x_1|}$. If it expedites one period less, $x_1 + 1$, the manufacturer's best response is either $x_0^*(x_1)$ or $x_0^*(x_1) - 1$. The former one yields the supplier a pay-off of $-c_1^1 - \ldots - c_1^{|x_1+1|}$. Note that $r_1^1 < c_1^1 \leq c_1^{|x_1|}$. The supplier has a higher pay-off at $x_1 + 1$ than that at x_1. The latter one is the same as the case discussed in the previous paragraph which is shown that the supplier could improve its pay-off from x_1 to $x_1 + 1$. In other words, at $x_1 + x_0^*(x_1) = -1$, the supplier could also improve its pay-off until $x_1 + x_0^*(x_1) = 0$.

When $x_1 < x_L$ and $x_1 + x_0^*(x_1) = 0$, the supplier's pay-off is $-c_1^1 - \ldots - c_1^{|x_1|}$. If the supplier expedites one period less, as long as $x_1 + x_0^*(x_1)$ is still equal to 0, the supplier always gets its pay-off improved.

In summary, x_L always exists and for any $x_1 < x_L$, we have $\pi_1(x_1, x_0^*(x_1)) < \pi_1(x_L, x_0^*(x_L))$. $\square$

Proof of Theorem 7

It is obvious that the manufacturer has a dominant strategy of "keep". Using a backward induction, the pay-off matrix between supplier 1 and supplier 2 is:

1\2	K	D
K	$0, 0$	$0, s_2 - p$
D	$s_1 - p, 0$	$s_1 - \beta_1 p, s_2 - \beta_2 p$

By Condition 4, [D, K] or [K, D] cannot be the equilibrium because $s_1 < p$ and $s_2 < p$. [D, D] cannot be the equilibrium either because $s_1 - \beta_1 p$ and $s_2 - \beta_2 p$ cannot be larger than 0 at the same time, otherwise $s_1 + s_2 < p$ from Condition 4 is violated. We could verify that [K, K] is the only equilibrium. Note that we do not have to specify β_1 and β_2 completely. $\square$

Proof of Theorem 8

Not every supplier would like to expedite because $-\sum_{j=1}^{|x_i|} c_i^j + \alpha_i \sum_{j=1}^{|x_s|} r^j$ is not positive for every i, otherwise we violate the assumption that the original schedule is the optimal schedule. So $x_s \geq 0$ and thus no supplier would like to expedite. On the other hand, no supplier would like to delay because those suppliers who delayed have to share the penalty. By Condition 5, at least one of them is losing money. Because this fact applies to any group of suppliers who delay, no supplier would like to delay and so "Keep" is the dominant strategy for every supplier.

Knowing that suppliers will always keep, the manufacturer's pay-off is: (1) $\sum_{i=1}^{|x_0|} r^i - \sum_{i=1}^{|x_0|} c_0^i$ if it expedites $|x_0|$; (2) 0 if it keeps; (3) $-\sum_{i=1}^{|x_0|} p^i + \sum_{i=1}^{|x_0|} s_0^i$ if it delays. By Condition 5(1), the pay-offs in (1) and (3) are all less than 0. Thus, the best strategy for the manufacturer is "keep". $\square$